Repetitive Patterns in the Juggler Sequence

By Gabriel Kanarek for G4G13, 4/12/2018

Juggler Sequence Fast Facts!

- Integer sequence like the famous Collatz
- First publicized by Clifford Pickover in 1992
- Starting with an integer $N_0 > 0$:
  - $N_{i+1} = \lfloor N_i^{1/2} \rfloor$ if N_i is even
  - $N_{i+1} = \lfloor N_i^{3/2} \rfloor$ if N_i is odd
  - Repeat until the sequence converges to 1 (or doesn't!)

No Trivial Loops?

A trivial loop would occur when you start with a number $N_0 = x$, and after applying the Juggler function twice, end up back at x: $J(J(x)) = x$.

It doesn't matter whether x is odd or even, because we can just substitute $x \rightarrow J(x)$: the loop would oscillate between the same two values, so let's assume x is odd.

Let's call the other value $J(x) = y$, so that $x^3 = y^2 + C$, where $0 \leq C < 2y + 1$. Then we know that $J(y) = x$.

We also know that y must be even, or the next number in the sequence would be larger, not smaller; therefore $y = x^2 + D$, where again $0 \leq D < 2x + 1$.

Substituting back into our earlier equation, we get $x^3 = (x^2 + D)^2 + C$, and we can expand the right-hand side to get $x^3 = x^4 + 2x^2D + D^2 + C$.

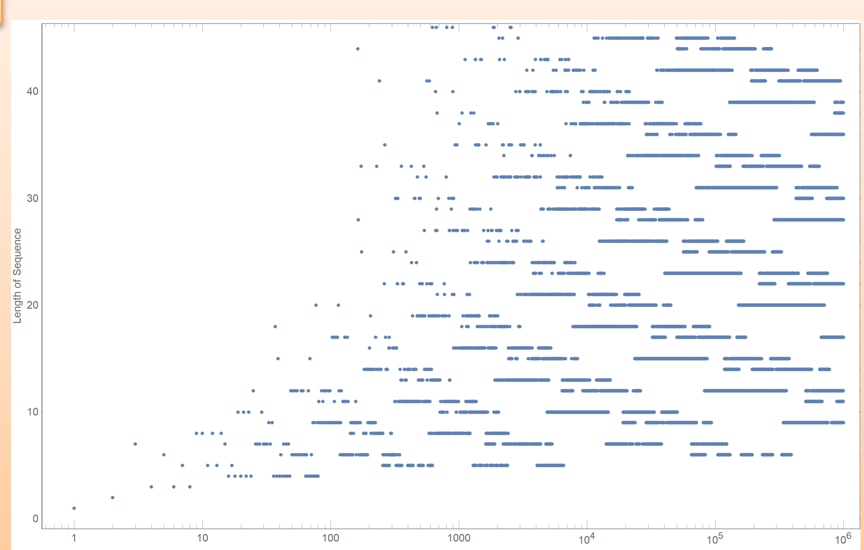
Even if $D = C = 0$, there is no integer value of $x > 1$ where this holds true, and therefore there can't be any trivial loops!

How Long Can You Juggle?

The length of Juggler sequences seems to have some patterns as well, but I haven't finished analyzing them.

They have long stretches of even numbers which all have the same length because they all get Juggled to the same number, because the Floor function puts everything between two perfect squares to the smaller perfect square.

I don't know why there are diagonal stripes. What do you think?



More Questions?

Email my dad at graykanarek@gmail.com, and he'll pass them along to me!