Deck Building Games with Playing Cards

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for G4G13

Abstract

Here are two new deck building games that may be played with an ordinary 52-card deck of playing cards (appropriately for G**4**G**13** as such a deck has **4** suits of **13** cards).

1   Introduction

I have long enjoyed playing deck building games such as Dominion or Ascension. In a deck building game, each player begins with a small individual deck of low level cards and over the course of the game acquires more (and better) cards. Some of the enjoyment comes from beautiful artwork and themes, but the game mechanics themselves are fun, and so I wanted to invent a similar game that could be played using an ordinary deck of cards, and would therefore be much more portable. In this article I will describe two such games, with variations.

2   Structure and Common Assumptions

Both games assign a numeric value to each card in a typical way, ace through king representing integer values between 1 and 13.

These games are best played at a table; each player should have space for an individual deck (face down), a discard pile, and separate pile (the "vault") of cards removed from the game by the player. In the center of the table will be the remaining cards (the "store") in three stacks with only the top card of each stack face up.

In both games each player's starting deck contains ten cards. The player draws up to a hand of five cards from his or her deck (shuffling the discard pile when necessary to replenish the deck), then plays cards in an attempt to "buy" a new card from the store by creating a numeric expression whose value is exactly that of the desired card. Cards that have been bought are immediately replaced by turning over the exposed card below. Under certain circumstances cards may also be removed from the player's deck and "banked" in the vault. The games end when the store has an empty stack.

	hearts	clubs	diamonds	spades	
1	A♥	A♣	A♦	A♠	ace
2	2♥	2♣	2♦	2♠	two
3	3♥	3♣	3♦	3♠	three
4	4♥	4♣	4♦	4♠	four
5	5♥	5♣	5♦	5♠	five
6	6♥	6♣	6♦	6♠	six
7	7♥	7♣	7♦	7♠	seven
8	8♥	8♣	8♦	8♠	eight
9	9♥	9♣	9♦	9♠	nine
10	10♥	10♣	10♦	10♠	ten
11	J♥	J♣	J♦	J♠	jack
12	Q♥	Q♣	Q♦	Q♠	queen
13	K♥	K♣	K♦	K♠	king

Figure 1: Card values.

Where the games differ is in the composition of the initial decks, the expression logic, and the scoring rules.

3 RPN

This game is named for *Reverse Polish Notation*. Suits represent arithmetic operations, and so each card has both an operand (value) and an operator, and the 52 cards in the deck are therefore functionally unique.

3.1 Setup

Use one deck for two players, two decks for three or four players, three decks for five or six players, et cetera. For your first game, if you have enough spare decks from which you can borrow cards, each player should start with the same set of ten cards: A♥ 2♥ 3♥ A♣ 2♣ A♦ 2♦ 3♦ A♠ 2♠ . Otherwise, just randomly deal each player six hearts and four clubs. See Sec. 5.6 for more ideas on starting decks.

Shuffle each player deck of ten cards. Shuffle the main deck and split into three stacks with only the top card of each stack face up. Determine who should go first.

value	cards	RPN	expression
1	7♠ J♣ 9♦ K♥	7 11- 3× 13+	$((7 - 11) \times 3) + 13$
2	K♥ J♣	13 11-	$13 - 11$
4	K♥ 9♦ J♣ 7♠	13 3× 11- 7÷	$((13 \times 3) - 11) \div 7$
5	9♦ J♣ K♥	3 11- 13+	$(3 - 11) + 13$
5	9♦ K♥ J♣	3 13+ 11-	$(3 + 13) - 11$
6	K♥ J♣ 9♦	13 11- 3×	$(13 - 11) \times 3$
9	7♠ J♣ K♥	7 11- 13+	$(7 - 11) + 13$
9	7♠ K♥ J♣	7 13+ 11-	$(7 + 13) - 11$
10	7♠ 9♦ J♣	7 3× 11-	$(7 \times 3) - 11$
11	9♦ J♣ K♥	9 11- 13+	$(9 - 11) + 13$
11	9♦ K♥ J♣	9 13+ 11-	$(9 + 13) - 11$

Figure 2: Sample expressions in RPN

3.2 The Turn

When it is your turn, if you haven't already done so, draw five cards from your deck into your hand. You may then buy a card from the store by forming an expression, using at least two of the cards in your hand, whose value is that of the card. The first card in the expression is simply its value, regardless of suit. Each subsequent card in the expression combines its value with an operation: hearts for addition, clubs for subtraction, diamonds for multiplication, and spades for division. Note that you must always use at least two cards in your expression.

In this game, cards in your hand with composite values may be treated as any of their factors greater than one (only aces are allowed to have the value 1). Therefore the most versatile cards are queens, since they may be treated as any of $\{2, 3, 4, 6, 12\}$.[1] See Fig. 2 for the possible values and derivations for the partial hand K♥ J♣ 9♦ 7♠ .

As soon as you buy a card from the store, the card below it in the stack is turned over. You then have an opportunity to buy it or one of the other face up cards with the cards remaining in your hand.

At the end of your turn, discard any unused cards.

3.3 Additional Rules

At any point during your turn, you may discard a heart from your hand (i.e., a heart that hasn't already been played that turn) to draw a replacement card from your deck. At any point during your turn, you may discard a club from your hand to bank another yet-unplayed card: adding it to your vault, where it shall remain until the game ends.

3.4 Scoring

The game ends when the store has at least one empty stack. Add up your victory points from the cards in your hand, discard pile, deck, and vault. Hearts are not worth any victory points; clubs are worth 1 victory point each, diamonds are worth 2, and spades are worth 3. See Sec. 5.7 for a more advanced scoring variant.

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[1]This only applies to cards in your hand, not the store; you may only buy a queen with an expression whose value is 12.

4 Golomb

I named this game after Solomon Golomb, mathematician and long time member of the G4G community.

4.1 Setup

Use one deck of cards for two players, but add a second deck for three or more. For your first game, try a starting deck of A♥ 2♥ 4♥ 8♥ J♥ in one suit (hearts, clubs, diamonds, or spades) and 3♣ 6♣ 10♣ Q♣ K♣ in a different suit. For these starting decks, only use as many suits as there are players (but keep unused suits in the main deck). For future games, each player should take an entire suit and choose five cards to pass to the player to the right, then choose three out of the thirteen cards (five from the player to the left and eight original cards) to return to the main deck.

Shuffle each player deck of ten cards. Shuffle the main deck and split into three stacks with only the top card of each stack face up. Determine who should go first.

value	cards	expression
1	K♣ Q♣	$(13 - 12)$
2	Q♣ 10♣	$(12 - 10)$
	Q♣ 10♣ J♥	$(12 + 10) \div (11)$
3	J♥ 8♥	$(11 - 8)$
	K♣ 10♣	$(13 - 10)$
	J♥ 8♥ K♣ Q♣	$(11 - 8) \times (13 - 12)$
4	Q♣ J♥ 8♥	$(12) \div (11 - 8)$
6	Q♣ 10♣ J♥ 8♥	$(12 - 10) \times (11 - 8)$
8	8♥ K♣ Q♣	$(8) \times (13 - 12)$
9	K♣ 10♣ J♥ 8♥	$(13 - 10) \times (11 - 8)$
11	J♥ K♣ Q♣	$(11) \times (13 - 12)$

Figure 3: Sample expressions in Golomb

4.2 The Turn

When it is your turn, draw from your deck into your hand until you have five cards in your hand (you may already have cards in your hand from the previous turn). Group your cards by suit. If you have two cards of the same suit, you may use either as its value, or you may use both together as their sum or difference. If you have more than two you can extend this in a natural way. Another way to put this is that you can, using the cards of a single suit, express any number than may be obtained by adding and/or subtracting the values of those cards. So, if you have A♥ 2♥ 4♥ in the same suit, you may express the values 1, 2, and 4 with individual cards, or $3 = 4 - 1$, $5 = 4 + 1$, $6 = 4 + 2$, or $7 = 4 + 2 + 1$. This logic is reminiscent of the *Golomb ruler*, hence the name of the game.

Given values derived from different suits, you may use multiplication and/or division to combine those values. See Fig. 3 for the possible values and derivations for the hand 8♥ J♥ 10♣ Q♣ K♣ . Note that you must always use at least two cards, so there's no way to get the value 12 even though that hand includes a queen.

4.3 Additional Rules

As soon as you buy a card from the store, the card below it in the stack is turned over. You then have an opportunity to buy it or one of the other face up cards with the cards remaining in your hand. If you were unable to play any cards this turn, you may discard as many as you like from your hand, but your turn ends there. If you played between two and four cards, you may choose at most one card remaining in your hand to bank in your vault, but keep any remaining cards in your hand for your next turn. If and only if you managed to play all five cards, draw five more and take another turn.

4.4 Scoring

At the end of the game, the winner is the player with the highest score, calculated as the sum of the following:

- 1 point per card in your deck, discard pile, hand, and vault

- 3 more points per card in your most numerous suit

- 4 more points per face card (jacks, queens, and kings)

- 5 more points per card in your vault

5 Variations

5.1 The Store

For a shorter game, divide the store into more than three stacks. For a longer game, play until all stacks are empty, not just one. Also, consider allowing a player to buy all store cards simultaneously if an expression may be formed using the store cards that matches the value of the expression formed by the player's cards.

5.2 Clock Arithmetic

Perform all operations modulo 13, or perhaps modulo 12 (and make the kings equivalent to aces).

5.3 Jokers

Add jokers to your deck(s) and treat them as having the value 0; choose suit each time you play one. Alternatively, allow jokers to add or subtract one; to square or take the square root; as factorial; to produce the additive or multiplicative inverse; any other unary operation you can imagine.

5.4 Kings

Give each king a special power. For example: discard K♥ to draw two replacement cards; discard K♣ to put up to two cards from your hand into the vault; discard K♦ to bring a card from the store directly into your hand (available to play in the same turn); discard K♠ to move a card (perhaps only the top card) from an opponent's discard pile into your own discard pile.

5.5 Scoring

In Golomb, make the final score the maximum of the four component scores instead of their sum, or make it the best component score minus the worst component score, or choose one randomly at the beginning of the game, or allow a player to change the criteria mid-game as the reward for a particularly difficult accomplishment.

5.6 Starting Decks

In RPN, since no two cards are alike (unless you have multiple decks) it is impossible for players to start with identical decks. To mitigate the unfairness of a purely random start, use drafting (which is a game skill in itself). To draft, shuffle and deal each player ten cards; choose one card to keep and pass the rest to the left. Repeat this step until you have a complete ten card deck.

If you have only one deck for two players, take all four aces, twos, threes, fours, and kings, and divide them randomly or draft them. For four players, use two decks and the same starting cards. For three players, use two decks and start with all eight aces, twos, threes, and all kings except the kings of spades (K♠). You could also choose which ranks to include in the starting draft randomly (this is analogous to the selection of available cards in Dominion).

Alternatively, if you have enough extra decks that each player can create his or her ideal deck, allow it but perhaps with some constraints such as a cap on the total number of victory points in the deck.

5.7 Final Operation

In RPN, do not use clubs as a mechanism for putting cards in your vault. Instead, whenever you buy a card from the store, you may choose to bank the last card in your expression. For scoring, only count the cards in your vault. This makes the king of spades (K♠) a less desirable card because the only way to claim its 3 victory points is to divide by 13.

5.8   Negative Numbers and Fractions

In RPN, treat red suits (hearts and diamonds) as positive integers and black suits (clubs and spades) as their additive and multiplicative inverses, respectively. In other words, clubs are negative numbers, and spades are unit fractions. Hearts and clubs both add their values; diamonds and spades both multiply their values. Since adding a negative number is the same as subtracting a positive number, and multiplying by a unit fraction is the same as dividing by its denominator, the expression logic is nearly the same as in the base game. The true differences lie in the beginning of the expression and in the target value. If the first card is a 3♣, treat it as a -3. If the first card is a 10♠, treat it as $\frac{1}{10}$, $\frac{1}{5}$, or $\frac{1}{2}$ as you choose. Similarly, to buy a club you will need at least one club in your expression because the expression's value must be a negative number; to buy any spade (other than A♠) you will need at least one spade because the expression's value must be a unit fraction. This variation may work well with the inclusion of a joker in each start deck with the joker serving to take the arithmetic or multiplicative inverse.

5.9   Complex Numbers

I need to think some more about how this would work, but you could treat some suits as real and others as imaginary.

Thanks