Deconstructing Magic Squares
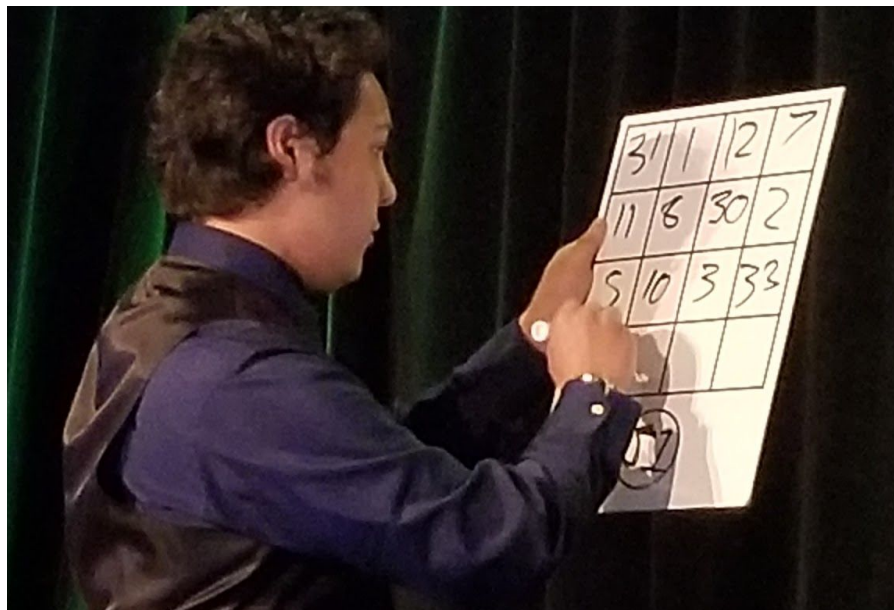
& The MATLAB Magic Show

G4G13 Gift Exchange Paper

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31	1	12	7
11	8	30	2
5	10	3	33
4	32	6	9

51

Magicians have performed magic squares for many years. The general routine involves the magician getting a random 2-digit number (N) that is the magic sum and then the 4x4 grid of numbers then gets filled out quickly to show that all combinations - rows, columns, diagonals, corners, and many others - add up to the given sum.

When I was in the 8th grade, after seeing a magician perform the magic square at a local venue, I was inspired to learn this impressive routine. After months of research and working to solve the square, I figured out that it was a template of numbers 1 - 12 and four variable numbers carefully arranged in one such way:

11	8	N - 21	2
N - 20	1	12	7
4	N - 19	6	9
5	10	3	N - 18

The main secret is that you simply subtract the appropriate numbers from the given sum and plug them in and you get a selected number magic square where nearly all combinations add up. By having 12 set numbers and 4 variable numbers, you are able to make a square with any sum greater than 34 (with no repeated numbers). Because I went about learning this the hard way, I gained a much deeper understanding of the square and the relationships that the numbers had. With the number of performances I studied, I came to notice certain repeating patterns that were just variations of the square above. I then tried to further my research and asked the question if it was possible then to be able to place any number (1 - 12) in any of the 16 squares.

The answer was yes. And what follows is the method for doing just that. I have been performing this informally for some time showing this to small groups or select magicians. This is the first time it is published since I created it nearly 10 years ago. I hope you find this method of interest or potentially some use to you.

The best way to think about the apparent nearly 200 combinations for this new square is to break it down into 5 separate cases.

Case #0: Base Case

I call this Case #0 since it is the simplest scenario since no additional work is required. Let's use the square I showed at the beginning as our standard. Memorize this backwards and forwards. Try to spot patterns or come up with mnemonics to help remember the placements of the numbers and their relationships. I generally recite the square going across the rows, but you should be able to do it from the columns as well.

If they told you to put the number 11 in the top left corner or 9 in the third row and fourth column, then you just need to fill out the general square you have memorized best.

11	8	N - 21	2
N - 20	1	12	7
4	N - 19	6	9
5	10	3	N - 18

Case #1: Rotate Square

Case #1 follows intuitively. This takes very little additional computational effort as all you need to do is rotate the square 90°, or any number of times necessary. So, if 11 if needed to be placed in the top right corner or 9 was placed in the fourth row and second column, then just one rotation will do.

11	8	N-21	2
N-20	1	12	7
4	N-19	6	9
5	10	3	N-18

90° Rotation →

5	4	N-20	11
10	N-19	1	8
3	6	12	N-21
N-18	9	7	2
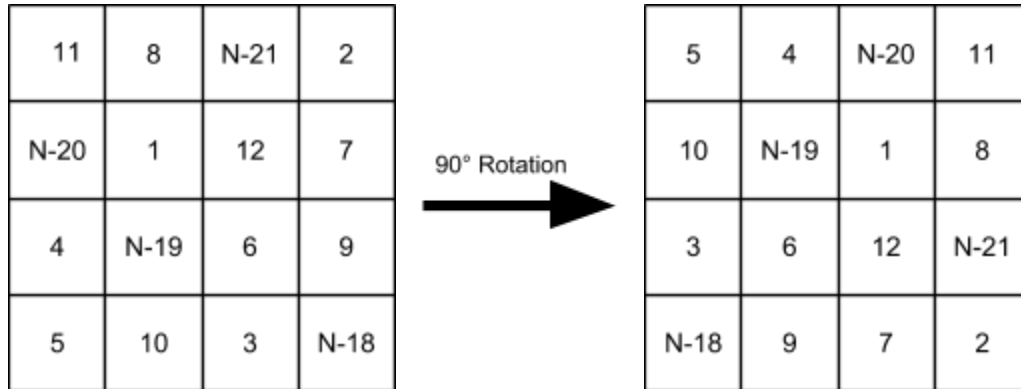
You can see an animation for this at bit.ly/SquareCase1.

Alternatively, you can use a mirrored version of the square, or fill out the square going across rows from a different starting point.

2	N-21	8	11
7	12	1	N-20
9	6	N-19	4
N-18	3	10	5

Mirrored

N-18	3	10	5
9	6	N-19	4
7	12	1	N-20
2	N-21	8	11

Bottom Right Corner
Starting Point

Case #2: Swap Rows

Case #2 becomes slightly more challenging. You will simply swap the first and second rows as well as the third and fourth rows. You are performing a symmetrical transformation of the square which retains the rigid nature of the sums. This is what I consider slightly more challenging than Case #1 and less than #3, as I have memorized them going from rows.
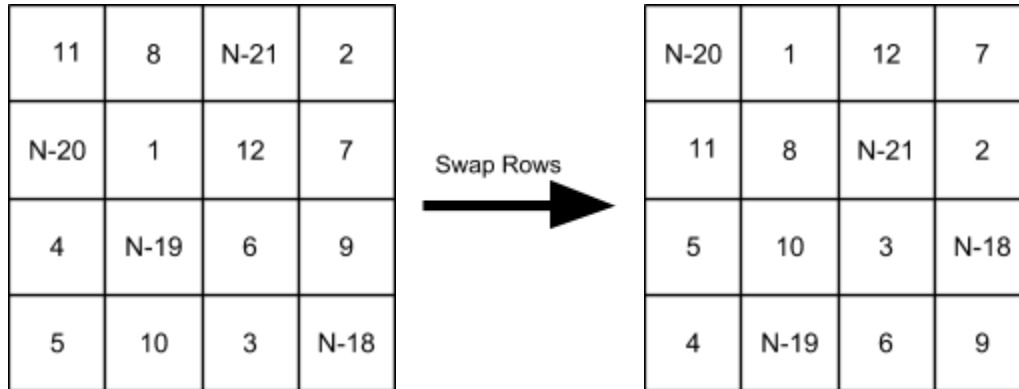
You can see an animation for this at bit.ly/SquareCase2.

Case #3: Swap Columns

Case #3 follows intuitively with the previous case, but this is a swapping of the columns. Simply switch the first two columns with the last two columns.
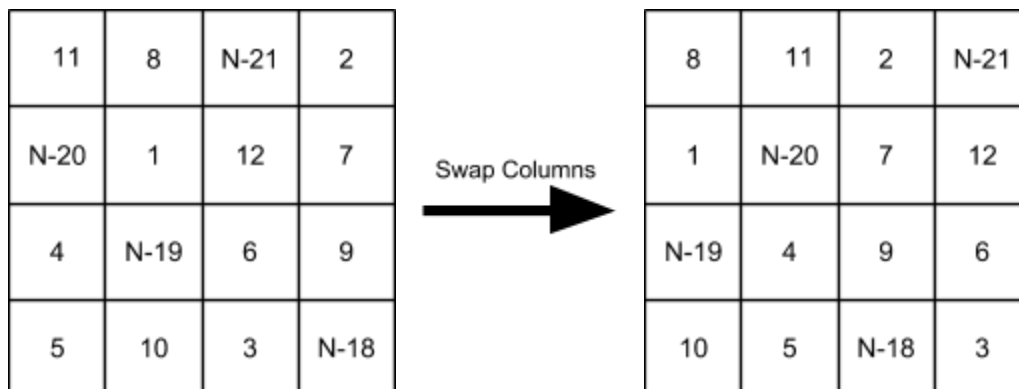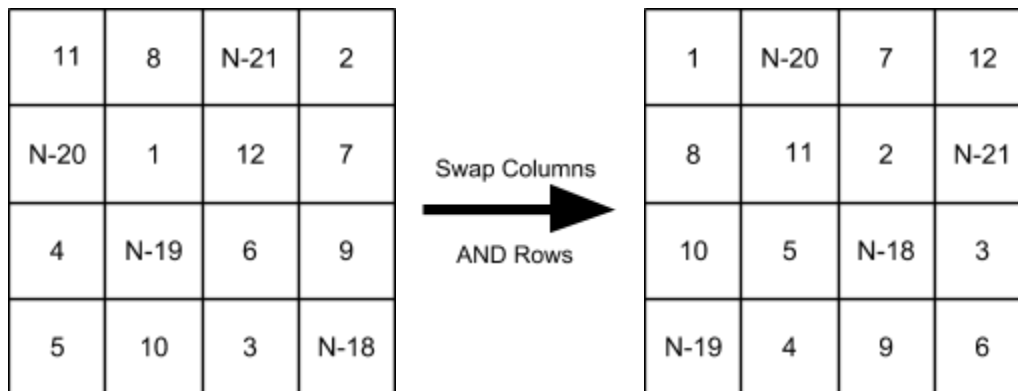

You can see an animation for this at bit.ly/SquareCase3.

Case #4: Swap Rows AND Columns

This is by far the hardest case. This is when any of the four corners need to be placed in any of the inside squares or vice versa. Our current methods will not allow us to simply rotate and swap the rows or columns. So, you will need to do both. This takes the most mental energy to map out (without simply memorizing). I carefully place each number to ensure that I have retained the paired relationships between each row and column. This method works because we are still applying symmetrical transformations to the square. With the many combinations, you will get sums from, for example, the four corners as well as the four inside squares. Case #4

is essentially turning the square inside out and the corners become the inside squares and vice versa.

11	8	N-21	2
N-20	1	12	7
4	N-19	6	9
5	10	3	N-18

Swap Columns

AND Rows

1	N-20	7	12
8	11	2	N-21
10	5	N-18	3
N-19	4	9	6

You can see an animation for this at bit.ly/SquareCase4.

These are the 4 basic cases to place any number 1-12 in any of the 16 spots on the magic square. The best thing to do is to practice this many times to get a feel for how you might fill in the square, as there often are more than one possible arrangement for the square that follows all rules set forth here. What I recommend is before going straight into a solve, take a second to think about what the best method to apply might be. Ask if instead of swapping columns you can get a correct solution by starting from a different point. Or, instead of swapping rows, columns, and rotating, if you can use one or two to get the same result. This then becomes a pretty fun game of strategizing and problem solving.

The square on the cover of the paper is a recreation of the one made during my presentation at G4G. To save time but also showcase my method, I incorporated the 4 and 13 aspects by using the number 4 in the 13th spot on the grid (as opposed to asking people to call them out for me) and the number 51 was freely called. You will easily see that this square was a simple application of Case #2 of switching the rows.

In terms of memorization, I came to know what I use as Case #0 as my main point of reference, mainly because I like to remember the numbers in rows and each variable number across rows is larger than the previous one. But, some people might prefer to have the simple calculation of N-20 be the first they do, and so you can figure out whichever square you want to be your point of reference and then apply my method to that square. If you are better at memorization than I am, you can memorize 4 squares (Case #0, #2, #3 and #4) and that will allow you to do the same effect, but only need to apply a rotation, mirroring, or alternative start point (Case #1) to the square. And, if you want to use no mental energy other than recall, then memorizing 16 squares (feel free to figure out which 16) will give you all possibilities for any

number 1-12 in any spot.

From a performance perspective, this is sadly hard to really sell as much more impressive than a standard magic square, at least from an extra effort vs. overall effect perspective. Perhaps you will be able to perform it more effectively than I have been able to, but what I have found is that it impresses magicians and it has about the same effect as the standard square does on the lay audience. If you are thinking of performing this to an audience, I recommend really practicing the different arrangements, and possibly more importantly, practice the strategizing aspect quickly and effectively. This could potentially save you a few seconds from your filling time as well as help you be more confident and less prone to mistakes. I also recommend asking them to place any number 1-9 (instead of 12) in any spot. Saying you can place any number 1-12 seems a bit arbitrary and might make the audience suspicious. It can be easily motivated by noting that you should keep the number to one digit or 1-10 since they are all supposed to add up to the selected sum. I also recommend keeping the sum that is given to a two-digit number. Though it will still work with numbers over 100, the distribution of the variable and set numbers becomes quite skewed and fairly easy to figure out from there.

Also, note that with this and the general method, there are two particular combinations that do not add up to the given sum. Looking at the middle two rows, they are the two partitions of the 2x2 squares. One contains two large variable numbers and the other contains none. If performing this, I recommend not calling attention to this and just focus on the sums that actually do add up. It is a bit of a pet peeve of mine when a performer will circle those two combinations along with all other sums to give the appearance of every possible combination working in the hopes that the audience will not catch on or not still checking combinations by then. I believe that some audience members can catch on, and it is best not to call any attention to this discrepancy and let the actual sums speak for themselves.

I am excited to be sharing this with such a special community. I do plan to publish this very soon, but for now you are of a select few who have the method behind my version of the magic square. I do hope you will take the time to learn it and have fun either challenging yourself or sharing this in performance. I would love to hear your thoughts or any questions that you might have. Feel free to email me at magicalnathaniel@gmail.com and I look forward to hearing from you!

Bonus for MATLAB users:

A few years back, I developed a magic show in MATLAB. It is 5 fully interactive (math based) magic tricks that I have added some fun presentation to. All you have to do is run the filename MagicShow and it will give you directions from there. And, one of the tricks is a fully coded version of the magic square that is outlined here. I hope you enjoy!

To download, visit bit.ly/MATLABMagic.

Here is the original sheet made figuring out the different permutations of the square, circa 2008.