# Scrabble® Seven-letter Words

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"It's a damn poor mind that can only think of one way to spell a word."
- Andrew Jackson

## **Abstract**

Scrabble® may be the world's most popular word game and is played in more than 30 different languages. Over 150 million Scrabble sets have been sold in over 120 countries. This paper analyzes various statistics for the English-language edition of the game of Scrabble. Specifically we investigate the mathematics of the seven-tile starting racks and seven-letter words, and determine the likelihood that a starting rack can make a seven-letter word.

#### Introduction

Scrabble<sup>1</sup> is a multiplayer word game where players compete for the highest score by using letter tiles to make words crossword-style. It was developed by Alfred Mosher Butts<sup>2</sup>, an American architect, in 1938 as a variant of his game Lexiko. Lexiko is played without a board, similar to dominoes. When Butts created Scrabble, he added a board.

The board contains 225 squares arranged in a 15 by 15 grid. There is a pool of 100 tiles. Each tile fits on a square and contains a letter and a point value. Each player has a rack that holds seven tiles randomly chosen from the pool. A play consists of placing tiles from your rack onto the board to form new words. The point values of each tile in the new word or words created are added to that player's score. If all seven tiles are used in one turn, this is known as making a "bingo" and the player receives a 50 point bonus. Making bingos is a key strategy of advanced players.

Certain squares on the board have bonus properties when newly covered by a tile. The four types of bonus squares are shown in Table 1.

bonus square	count
Double Letter Score	24
Triple Letter Score	12
Double Word Score	17
Triple Word Score	8

**Table 1 - Bonus Square Counts** 

Bonus squares are only mentioned for completeness; this paper will not be addressing tile placement.

Alfred Butts determined a point value for each letter based on its frequency in written English words. He eventually settled on the tile distribution shown in Table 2, which is still in use today.

**Table 2 – Tile Distribution** 

letter	point value	tile count
A	1	9
В	3	2
С	3	2
D	2	4
E	1	12
F	4	2
G	2	3
Н	4	2
I	1	9
J	8	1
K	5	1
L	1	4
М	3	2

letter	point value	tile count
N	1	6
0	1	8
P	3	2
Q	10	1
R	1	6
S	1	4
Т	1	6
U	1	4
V	4	2
W	4	2
Х	8	1
Y	4	2
Z	10	1
?	0	2

There are two blank tiles in the pool which are the equivalent of "wild cards" and which may be used to represent any letter. In this paper, a blank tile is represented by a question mark.

In an interview Stefan Fatsis said, "... the game is all about math. There are 100 tiles, 98 letters and two blanks. It's all about combinations, and they are mathematical." In the present paper, we investigate the mathematics of seven-letter letter words and starting racks. We are not going to be concerned with tile point values and word scores.

# **The Dictionary**

Acceptable words are determined as those contained in some chosen reference source. Casual players may simply agree to use any dictionary at hand. Many players agree to use the Official Scrabble Players Dictionary or OSPD. Tournament and club players in the United States and Canada use the more comprehensive Official Tournament and Club Word List. In this paper, we will use the third and current edition of this reference source: OTCWL2014, also known as OWL2014.

We refer to OWL2014 as the "dictionary," even though it is just a word list and does not contain pronunciations, etymologies, or definitions. OWL2014 contains 24,029 seven-letter words.

## **Alphabetical Bingos**

The dictionary was checked for bingos with their letters in alphabetical order. Two lists of the 24,029 bingos in the dictionary were created. In the second list, the letters in each bingo were sorted alphabetically left to right. A line by line comparison was performed of the original list of bingos against the list of alphabetized bingos, checking for matches.

The only bingos that have their letters in alphabetical order are:

BEEFILY BILLOWY

#### **Palindromic Bingos**

The dictionary was checked for bingos that are palindromes. A palindrome is a word or phrase that reads the same forwards and backwards. An example of a palindromic phrase is "Never odd or even." Examples of some common words that are palindromes include CIVIC, NOON, and SEXES.

Two lists of the 24,029 bingos in the dictionary were created. In the second list the letters in each bingo were reversed. A line by line comparison was performed of the original list of bingos against the list of reversed bingos, checking for matches.

The only palindromic bingos are:

DEIFIED HALALAH REIFIER REPAPER REVIVER ROTATOR SEMEMES

The commonly known seven-letter palindrome, RACECAR, is considered two separate words in our dictionary (RACE and CAR), so it is not in the above list.

## **Bingos with Duplicated Letters**

The 24,029 bingos in the dictionary were checked for words with duplicated letters. Each letter in a word may appear once, twice, three times, or four times. There are no bingos with more than four instances of the same letter.

For this analysis, our algorithm used four counters to track the number of single, double, triple, and quadruple letter occurrences. For each bingo, the letters were sorted alphabetically left to right. This sorting brings the repeated letters together, making it easier to count how many single, double, triple, and quadruple letters occur. The eleven possible letter-repetition patterns are shown in Table 3.

**Table 3 – Letter-Repetition Patterns** 

	letter count				mples
single	double	triple	quadruple	bingo	alphabetized bingo
7				CINEMAS	ACEIMNS
5	1			BLUFFED	BDEFFLU
3	2			TUITION	IINOTTU
1	3			GRAMMAR	AAGMMRR
4		1		LOWBALL	ABLLLOW
2	1	1		EELLIKE	EEEIKLL
	2	1		ALFALFA	AAAFFLL
1		2		LESSEES	EEELSSS
3			1	ASSISTS	AISSSST
1	1		1	REFEREE	EEEEFRR
		1	1		

There are no bingos with both a triple and a quadruple letter.

The letter repetitions in each alphabetized bingo were examined and the appropriate counters were incremented. The number of bingos for each of the eleven patterns is presented in Table 4.

**Table 4 – Bingo Counts by Letter-Repetition Patterns** 

	number of bingos						
single	double	triple	triple quadruple				
7				7,940			
5	1			11,040			
3	2			3,413			
1	3			195			
4		1		989			
2	1	1		399			
	2	1		15			
1		2		5			
3			1	24			
1	1		1	9			
		1	1	0			
	24,029						

The number of bingos containing various repetitions of each letter was examined. For example, how many bingos contained exactly three A's?

We created 104 counters - four counters for each of the 26 letters. These counters tracked the number of single, double, triple, and quadruple occurrences of that letter. For each of the 24,029 bingos, the letters were again sorted alphabetically left to right, bringing the repeated letters together and making it simpler to count how many singles, pairs, triplets, and quadruplets occur for each letter. The letter repetitions in each alphabetized bingo were examined and the appropriate counters were incremented.

For example, the word ALFALFA would be alphabetized as AAAFFLL and the "triple A," "double F," and "double L" counters would each get incremented. The results are presented in Table 5.

Table 5 - Bingo Counts by Letter Repetitions

	number of bingos				
letter	single	gle double tripl		quadruple	
A	9,328	1,727	184	1	
В	3,063	347	27		
С	5,029	476	6		
D	5,453	660	74	2	
E	11,888	3,426	431	11	
F	1,877	315	12		
G	4,181	497	65	2	
Н	3,689	146	3		
I	9,828	1,545	30		
J	428	9			
K	2,100	101			
L	7,155	1,017	41		
М	3,947	356	25		
N	7,773	1,020	63		
0	6,907	1,318	71	10	
P	3,965	476	28		
Q	308				
R	9,337	1,465	36		
S	10,253	2,278	238	5	
Т	7,141	1,083	67		
U	5,534	366	6	1	
V	1,385	45			
W	1,827	48	2		
Х	528	3			
Y	2,592	66			
Z	538	99	4	1	

## **Bingos That Require Blanks**

Some of the 24,029 bingos in the dictionary require one or more blanks because of the distribution of letter tiles in the pool. For example KNOCKED requires a blank to be used for one of the two K's, MAXIMUM and MINIMUM both require a blank to be used for one of the three M's, and ZIZZLES requires two blanks to be used for two of the three Z's.

To calculate the words that require blank tiles, we started once again by sorting the letters alphabetically left to right for each bingo, bringing the repeated letters together. The number of times each letter appears in the word was checked against the tile pool to see if the pool contains enough tiles of that letter. If the pool doesn't contain enough tiles of that letter, then one or more blank tiles would be needed to make that bingo. Each alphabetized bingo was examined to determine how many blanks were required to make that word with the distribution of letter tiles in the pool.

blanks required	number of bingos
0	23,707
1	317
2	4
3	1
total	24,029

**Table 6 - Bingos That Require Blanks** 

The last line in Table 6 indicates that there is a seven-letter word in the dictionary that requires more blanks than are available in the tile pool - and there is: the word is PIZZAZZ!

## **Letter Combinations That Can Not Bingo**

The question of letter combinations that do not occur on any of the 24,029 bingos in the dictionary was explored. Were there any two-letter combinations that could be drawn from the tile pool which would prevent making a bingo? Were there any three-letter combinations?

A list of all 351 two-letter combinations: AA, AB, AC, etc. was created. Letter combinations that can not be drawn from the tile pool were eliminated from the list. For example, JJ was eliminated, as there is only one J in the tile pool. After eliminating 5 combinations (JJ, KK, QQ, XX, and ZZ), there remained 346 two-letter combinations that can be drawn. For each of these two-letter combinations, the list of bingos was checked until a word that contains those two letters was found. A matching word was found for every pair of letters that can be drawn, so there are no two-letter combinations that will prevent making a bingo.

Next the three-letter combinations were considered. Similarly, a list of all 3,276 three-letter combinations was created and those that can not be drawn from the tile pool were eliminated. After eliminating 139 combinations, there remained 3,137 three-letter combinations that can be drawn. For each of these three-letter combinations, the list of bingos was checked until a word that contains those three letters was found. If no matching word was found, those three letters block the possibility of making a bingo. The 201 three-letter combinations that prevent making a bingo are shown in Table 7.4

Table 7 - Three-Letter Bingo Blockers

A         J         Q         A         J         X         A         Q         N         B         C         Q         B         D         Q         B         F         Q         B         G         Q         B         J         V         B         G         Q         D         J         X         D         B         G         Q         D         J         X         D										
B         V         Z         B         X         Z         C         F         Q         C         J         Q         C         J         V         C         J         X         C         Q         V         D         J         V         D         J         X         D         J         C         V         D         J         X         D         J         X         D         J         X         D         J         X         D         J         X         D         J         X         D         Q         V         D         Q         V         D         J         X         D         Q         V         D         J         X         D	АJQ	A J X	A Q X	B B V	B C Q	B D Q	B F J	B F Q E	3 G Q	ВЈР
C W Z C X Z D G Q D J Q D J V D J X D J Z D K X D Q V D Q W D Q X D Q Y E J Q F G Q F H Q F H V F J K F J Q F J V F J X F J Z F K Q F K V F M Q F N Q F D Z F V X F V Z F W Z F X Z G G X G J Q G J V F Q W F Q X F Q Z F V X F V Z F W Z F X Z G G X G J Q G J V G K X G L Q G M Q G Q V G Q W G Q X G Q Z G V Z G X G Q Z G X Z H H Q H X H X H J L H J Q H Q X H Q X H Q X H Q X H J X H J X H J X H W X H X Z I Q W J K Q J K V J L X J M Q J M Q X H Q X H Q X H Q X H Q X H Q X H Q X H X Z J Q W J K Q J K V J L X J M Q J M Q X H Q X H Q X H Q X H Q X H X X J Y X X X X X X X X X X X X X X X X	вјо	вју	ВКО	B M Q	B P Q	B Q V	B Q W	BQX	3 V W	вVХ
D Q X D Q Y E J Q F G Q F H Q F H V F J K F J Q F J V F J X F J X F J Q F J X	B V Z	в х г	C F Q	C G Q	СJQ	СJV	СЈХ	CQV	C Q W	C Q X
F J Z F K Q F K V F M Q F N Q F O Q F P Q F P V F P Z F Q T Q T Q T Q T Q T Q T Q T Q T Q T Q	C W Z	C X Z	D G Q	D J Q	D J V	D J X	D J Z	D K X I	V Q C	D Q W
F Q V F Q W F Q X F Q Z F V X F V Z F W Z F X Z G G X G Q Z G X G Q Z G X G Z G X X Z G X X Z G X Z G X	D Q X	D Q Y	E J Q	F G Q	F H Q	F H V	F J K	F J Q I	JV	FJΧ
G J V G K X G L Q G M Q G Q V G Q W G Q X G Q Z G V Z G X X Z G X X X Z G X X X Z G X X X Z G X X X Z G X X X Z G X X X Z G X X X X	F J Z	F K Q	F K V	F M Q	F N Q	F O Q	F P Q	F P V I	PZ	F Q T
H H Q H H X H J L H J Q H H J V H J X H J V H J X H J V H J Z H K Q H K X H P Q H Q T H Q V H Q W H Q X H Q Z H V V H V X H W X H W X H X Z I Q W J K Q J K V J L X J M Q J M V J M W J M X J P Q J J P V J P W J P X J P Z J Q R J Q S J Q T J Q V J Q W J Q X J Q X J Q Y J Q Z J R X J S X J T Z J W W J V V J V W J V X J V Z J W W J W X J W Z J X Y J X Z J Y Y K M X K P Q K Q V K Q X K Q Z K T X K U V K V W K V X K X Y K X Z L Q V L Q V L Q W L Q X L X Z M M Q M P Q M Q V M Q W M Q X M V V M V W M W Z M X Z N Q V N Q W N Q Z Q X Y Q X Z Q X X Q X X Q V V Q V W Q V X Q V Z Q W W Q W X Q W Z Q X Y Q X Z Q Y Y Q Y Z R X Z S X Z T X Z U W W U X Z V V W V V W V V X V V X V W X Z	F Q V	F Q W	F Q X	F Q Z	F V X	F V Z	F W Z	F X Z	G G X	GЈQ
H P Q H Q T H Q V H Q W H Q W H Q X H Q X H Q Z H V V H V X H W X H X Z  I Q W J K Q J K V J L X J M Q S J M V J M W J M X J P Q J P V  J P W J P X J P Z J Q R J Q S J V V W J V W J V W J V X J V X J V Z J W W  J Q Z J R X J S X J T Z J U W J V V J V W J V W J V X J V Z J W W  J W X J W Z J X Y J X Z J X X K X X X K X X X K X Z L Q V L Q W L Q X L X Z  M M Q M P Q M Q V M Q W M Q W M Q X M V V M V W M V W M W Z M X Z N Q V  N Q W N Q Z O W O Q Z P Q V P Q W P Q X P Q X P Q Y P Q Z P V X  P V Z P W Z P X Z Q X Y Q X Z Q X X Q Y X Q Y X Z W X Z W X Z W X Z  U X Z V V W V V W V V X X V X X X X X X X X X	GJV	G K X	G L Q	G M Q	G Q V	G Q W	G Q X	G Q Z	3 V Z	G X Z
I Q W       J K Q       J K V       J L X       J M Q       J M V       J M W       J M W       J M X       J P Q       J P V         J P W       J P X       J P X       J P Z       J Q R       J Q S       J Q T       J Q V       J Q W       J Q X       J Q X       J Q Y         J Q Z       J R X       J S X       J T Z       J U W J V Y       K M X K P Q K Y       K Q V K Q X       K Q X       K Q Z         K T X K W V V K V W K V W K V X K X Y K X Y K X Z L Q V L Q V L Q W L Q X       L X Z       L X Z         M M Q M R Q W M Q W M Q X M V V W M V W M W X W M W Z M X Z D W Y Q W W       M X Z M X Z D W Z D W W       M X Z M X Z D W X D W W         P V Z P W Z P W Z Q X Y Q R X Q R X Q S X Q V V Q V W Q V W Q V X Q V Z Q W W       W W Z W W         U X Z V V W W V V W V X V X V X X V V Z Z V W W W       V W X Z W Z W Z W Z W Z W Z W Z W Z W Z W	ннQ	ннх	нјь	НJQ	нJV	нјх	нју	H J Z I	H K Q	нкх
J P W J P X J P X J P Z J Q R J Q S J Q T J Q W J Q W J Q X J Q X           J Q Z J R X J J S X J T Z J W W J V V J V W J V W J V X J V Z J W W           J W X J W Z J X Y J X X J X X Z J X Y X X X X X X X X X X X X X X X X X	H P Q	НQТ	H Q V	H Q W	H Q X	H Q Z	H V V	H V X I	X W F	H X Z
J Q Z         J R X         J S X         J T Z         J W W J V V J V W J V W J V X J V Z J W W         J V Z J W W           J W X J W Z J W Z J X Y J X X Y J X X Y K X Y K M X K P Q K R Q V K Q X K Q X K Q Z         K T X K W V W K V W K V W K V X K X Y K X Y K X Z L Q V L Q W L Q W L Q X L X Z         L X Z           M M Q M P Q M Q V M Q V M Q W M Q X M Q X M V V M W X V M W X Z M X Z M X Z M X Z M Y X X         F Q X F Q X F Q X F X Q X X Z M Y X X X X Y M Y X X X X X X X X X X X	I Q W	J K Q	J K V	J L X	J M Q	J M V	J M W	J M X	J P Q	J P V
J         W         X         J         X         Y         J         X         Z         J         Y         Y         K         M         X         R         P         Q         K         Q         X         K         Q         Z         X         Z         L         Q         V         L         Q         X         L         Q         X         L         Q         X         L         Q         X         L         Q         X         L         Q         X         L         Q         X         L         Q         X         L         X         Z         L         Q         V         L         Q         X         L         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         Z         I         X         X         Z         I         X         X         Z <td< td=""><td>J P W</td><td>J P X</td><td>J P Z</td><td>J Q R</td><td>J Q S</td><td>J Q T</td><td>J Q V</td><td>J Q W S</td><td>J Q X</td><td>J Q Y</td></td<>	J P W	J P X	J P Z	J Q R	J Q S	J Q T	J Q V	J Q W S	J Q X	J Q Y
K T X K U V K V W K V X K X Y K X Z L Q V L Q W L Q X L X Z N Q V N Q V N Q V N Q V N Q V N Q V N Q V N Q V P Q W P Q X P Q Y P Q Z P V X X X Q X X Q V X Q Q X Q X	J Q Z	J R X	J S X	J T Z	J U W	JVV	J V W	J V X	J V Z	JWW
M M Q M P Q M Q V M Q W M Q X M V V M V W M V Z M X Z N Q V X A Q X A Q X	J W X	J W Z	J X Y	J X Z	J Y Y	K M X	K P Q	K Q V	X Q X	K Q Z
N Q W N Q Z O Q W O Q Z P Q V P Q W P Q X P Q X P Q Z P V X P V Z P W Z P X Z Q R X Q S X Q V V Q V W Q V X Q V Z Q W W Q W X Q W Z Q X Y Q X Z Q X Y Q X Z Q Y Y Q Y Z R X Z S X Z T X Z U W W U X Z V V W V V X V Z V W W V W W V W X V W Z V X Z W W Z W Z W X Z	КТХ	K U V	K V W	K V X	K X Y	K X Z	L Q V	L Q W I	ΧQΣ	L X Z
P V Z       P W Z       P X Z       Q R X       Q S X       Q V V Q V W Q V X       Q V Z       Q V Z       Q W W         Q W X       Q W Z       Q X Y Q X Z       Q X Z Q X Y Q X Z       Q Y Y Z R X Z S X Z T X Z U W W       X Z X Z X Z X Z X Z X Z X Z X Z X Z X Z	M M Q	M P Q	M Q V	M Q W	M Q X	M V V	M V W	M W Z N	1 X Z	N Q V
Q W X       Q W Z       Q X Y       Q X Z       Q Y Y       Q Y Z       R X Z       S X Z       T X Z       U W W         U X Z       V V W       V V X       V V Z       V W W       V W X       V W Z       V X Z       W W Z       W W Z       W X Z	N Q W	N Q Z	O Q W	O Q Z	P Q V	P Q W	P Q X	P Q Y I	QZ	P V X
U X Z V V W V V X V V Z V W W V W X V W Z V X Z W W Z W X Z	P V Z	P W Z	P X Z	Q R X	Q S X	Q V V	Q V W	QVXQ	Q V Z	Q W W
	Q W X	Q W Z	Q X Y	Q X Z	Q Y Y	QYZ	R X Z	SXZ	r X Z	U W W
X Y Z	U X Z	V V W	V V X	V V Z	V W W	V W X	V W Z	V X Z V	V W Z	WXZ
	X Y Z									

# **Starting Racks**

A starting rack consists of seven randomly drawn tiles from the pool of 100 tiles. We calculated the total number of possible starting racks.

We used combinations instead of permutations since the order in which the tiles were chosen doesn't matter. The formula for choosing k items from a set of n items without replacement is:

$$C(n, k) = n*(n-1)*...*(n-k+1) / [k*(k-1)*...*1]$$
(1)

By using factorials, this formula can be written as:

$$C(n, k) = n! / [(n-k)! * k!]$$
 (2)

For a starting rack, this is simply the number of ways to choose seven tiles from the 100-tile pool.

$$n(\text{starting racks}) = C(100,7) = 100! / (93! * 7!) = 16,007,560,800$$
 (3)

#### **Blanks and Starting Racks**

The 100-tile pool contains two blank tiles.

Number of starting racks with no blanks:

$$n(\text{starting racks}_{\text{blanks}=0}) = C(98,7) = 13,834,413,152$$
 (4)

Number of starting racks with 1 blank:

$$n(\text{starting racks}_{\text{blanks}=1}) = C(2,1)*C(98,6) = 2,105,236,784$$
 (5)

Number of starting racks with both blanks:

$$n(\text{starting racks}_{\text{blanks}=2}) = C(98.5) = 67.910.864$$
 (6)

## **Vowels and Consonants and Starting Racks**

As shown in Table 2, there are 42 vowel tiles, 56 consonant tiles, and two blank tiles in the pool.

Let n(starting racks bvc) be the number of starting racks with b blanks, v vowels, and c consonants. The formula for calculating them is:

$$n(\text{starting racks}_{\text{bvc}}) = C(2, b) * C(42, v) * C(56, c)$$
 (7)

The number of starting racks for all combinations of blanks, vowels, and consonants are tabulated in Tables 8 through 10.

Table 8 - Possible Starting Racks with 0 Blanks

vowels	consonants	combinations
7	0	26,978,328
6	1	293,764,016
5	2	1,310,028,720
4	3	3,102,699,600
3	4	4,216,489,200
2	5	3,288,861,576
1	6	1,363,674,312
0	7	231,917,400
	total	13,834,413,152

Table 9 - Possible Starting Racks with 1 Blank

vowels	consonants	combinations
6	0	10,491,572
5	1	95,274,816
4	2	344,744,400
3	3	636,451,200
2	4	632,473,380
1	5	320,864,544
0	6	64,936,872
	total	2,105,236,784

Table 10 - Possible Starting Racks with 2 Blanks

vowels	consonants	combinations
5	0	850,668
4	1	6,268,080
3	2	17,679,200
2	3	23,866,920
1	4	15,426,180
0	5	3,819,816
	total	67,910,864

The totals for each table provide a check against the previous rack calculations for various numbers of blanks and ensure that all of the cases were covered.

## **Unique Starting Racks**

There may be several ways of selecting seven tiles from the pool that result in the same starting rack. As an example, there are multiple ways of drawing the AAAAAA starting rack from the 100-tile pool, since there are nine A tiles.

We defined a rack to be "unique" if it has a unique collection of letters and blanks, independent of their order. To simplify determining if a rack is unique, the seven tiles were kept in alphabetic order with the blanks on the right. There may be multiple ways of drawing each unique rack from the 100-tile pool, since there are multiple tiles for many letters.

In the above example of drawing AAAAAA, the number of ways of drawing this starting rack is simply the number of ways of choosing seven tiles from the pool of the nine A tiles:

n(starting rack 
$$_{AAAAAAA}$$
) = C(9,7) = 36 (8)

We wanted to determine the number of unique starting racks that can be created by drawing tiles from the tile pool. All of our above results on starting racks were obtained using combinatorics,

but to determine the number of unique starting racks, we wrote a C program with a recursive subroutine.

An alphabetized list of the tiles in the 100-tile pool was created. The blanks, represented by question marks, are at the end of the list:

#### AAAAAAABBCC...VVWWXYYZ??

The program has seven pointers. The first pointer represents the first tile in the rack, the second pointer represents the second tile in the rack, etc. The first pointer starts by pointing to the first tile in the list, second pointer points to the second tile in the list, etc. So initially, the seven pointers produce:

#### AAAAAA

This is the first unique rack.

The seventh pointer was advanced through the list of tiles in the pool. Each time the pointer advances to a tile that has a different letter from the previous tile, a new unique rack was created, and the count of unique racks was incremented. When the seventh pointer reaches the 100<sup>th</sup> tile in the list, the sixth pointer was advanced through the list until it encounters a tile that has a different letter from the previous tile it was pointing at. The seventh pointer was then repositioned to the tile to the right of where the sixth pointer is now pointing, creating a new unique rack, and the count of unique racks was incremented.

The process repeated with the seventh pointer advancing through the list counting the number of unique racks. Every time the seventh pointer reaches the last tile in the list, the sixth pointer was advanced through the list to a tile that has a different letter and the process repeats.

When the sixth pointer reaches the 99<sup>th</sup> tile in the list (the seventh pointer is at the 100<sup>th</sup> tile), the fifth pointer was now advanced through the list to a tile that has a different letter, the sixth and seventh pointers were repositioned to the two tiles to the right of the new fifth-pointer tile and the count of unique racks was incremented. The process begins again with the seventh pointer advancing through the list.

When a pointer reaches the other pointers at the end of the list, the next lower pointer advances to a tile that has a different letter. Eventually the first pointer will advance to the 94<sup>th</sup> tile, the process will stop and each unique rack will have been counted.

Our program counted 3,199,724 unique starting racks.

# **Starting Racks and Bingos**

After analyzing the properties of the dictionary, tile distribution, and starting rack composition, we then analyzed the relationship between the seven tiles on the starting racks and the seven-letter words in the dictionary.

## Starting Racks That Can Make the Most Bingos

We determined which starting racks made the most bingos. A list of the 24,029 bingos in the dictionary was created and the letters in each bingo were again sorted alphabetically left to right.

In addition to letter tiles, we have the two blank tiles, which can be used for any letter. There are additional racks that can be made by substituting blanks for one or two letters for each alphabetized bingo. There are three ways to add one or two blanks. First, each unique letter in the bingo can replaced with a blank. Second, each letter that occurs two or more times in the bingo can be replaced with double blanks. Third, each unique pair of different letters in the bingo can be replaced with two single blanks. We present an example to clarify this concept.

The bingo EELLIKE when alphabetized yields EEEIKLL. The following racks can be made by substituting blanks for one or two tiles:

no blanks	one single blank	double blanks	two single blanks
EEEIKLL	?EEIKLL	??EIKLL	?EE?KLL
	EEE?KLL	EEEIK??	?EEI?LL
	EEEI?LL		?EEIK?L
	EEEIK?L		EEE??LL
			EEE?K?L
			EEEI??L

We added blanks as described above to create the expanded rack list and again sorted each rack alphabetically left to right, with the blanks on the far right.

Some of these racks can not be drawn because they have more occurrences of a letter than there are tiles of that letter in the pool. This happens because there are words in the dictionary with letters occurring more times than there are tiles for that letter, and blanks must be used for those letters. When our algorithm did not replace these letters with blanks, it generated a rack that could not be drawn from the tiles in the pool. For example the word PUZZLES can be made from a hypothetical rack of ELSUZZ?, but this rack can not be drawn because there is only a single Z tile in the pool. We eliminated those hypothetical racks that can not be drawn from the tile pool.

Then the list was sorted alphabetically top to bottom. Now any starting rack that can be used to make more than one bingo will be repeated on adjacent lines in the list (e.g. FLATCAR and FRACTAL will both be sorted to produce AACFLRT). As the number of times each line is duplicated corresponds to the number of words in the dictionary created from that starting rack, we counted the repeated lines to see which starting racks match the most words.

The starting racks that can make the most bingos based on the number of blanks are shown in Table 11.

Table 11 – Starti	ng Racks That Can	Make the Most Bingo	os
blank tiles	starting racks	bingos	

blank tiles	starting racks	bingos
0	AEINRST	9
1	AEIRST?	70
2	AERST??	406

#### **Starting Racks That Can Bingo**

We wanted to know the probability of drawing a starting rack that can make a bingo. The brute force solution would be to check each of the over 16 billion possible starting racks to see if those tiles could be used to make any of the 24,029 bingos in the dictionary. We devised a more efficient approach.

We were not concerned with the words themselves; we were only concerned with the combinations of letters that make up the word. First, we started with the list of bingos and used it to build a list of all the unique starting racks that can make a bingo. Then we determined how many ways each of these unique starting racks could be drawn from the tile pool.

A list of the 24,029 bingos in the dictionary was created and the letters in each bingo were again sorted alphabetically left to right. Then the list was sorted alphabetically top to bottom so that any combination of letters that can be used to make more than one bingo will be repeated on adjacent lines in the list. By eliminating these duplicate lines from the list, we determined that there are 20,134 unique letter combinations needed to make all 24,029 bingos.

We now had to account for the two blank tiles. To each unique letter combination, we added zero, one, or two blanks as described above to create an expanded rack list. We applied the same technique used previously of sorting the tiles left to right with the blanks on the far right. Next the list was sorted alphabetically top to bottom and the duplicates were eliminated to leave only the unique racks.

Now that we had a list of all unique blanks-added racks that can produce a bingo, the next step was to count the number of possible ways each of these unique racks can be drawn from the tile pool. For example if our rack is EEEIKLL, we compute the number of ways to draw three E's from the 12 available E's, draw one I from the nine available I's, draw one K from the one available K, and draw two L's from the four available L's. We multiply these numbers together to obtain the possible ways to draw the rack.

n(starting rack EEEIKLL) = 
$$C(12,3) * C(9,1) * C(1,1) * C(4,2) = 11,880$$
 (9)

We created a list of these racks and the number of possible ways they can be drawn from the tile pool. For any racks that could not be drawn due to the tile distribution, this number was zero. These impossible racks were eliminated and the list of unique bingo racks was what remained.

$$n(\text{unique bingo racks}) = 120,828 \tag{10}$$

We summed all the counts for the possible ways to draw each of these unique bingo racks to determine:

n(starting racks that can make a bingo) = 
$$2,068,621,350$$
 (11)

Dividing the number of starting racks that can make a bingo by the total number of starting racks yields:

$$2,068,621,350 / 16,007,560,800 = 0.1292$$
 (to four decimal places) (12)

Thus 12.92% of randomly drawn starting racks can make a bingo.

## **Blanks and Starting Racks That Can Bingo**

Our list of starting racks that can make a bingo were analyzed with respect to the number of blank tiles on the rack.

**Table 12 – Blank Analysis** 

	combinations	Percentages of	
blank tiles	starting racks that can bingo	all starting racks that can bingo	all starting racks
0	1,075,220,956	52.0%	6.72%
1	938,048,008	45.3%	5.86%
2	55,352,386	2.7%	0.35%
totals	2,068,621,350	100.0%	12.92%

The totals provide a check with our previous work.

#### Monte Carlo Simulation of Starting Racks That Can Bingo

Since the authors were quite surprised by the high probability that a starting rack could make a bingo, we decided to check our results against a computer simulation. We created a program which randomly selected seven tiles from the 100-tile pool one million times and checked to see if those seven tiles could make a bingo. This Monte Carlo simulation was run ten times with different random seeds. Our result was 1,292,224 bingos for ten million starting racks.

This confirms our calculated 12.92% of starting racks can make a bingo.

# **Conclusions**

There are 24,029 bingos in the dictionary. There are 20,134 unique letter combinations that will make all 24,029 bingos. Two of the bingos have their letters in alphabetical order and seven of the bingos are palindromes.

The most common letter-repetition patterns for bingos are five unique letters plus a pair, followed by seven unique letters. The least common letter-repetition pattern for bingos is two sets of triple letters (only five words). The dictionary contain 33 seven-letter words with a quadruple letter.

Examination of Table 5 reveals that the dictionary contains no bingos with two or more Q's and no bingos with three or more J's, K's, V's, X's, or Y's. The only letters that occur four times in a bingo are A, D, E, G, O, S, U, and Z.

Most bingos can be made without using a blank tile, but 317 bingos require one blank tile and four bingos require both blank tiles. One of the bingos in the dictionary, PIZZAZZ, can not be made with the tiles available, as it would require three blanks.

There are 16,007,560,800 ways to choose a seven-tile starting rack from the 100-tile pool. A starting rack will contain no blanks 86.4% of the time, one blank 13.2% of the time, and both

blanks 0.4% of the time. A starting rack will contain all consonants 1.5% of the time, and all vowels 0.2% of the time. The three most common starting racks are three vowels and four consonants (26.3%), two vowels and five consonants (20.6%), and four vowels and three consonants (19.4%).

The starting racks that can make the most bingos are AERST?? (406 bingos) and EIRST?? (340 bingos). The starting rack that can make the most bingos without a blank is AEINRST (9 bingos). Of all the racks that can bingo, the most likely to be drawn is AEEINRT, which has 1,154,736 ways it can be drawn. This rack can make three bingos, namely ARENITE, RETINAE, and TRAINEE. Of all the racks that can bingo, the least likely to be drawn are BBCCK?? and CCHHK??, each with only one way to draw these tiles. These racks can make the bingos BIB—COCK and CHACHKA respectively.

The answer to the question which started us on all of this Scrabble research surprised us. The probability that a randomly drawn starting rack can make a bingo is an unexpectedly high 12.92%.

A future paper will present our research using tile point values to analyze the scoring potential of starting racks and bingos.

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The author order is alphabetical reflecting equal contribution by both authors and in deference to the alphabetical sorting used in many of the algorithms. We thank Donna Atwood, Rod Bogart, Robert Follek, David Klein, and Robyn Stukalin for comments on early drafts of this paper.

## **Notes**

- 1. Scrabble® is a registered trademark of Hasbro, Inc. in the United States and Canada. Outside of the United States and Canada, it is a trademark of J.W. Spear & Sons Limited, a subsidiary of Mattel, Inc.
- 2. The history of Scrabble and Lexiko is from "Word Freak: Heartbreak, Triumph, Genius, and Obsession in the World of Competitive SCRABBLE Players" by Stefan Fatsis (2001)
- 3. The Pennsylvania Gazette, September/October 2001; http://www.upenn.edu/gazette/0901/fatsissidebar.html
- 4. Although not a true bingo blocker, VYZ can only make one bingo. All other three-letter combinations not shown in Table 7 can make at least four bingos. The only bingo containing VYZ is ZYZZYVA, which is a South American weevil and also happens to be the last word in many English language dictionaries.