

# Are You Smarter Than Google?

Chapter 8 of *Can You Outsmart an Economist?* by Steven E. Landsburg

## Chapter 8

### ARE YOU SMARTER THAN GOOGLE?

In the years I've been blogging at TheBigQuestions.com, I haven't shied away from controversy. Religion, politics, and manners are standard fare, though I try not to post unless I have something at least a bit novel to say. As a result, I'm usually not preaching to any particular choir, which means I risk offending every variety of knee-jerker.

I've gotten used to being called a radical socialist, a mindless liberal, a heartless conservative, and a reactionary mooncalf. But in all my years of blogging, no post has inspired more vitriol than one titled "Are You Smarter Than Google?".

In fact, it's not even close. This post generated many thousands of responses, both on my own blog and others, a great many of them demanding that I be fired, publicly humiliated, and/or banned from the Internet. I don't delete comments, even when they're very strongly worded, unless they're extremely abusive and/or quite thoroughly devoid of intellectual content. In this case, I deleted many hundreds.

What was the content of this post? It was the following brain teaser:

1

**ARE YOU SMARTER THAN GOOGLE?**

There's a certain country where everybody wants to have a son. Therefore each couple keeps having children until they have a boy; then they stop. What fraction of the population is female?

Well, of course you can't know for sure, because maybe, by some extraordinary coincidence, the last 100,000 couples in a row have gotten boys on the first try, or maybe, by an even more extraordinary coincidence, the last 100,000 couples have had to try eight times before succeeding.

Therefore (as I told my readers in the original blog post), the question is meant to be answered *in expectation*, which means this: If there are a great many countries just like this one, what fraction of the population is female in the average country?

This problem has been around, in many forms, for at least half a century, but it keeps finding new life. I found it in a children's puzzle book when I was about ten years old, and (much more recently) Google has used it to screen job candidates. The official answer — that is, the answer I found in the back of that

puzzle book, and the answer Google reportedly expected from its job candidates — is simple, clear, and wrong.

And no, it's not wrong because of small real-world discrepancies between the number of male and female births, or because of anything else that's extraneous to the spirit of the problem. It's just wrong. The correct answer, unlike the expected one, is not so simple.

So: are you smarter than the folks at Google? Before you read ahead, what's your answer?

I'll wait....

Ready now?

Okay, let's continue.

The answer Google seems to have expected is the same answer I gave when I first saw this problem long long ago. It goes like this:

Each birth has a 50% chance of producing a girl. Nothing the parents do can change that. So each individual child is equally like to be male or female, and therefore, in expectation, half of all the children are girls.

I'll give you another chance to take a break. Before you read ahead, what's wrong with that reasoning?

Ready?

Okay, then:

Actually, most of it is right. Each birth has a 50% chance of producing a girl — check! Nothing the parents do can change that — check! So, each individual child is equally likely to be male or female — check!

But it does not follow — and in fact is not true! — that in expectation, half of all children are girls.

What *does* follow is that, in expectation, the number of boys and the number of girls are equal. But that's not at all the same thing.

To see why not, try this much easier problem:

## 2

### EGGS AND PANCAKES

Every day I flip a coin to decide what to have for breakfast. If the coin comes up heads, I have two eggs and one pancake. If it comes up tails, I have two eggs and three pancakes. On average, what fraction of my breakfast items are pancakes?

THE WRONG SOLUTION: On the average day (in fact each and every day day!) I have exactly two eggs.

On average day, I also have two pancakes (two being the average of one and three). So on average, the number of pancakes is equal to the number of eggs.

Therefore on average, half my breakfast items are pancakes.

Except for the final sentence, all of that is true but most of it is irrelevant. I do in fact have two pancakes on the average day, but *that has nothing to do with the question*. Here's the right answer:

THE RIGHT SOLUTION: Whenever I flip heads,  $1/3$  of my breakfast items are pancakes. Whenever I flip tails,  $3/5$  of my breakfast items are pancakes. The average of those two numbers is  $7/15$ . The answer to the question, then, is that on the average day,  $7/15$  of my breakfast items are pancakes.

Here's the analogy:

Imagine many breakfasts	Imagine many countries
At the average breakfast, the number of pancakes is equal to the number of eggs (TRUE!)	In the average country, the number of girls is equal to the number of boys (TRUE!)
Therefore at the average breakfast, the fraction of items that are pancakes is $1/2$ (FALSE!)	Therefore in the average country, the fraction of children that are girls is $1/2$ (FALSE!)

**MORAL: Two things (be they eggs and pancakes or boys and girls) can be equal in expectation,<sup>1</sup> but that tells you nothing about their expected *ratio*.**

The gist of that moral is that the official answer to the Google problem is wrong. But we still have to figure out what's right.

It turns out that the correct answer depends on the size of the

---

<sup>1</sup> Remember that "in expectation" means the same thing as "on average".

country. This is easiest to think about when the country is so tiny that it has just one family. Let's solve that case first; then we'll move on to bigger countries.<sup>2</sup>

Here are some possible configurations for that one family:

PROBABILITY	CONFIGURATION	FRACTION FEMALE
1/2	B	0
1/4	GB	1/2
1/8	GGB	2/3
1/16	GGGB	3/4

From this, we can see that the number of boys is always exactly 1.

The number of girls, of course, can be anything at all, but we want to know what it is on average. For that, we take each possible number, multiply it by the corresponding probability, and add up, as follows:

---

<sup>2</sup> If you — like many of my blog readers — are prepared to object that the one-family assumption is contrary to the spirit of the problem, let me assure you that I agree with you. I'm solving this case first not because it's the most important case, but because it's the easiest. I hope that you, unlike some of my more impatient blog readers, will bear with me.

	Number of girls			
Probability of 0 girls	→ $\frac{1}{2}$	x	0	= 0
Probability of 1 girl	→ $\frac{1}{4}$	x	1	= $\frac{1}{4}$
Probability of 2 girls	→ $\frac{1}{8}$	x	2	= $\frac{1}{4}$
Probability of 3 girls	→ $\frac{1}{16}$	x	3	= $\frac{3}{16}$
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
				1

The numbers in the infinitely long column on the right add up to 1. (If you don't believe me, try adding several terms and you'll see them approaching closer and closer to 1.) That is, the expected number of girls is equal to the expected number of boys — just as we knew it must be.

But to get the expected *fraction* of girls, we need to do the same calculation with fractions of girls instead of numbers of girls. And it comes out like this:



	Fraction of girls			
Probability of 0 girls	→ $\frac{1}{2}$	$\times 0$	$=$	0
Probability of 1 girl	→ $\frac{1}{4}$	$\times \frac{1}{2}$	$=$	$\frac{1}{8}$
Probability of 2 girls	→ $\frac{1}{8}$	$\times \frac{2}{3}$	$=$	$\frac{1}{12}$
Probability of 3 girls	→ $\frac{1}{16}$	$\times \frac{3}{4}$	$=$	$\frac{3}{64}$
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
				.306

The numbers in the right hand column add up to just about .306, or 30.6%<sup>3</sup>

Now again — that calculation is only correct for a country with just one family. For a country with two families, a similar but more complicated calculation gives an expected fraction of about 38.63%. If you want to see that calculation, you can look in the appendix to this book.<sup>4</sup>

For a country with 10 families, the expected fraction is about

---

<sup>3</sup> Where did the 30.6% come from? It takes a bit of work. If you remember your calculus, you might be able to show that the sum of the infinite series is actually  $\log(2) - 1$ , which is just about 30.6%. If you don't remember your calculus, I hope you'll take my word for this.

<sup>4</sup> The calculation in the appendix will appeal to the sort of readers who find that thing appealing and not to others. Fortunately, the main point here is not to follow the complicated calculation, but to understand why it's necessary. That is, the important thing is not so much to understand the right answer as to understand why the "obvious" answer is wrong.

47.51%. For a country with 100 families, it's about 49.75%. For a country with 1000 families, it's about 49.98%. For a country with 5000 families, it's about 49.995%. For a country comparable to the United States, with about 100,000,000 families, the expected fraction is about 49.99999975%.

You might be tempted to say, "Aha! Surely there's no important difference between 49.995% and 50%. So the official reasoning is correct after all!"

Hold on there! First of all, even if the correct answer were *exactly* 50%, the official reasoning would still be entirely wrong. We don't generally give full credit (or even partial credit) for bad reasoning that just happens to get the right answer.

Besides, who says there's no important difference between 49.995% and 50%? Try telling that to Al Gore, who got 49.995% of the Bush/Gore vote in Florida in the year 2000, and thereby lost the presidency of the United States.

Or if that doesn't convince you, try this variation, where the official reasoning will lead you neither slightly astray nor moderately astray or even hugely astray, but *infinitely* astray:

3

THE GOOGLE PROBLEM REDUX

There's a certain country where everybody wants to have a son. Therefore each couple keeps having children until they have a boy; then they stop. What is the ratio of boys to girls?

This differs from the original Google problem by asking about the ratio of boys to girls, rather than the fraction of girls in the population.

Again, the answer in any one country could of course be just about anything, so we need to specify that the question is to be answered *in expectation*, or in simpler words *on average* over many such countries.

SOLUTION: There's always some chance — perhaps a tiny chance, but still some chance — that every single family has a boy on the first try. If that happens, there are no girls, so the ratio of boys to girls is infinite.

To get the *expected* ratio, we have to average over all possible ratios, including infinity. That average is infinity.

If you said that the answer was  $1/2$ , you were infinitely wrong.

It turns out that a lot of people — and especially, I suppose, the sort of people who like to solve brain teasers on the Internet — have seen some version of this problem before, and have had the (correct) insight that in expectation the number of boys and the

number of girls must be equal. Some of them tend to feel pretty proud of that insight, which makes them exceptionally reluctant to admit that it fails to solve the problem.

I'd intended to blog twice on the subject — once to pose the puzzle and once to reveal the answer. Instead, the discussion ended up stretching over six blog posts. You can find links to all of them at [www.TheBigQuestions.com/google.html](http://www.TheBigQuestions.com/google.html).

A lot of readers fell into the trap. A lot of those defended their answers vigorously, then gradually saw the light as I and other commenters pointed out their errors. Those people learned something, and many of them were delighted. That delighted me, too.

Others brought up interesting and valid new twists. Here are a few examples:

- My analysis assumes that all families have finished reproducing. What if we take a snapshot *before* the last family gets its son? (Answer: It depends on when you take the snapshot. But in no case is the expected fraction of girls equal to  $1/2$ .)
- What if you count the parents and not just the kids? (The answer changes, but it's still not  $1/2$ .)
- What if the country's population is literally infinite? (Answer: Then there are infinitely many girls and infinitely many boys,

giving a fraction of infinity over infinity, which is not a number at all, and certainly not  $1/2$ . Besides, who ever heard of a country with an infinite population?)<sup>5</sup>

But others kept returning to the comments section to defend the wrong answer, while a great many others jumped into the fray to help point out their errors — help that was not always appreciated.

The whole thing might have died down in a few days had it not caught the attention of an Internet phenomenon named Lubos Motl. It's been said of Lubos that he's hard to ignore, but it's always worth the effort. I eventually took this advice to heart, but not before we had several rounds of increasingly bizarre correspondence.

Lubos is a physicist by training and a crank by choice. He appears to haunt the Internet twenty-four hours a day from his home in the Czech Republic. When he blogs about physics, he's often clear, accurate and generous with his explanations. The rest of the time he burnishes his reputation as a nut.

That's what he was doing when he announced on his blog that

---

<sup>5</sup> In a delightfully ironic twist, many of the readers who insisted on assuming the population was literally infinite were the same readers who excoriated me for working through the case of a single-family country, even for illustration, on the grounds that a single-family country is “unrealistic”.

the only acceptable answer to the Google problem is 50%, and that you (or in this case I) would have to be a complete idiot to believe otherwise. He gave absolutely no argument to support this position, and repeatedly asserted that no argument was necessary. Those who know him will recognize this as classic crank-mode Lubos.

Because Lubos was quite unsusceptible to reason (completely ignoring, for example, a series of simple numerical examples that proved him wrong, and refusing ever to state the secret additional assumptions that he claimed would support his 50% answer), I went a different route and publicly offered to bet him up to \$15,000 (and anyone else up to \$5000) that a computer simulation (for a country with four families reproducing for 30 generations) — with disputes over interpretation to be settled by a panel of randomly chosen statistics professors from top departments — would prove me right.

At first a dozen readers stepped up to get in on this bet, but they all soon either changed their minds or mysteriously stopped responding to emails.

Then I screwed up.

A reader named Larry suggested a slightly different bet, which I accepted without carefully reading his terms. This bet turned out

to be stacked against me.

I knew that in a country with four families, the expected fraction of girls is about 44%. I therefore agreed to Larry's bet that a series of simulations would show it to be less than 46.5%, leaving a little room for statistical anomalies. But I overlooked Larry's stipulation that we include the parents in the count. This turns out to drive the expected ratio up over 46.5% (though it's still less than 50%).

Having rashly accepted Larry's challenge, I was legitimately on the hook for a \$5000 bet I was almost sure to lose. I'd have paid up if necessary, but Larry most graciously suggested that he'd settle for some autographed books.

Hundreds of others refused to take the bet but continued to defend the wrong answer. A happy exception was a reader known to me only as Tom, who started out as a serial repeater of false and tired pro-50% arguments. I (and others) tried patiently pointing out his errors, but he seemed hell-bent on ignoring everything we said — to the point where I eventually lost my patience and said “I'm sorry, but it appears that you are too stupid to think about this brain teaser”. To his great credit, Tom responded not by digging his heels in further, but by taking a little time to think — and then returning a few days later with a beautifully reasoned

essay that not only explained the right answer but offered a whole new (and completely correct) explanation of why the answer cannot possibly be 50%. He graciously allowed me to share his essay with my readers as a guest poster, and I know from my email that it helped a lot of people see the light.

That happy experience aside, I remain astonished that so many became so emotionally invested in defending the wrong answer to a simple brain teaser. Clearly, the right answer comes as a surprise to many people. It came as a surprise to me at first! But I still don't quite get why so many people are so resistant to being surprised. Or more to the point: How does someone get so emotionally invested in a simple brain teaser that he is willing to make the same false arguments over and over and over and over and over and over and over again, but not care enough to read and digest the right answer? Perhaps that would be a good puzzle for a book called *Can You Outsmart a Psychologist?*

\* \* \* \*

Over many years of teaching, one thing I've learned is that when students don't see the point of pure theory, you can usually snag their attention with an application to sports. (Interestingly, this works best with students who are inclined to dismiss pure theory as "just a game".)



Let us, then, turn to the age-old issue of “hot hands” in basketball. The question is whether basketball players experience good and bad streaks beyond what you’d expect from pure chance. Of course we’ve got a lot of data on this, but historically a great many people have misinterpreted those data—*precisely* because they didn’t understand the issues in the great Google problem controversy.

I’ll tell you that story in a moment. But first, let me show you how to make some money.

We’ll play a game: One of us flips a coin four times in a row to get three pairs of consecutive  $H$ ’s and  $T$ ’s. For example, if you flip  $HHTH$ , your three pairs are  $HH$ ,  $HT$ , and  $TH$ . If you flip  $THTT$ , your pairs are  $TH$ ,  $HT$ ,  $TT$ .

Now: I’ll give you a dollar for each  $HH$ , and you give me a dollar for each  $HT$ .

This is a perfectly fair game, because  $HH$  and  $HT$  are equally likely. If you doubt me, try writing down all sixteen possible outcomes, and count all the  $HH$ ’s and all the  $HT$ ’s. There are exactly twelve of each:

	Number of HH	Number of HT
<i>HHHH</i>	3	0
<i>HHHT</i>	2	1
<i>HHTH</i>	1	1
<i>HHTT</i>	1	1
<i>HTHH</i>	1	1
<i>HTHT</i>	0	2
<i>HTTH</i>	0	1
<i>HTTT</i>	0	1
<i>THHH</i>	2	0
<i>THHT</i>	1	1
<i>THTH</i>	0	1
<i>THTT</i>	0	1
<i>TTHH</i>	1	0
<i>TTHT</i>	0	1
<i>TTTH</i>	0	0
<i>TTTT</i>	0	0

*TOTAL : 12      TOTAL : 12*

If you play this game against an experienced gambler, he or she will quickly realize that it's fair. First, experienced gamblers have a very good sense of facts like "*HH* and *HT* are equally likely". Second, if you play long enough, you'll probably both come pretty close to breaking even on average, which tells you that the game is probably fair.

Now try a variation:

### WANNA PLAY?

Once again, we'll flip four times to get three sequences. We'll count the  $HH$ 's and the  $HT$ 's. I'll give you a number of dollars equal to the *percentage* of those sequences that are  $HH$ , and you give me a number of dollars equal to the percentage that are  $HT$ .

Does that game strike you as fair?

SOLUTION: If you think like so many of my blog commenters, you'll say "Well,  $HH$  and  $HT$  are equally likely, so on average half of all the  $HH$  and  $HT$  flips will be  $HH$  and the other half will be  $HT$ . This is another fair game."

If you do think that way, please contact me. I'd like to play this game against you. Because here are the relevant percentages:

Are You Smarter Than Google?

	Number of HH	Number of HT	Percentage of HH
<i>HHHH</i>	3	0	100
<i>HHHT</i>	2	1	67
<i>HHTH</i>	1	1	50
<i>HHTT</i>	1	1	50
<i>HTHH</i>	1	1	50
<i>HTHT</i>	0	2	0
<i>HTTH</i>	0	1	0
<i>HTTT</i>	0	1	0
<i>THHH</i>	2	0	100
<i>THHT</i>	1	1	50
<i>THTH</i>	0	1	0
<i>THTT</i>	0	1	0
<i>TTHH</i>	1	0	100
<i>TTHT</i>	0	1	0
<i>TTTH</i>	0	0	—
<i>TTTT</i>	0	0	—

*TOTAL : 12    TOTAL : 12    AVERAGE : 40.5*

In each row, the percentage shown is the percentage of all pairs starting with *H* that are *HH*. (In the last two rows, there are no pairs starting with *HH*, so the ratio can't be computed. If our flips produce either of those patterns, no money changes hands.)

The average of all the percentages is 40.5%, which is definitely not at all the something as 50%. On the average play of this game, I will pay you \$40.50 and you will pay me \$59.50. In the long run, I make an average of \$19 each time we play.

The moral? Two things (in this case  $HH$  and  $HT$  can be equal in expectation — that is, they occur equally often — but that tells you nothing about their expected *ratio*. Perhaps that moral rings a bell by now.

Now back to the hot hands.

Take a ball player. Put him at a distance from the basket where he makes just about half his free throws. (This distance will be different for different players.)

Have him take four shots. Write down an  $H$  for each success and a  $T$  for each miss. Repeat with many different players.

If there are no hot hands, then  $HH$  and  $HT$  should occur about equally often. (That is, after a successful shot, a second success should be no more likely than a miss.) So a good test of the hot hand theory would be to count the  $HH$ 's and the  $HT$ 's for all the players, and see whether the totals are roughly equal.

In 1985, a group of researchers (let's call them GVT, because those were their initials) set out to analyze exactly this experiment. Unfortunately, they kept track of the wrong statistic. Instead of asking whether, on average, there are equal numbers of  $HH$  and  $HT$ , they figured they might as well ask whether, on average, there are equal *fractions* of  $HH$  and  $HT$ . After all, equal numbers should be the same thing as equal fractions, right? At least that's what so

many of my blog commenters thought — and GVT made exactly the same mistake.

So they counted pairs and computed fractions. And, coincidentally, they discovered that among all pairs that start with  $H$ , on average just about half were  $HH$  and half were  $HT$ .

**Here's what they figured:** The percentages for  $HH$  and  $HT$  are about fifty-fifty. That's just what you'd expect from a series of coin flips. So foul shots are like coin flips. There are no hot hands.

**Here's what they should have figured:** The percentages for  $HH$  and  $HT$  are about fifty-fifty. If these were coin flips, we'd expect them to be about 40.5 and 59.5. We're getting a *much* bigger fraction of  $HH$ 's on the basketball court than we'd get from a coin flip. The hot hand must be real.

It took almost twenty years before another group of researchers noticed this mistake. Meanwhile, GVT had fooled not only themselves, but a substantial fraction of the economics profession, into believing that their study had rejected the hot hand hypothesis, when in fact it had confirmed it.<sup>6</sup>

---

<sup>6</sup> This doesn't necessarily mean that the hot hand hypothesis is *true* — it means only that this particular bit of evidence points in that direction. There is other important evidence in both directions, some of it collected (and correctly interpreted) by the same GVT team.

Once again, the obvious can be the enemy of the true. It's "obvious" that if boys and girls are equally likely to be born, then on average, the fraction of girls should be  $1/2$ . It's equally obvious that if you're equally likely to flip  $HH$  and  $HT$ , then on average, the fraction of  $HH$ 's should be  $1/2$ . Neither of those things is true. If you insist on believing them, you're an easy mark for coinflipping con men.