

# Knight Mazes

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## Abstract

Knight mazes are a set of squares on a square lattice upon which a chess knight may move. We examine elements of mazes which can be both attractive and puzzling, and discuss two methods of creating mazes.

## Knight Mazes I - Elements

A chess knight sits alone on a small island in a peaceful pond. Across the pond, a trophy awaits on another island (Figure 1). A moment's reflection may reveal that the scene is a puzzle – the knight is free to hop from island to island, moving as a chess knight does, with the goal of reaching the trophy. The route to the trophy is riddled with topological surprises, and you are invited to try out the puzzle before reading further.

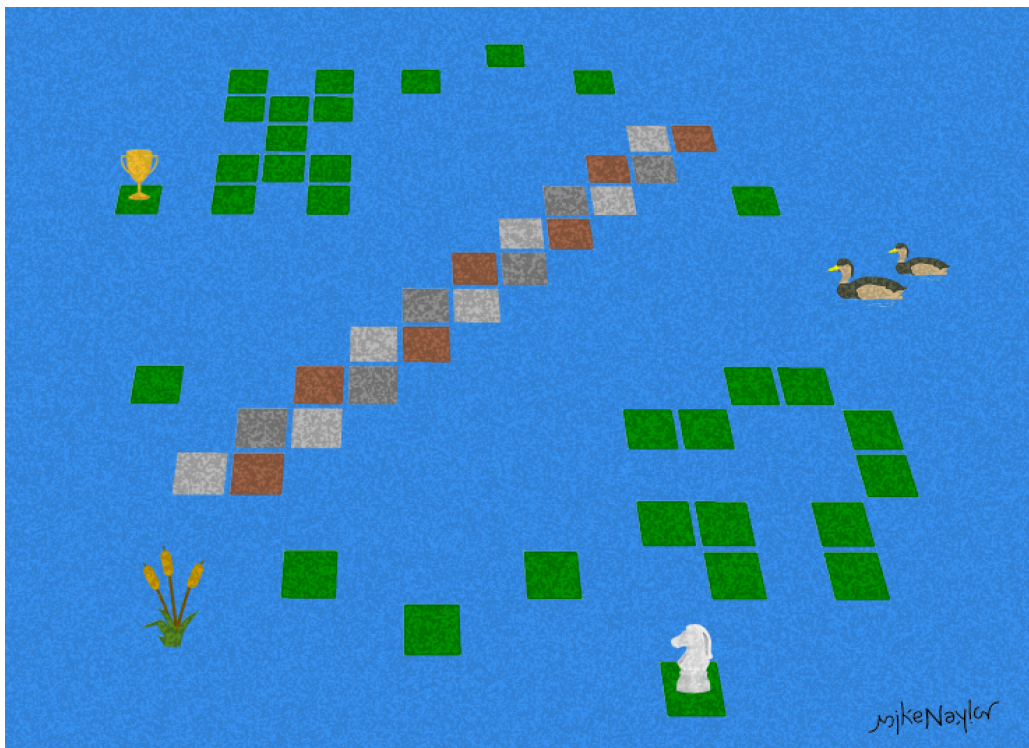
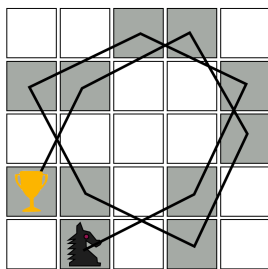
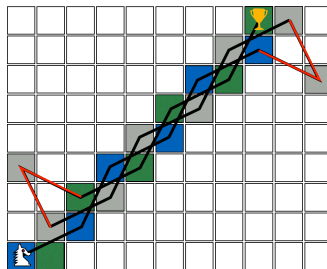


Figure 1: *Knight Maze I - Relax*

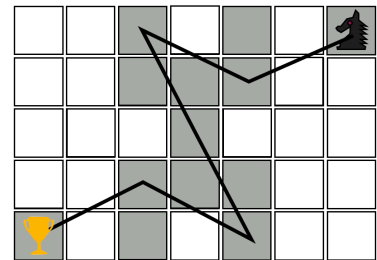
The image is an example of a knight maze and is composed of 3 main elements. The first shape to the lower right is a *double loop-the-loop* (see also Figure 2). The knight must hop in a counter-clockwise loop and travel two times around the loop before leaving the area. The central diagonal area is a *triple braided ladder* (see also Figure 3) with the squares are colored in three different colors to help the puzzle-solver distinguish between the three routes. The knight must travel up, down, and up the ladder again before moving on. Squares to side on the top and bottom facilitate switching between the three interwoven pathways. The final element is a *double-Y* (see also Figure 4) which contains a mix of possibilities for jumping, making for a enjoyable and puzzling finish to the maze.



**Figure 2:** *double loop*

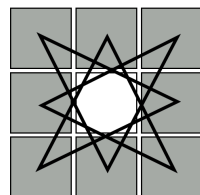


**Figure 3:** *triple braided ladder*

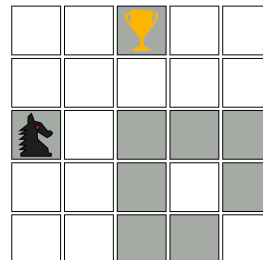


**Figure 4:** *double-Y*

Knight maze elements are fun to design, and it can be challenging to create shapes that are both attractive and interesting to solve. Figure 5 shows the design of an element based on a square. A knight can travel in a loop of 8 positions that form the outside of a square. By removing one of these positions from the route we break the loop, creating starting and ending points on the square that can be connected to outside positions.

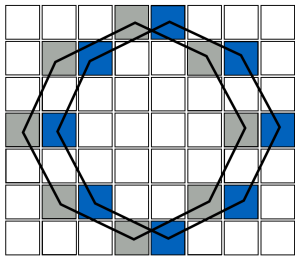


**Figure 5:** The square

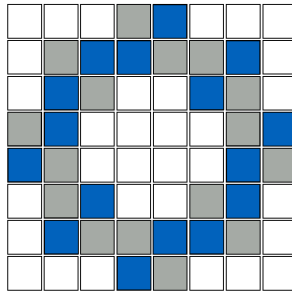


**Figure 6:** *The square as a maze element*

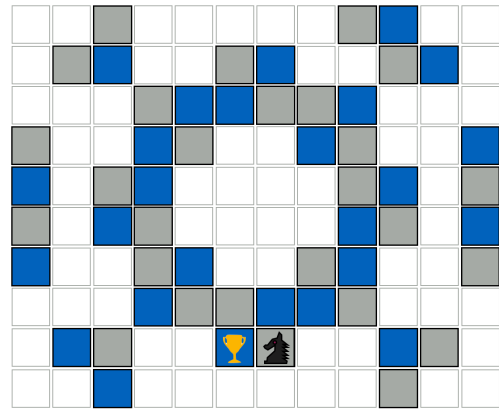
The movement of the knight allows independent paths to cross and weave around each other. Figure 6 show two octagonal paths beside each other with no connection between them. If the paths are duplicated and shifted down one square, we now have 4 octagons. The octagons are connected pairwise – it is possible to jump between a pair of octagons but not possible to jump to the other pair of octagons. Figure 7 shows the two independent sets of squares colored accordingly. This shape can be used as the basis for a puzzling maze such as the one shown in Figure 8. The knight starts on one color and the goal is on the other color. An extra square is added in a subtle position allowing for transition from one set of colors to the other. Can you discover the square that is the key to solving this maze? (Hint: look for a break in symmetry.)



**Figure 6:** two octagons



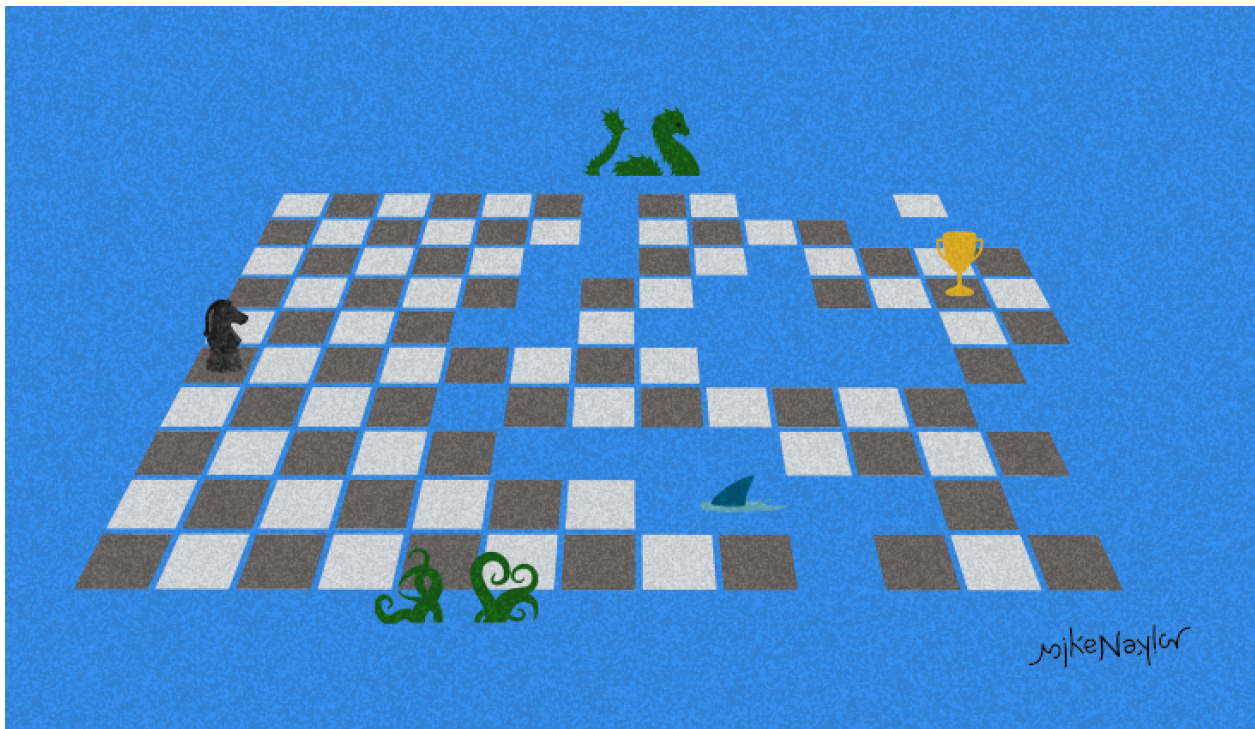
**Figure 7:** independent paths



**Figure 8:** Maze built on four octagons

### Knight Mazes II - Destructive construction

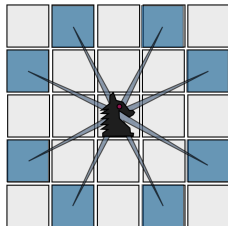
Figure 9 shows *Knight Maze II - Danger!*, an artwork maze with a rather difficult solution. The reader is encouraged to attempt a solution before reading the details of its construction and thus the key to solving this puzzle.



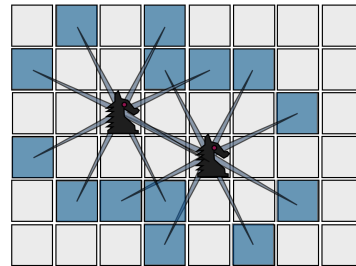
**Figure 9:** *Knight Maze II – Danger!*

While the earlier knight's mazes are built constructively, adding squares to create interesting routes, we can make a difficult puzzle by building *destructively*, removing squares in order to limit the knight's movements.

In the middle of a blank board, a knight has 8 positions it can move to (Figure 10). If we block all of these positions, a knight is trapped in the center. We then remove one of these blocks and then block all of the positions the knight can move to from this new position as shown in Figure 11. A knight on one of the two squares in the center is free to move back and forth between the two, but cannot go further anywhere else on the board.

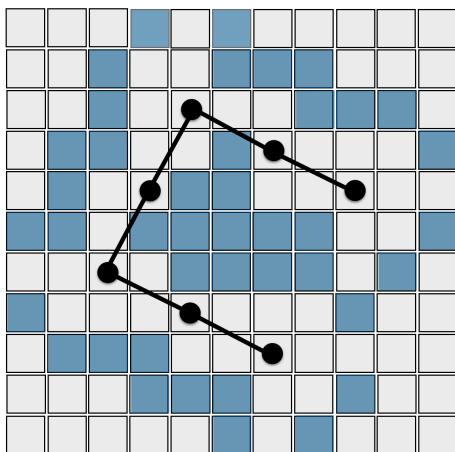


**Figure 10:** blocked

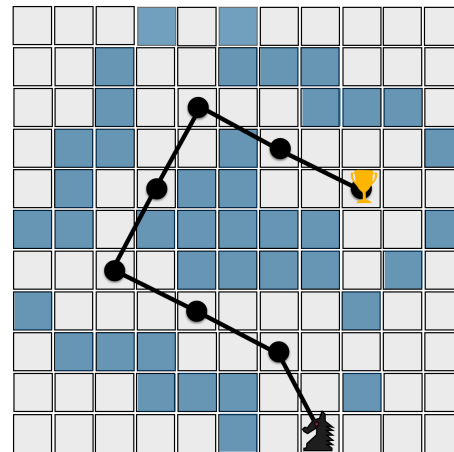


**Figure 11:** the start of a forced path

By stringing together a chain of such elements, we can create a long pathway with no exits or entrances. Figure 12 shows a chain of seven linked but isolated positions (marked with black dots). The colored squares are both sufficient and necessary to isolate the path.



**Figure 12:** Seven linked but isolated positions



**Figure 13:** Implementing the chain in the maze

To complete the maze, we add the goal at one end of the chain and remove one of the squares blocking access to the other end of the chain (Figure 13). The chain is now open to the rest of the board. This is the pattern used to create *Knight Maze II – Danger!* The board is extended to the left with plenty of open squares to give a feeling of freedom, but the goal can be reached only by first coming to the key position at the start of the chain.

## Endgame

Knight mazes can be fun and surprising. They are also ideal for garden mazes or in public spaces, where participants can hop from tile to tile. A human-size knight maze is currently being built at Amborneset Math Creativity Center in Norway.