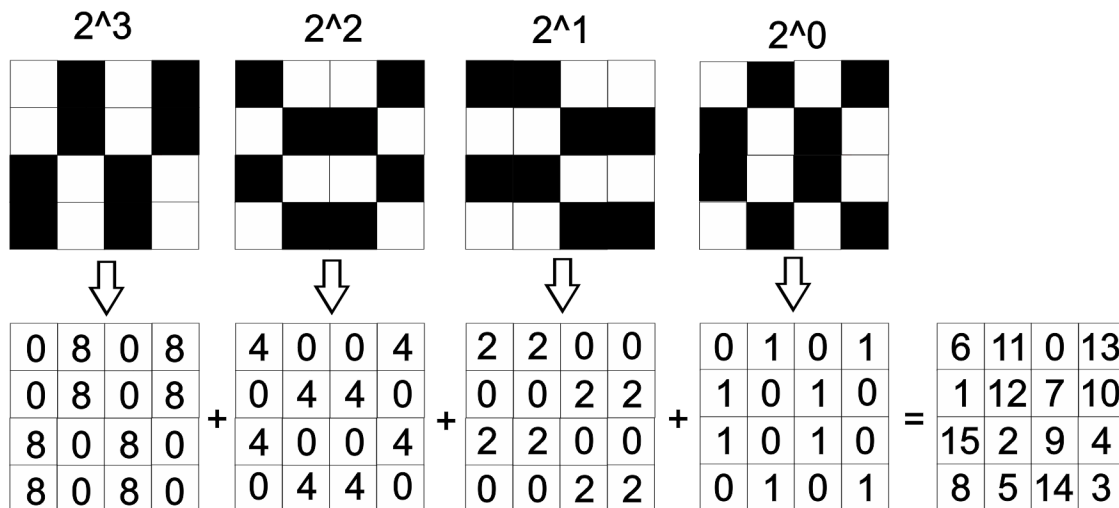


**Jain magic square space number analysis.** Ref. link: [https://en.wikipedia.org/wiki/Most-perfect\\_magic\\_square](https://en.wikipedia.org/wiki/Most-perfect_magic_square) .

The Parshvanath Jain temple in Khajuraho India has a most perfect magic square, meaning a magic square where the greatest number of possible magic sums appear. It was produced in about the year 1000. It is a 4x4 square with numbers 1 thru 16 placed in its 16 interior squares. The numbers are arranged so that all columns, main diagonals and 2x2 corner sub-squares all add up to the magic constant 34. Many other symmetrical patterns also add up to 34. It is also known as a diabolical square since it has so many ways to derive the magic constant.



The 1000 AD Jain magic square found in a temple in India is 'most perfect' and diabolical. This space # produces 4! possible shufflings,  $2^3$  reverses(blk/wht exchange) (since  $2^4$  would include reversing all 4, equivalent to a 180 deg rotation), no rotations since the two pairs are 90 deg rotations of each other and a 180 deg rotation is the same as a reverse. This equals  $24 \times 8 = 192$  Jain mg'c sq's(est.). Its space # magic constant is 30.

Since it is a 4x4 square and  $2^4=16$ , it can be seen as four two dimensional power patterns,  $2^0$ ,  $2^1$ ,  $2^2$ ,  $2^3$  as shown above by the black and white 4x4 cells. Think of each power pattern as having zeroes in the white squares and the power number in the black squares. Thus the  $2^3$  pattern has 0's and 8's. The numbers of the magic square are produced by adding the power numbers in each position together. Then the lower right Jain square numbered cell would equal  $0+0+2+1=3$ . This representation is called a space number. Space numbers subtract 4 from the magic constant changing it from 34 to 30(numbers 0 thru 15) but it remains just as magic. Looking at the four power patterns you see immediately why it is so magical. For instance you can see that all the columns and rows have 2 black squares and two white squares and each 2x2 corner subsquare has two black squares and two white squares and similarly for the two main diagonals, and so on for the other magic patterns of the Jain magic square. In addition we can shuffle the power patterns(such as let the  $2^0$  and  $2^2$  power patterns exchange positions) however we like and the square stays most perfect, only the numbers change positions. We can also reverse the black and white of any power pattern and the magic is retained.

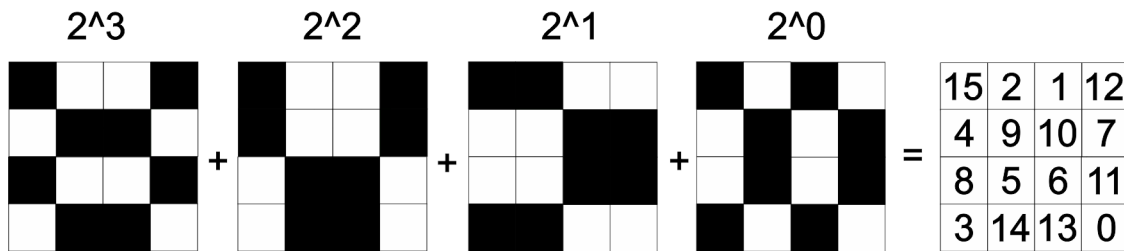
Rotation of individual power patterns is not allowed as the two pairs of the power patterns are already ninety degree rotations of each other. Thus with shuffling and reversing we can have  $24 \times 16 = 384$  different number arrangements of the Jain magic square. Of course many of these will be rotations or reflections of others, but it shows the ease with which space numbers can be used to make more magic squares. All of these different magic squares can be thought of as a single space number. The  $4 \times 4$  magic square shown in Albrecht Durer's 1514 engraving 'Melancholia' can be analysed the same way and produces a very similar space number with 384 number arrangements. Other magic squares could be produced using space numbers. For instance a  $12 \times 12$  magic square would need to use power patterns in base 2 and base 3.

**The Durer magic square as a space number**

Here is an illustration of Albrecht Durer's magic square from his 1514 engraving Melancholia. It is not quite as magic as the Jain magic square but has several interesting features detailed here

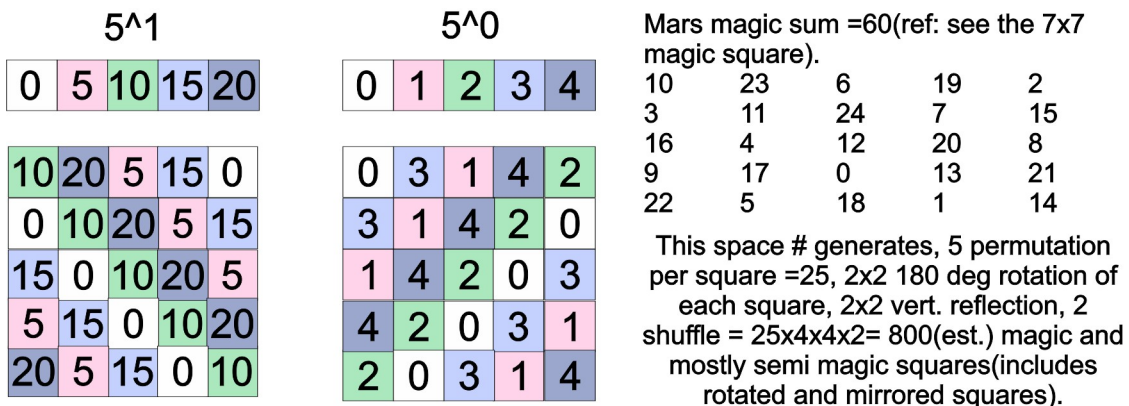
[https://en.wikipedia.org/wiki/Magic\\_square#Albrecht\\_D.C3.BCcrer.27s\\_magic\\_square](https://en.wikipedia.org/wiki/Magic_square#Albrecht_D.C3.BCcrer.27s_magic_square)

The four power patterns can be exchanged, and black and white reversed for a total of  $4! \times (2^4) = 192$  new magic squares. There may be some duplicates since reversing all four is the same as a 180 degree rotation.



Albrecht Durer,s magic square, from his engraving Melancholia as a  $4 \times 4$  space number(space # magic constant = 30) produces 192(est.) Durer magic squares.

The figure below shows the Mars magic square as a base 5 space number. Since 5 is prime this is the only way it can be shown as a space number. It shows how much simpler an odd order magic square can be as opposed to an even order magic square.



Below we show the Sol magic square as a space number. Since it's prime factors are 2 and 3 it is

necessary to use both binary and trinary base numbers to break it into a space number. From this you see it has some complexity since it contains both even order and odd order properties. All even order magic squares are more complicated than odd order magic squares.

$3^1 \#s \times 4$

0	12	24
---	----	----

0	24	0	24	24	0
0	0	24	24	0	24
12	12	12	12	12	12
12	12	12	12	12	12
24	24	0	0	24	0
24	0	24	0	0	24

$3^0 \#s \times 4$

0	4	8
---	---	---

4	4	0	8	8	0
4	8	0	0	4	4
4	0	0	0	8	8
4	4	8	8	4	0
0	4	8	8	0	8
8	4	8	0	0	4

$2^1$

0	2
---	---

0	2	2	0	2	0
2	2	2	2	2	0
2	0	2	2	2	2
0	2	0	0	0	0
0	0	0	0	0	2
2	0	0	2	0	2

$2^0$

0	1
---	---

1	1	0	1	0	0
0	0	0	1	1	1
0	1	1	0	0	1
1	1	1	0	0	0
0	0	1	0	1	1
1	0	0	1	1	0

The 6x6 Sol magic square(see the ref. given on the binary sn 8x8). Magic constant=105. This base 2 and base 3 space # generates 1,,990,656(est.) magic and semi magic sq's.  
[https://en.wikipedia.org/wiki/Magic\\_square](https://en.wikipedia.org/wiki/Magic_square)

5	31	2	33	34	0
6	10	26	27	7	29
18	13	15	14	22	23
17	19	21	20	16	12
24	28	9	8	25	11
35	4	32	3	1	30

Next we show the Venus 7x7 magic square below. It can only be shown as a base 7 space number since 7 is prime. You can see it is very simple with a diagonal symmetry of numbers similar, to the order 5 Mars magic square.

$7^1$

0	7	14	21	28	35	42
---	---	----	----	----	----	----

21	42	14	35	7	28	0
0	21	42	14	35	7	28
28	0	21	42	14	35	7
7	28	0	21	42	14	35
35	7	28	0	21	42	14
14	35	7	28	0	21	42
42	14	35	7	28	0	21

$7^0$

0	1	2	3	4	5	6
---	---	---	---	---	---	---

0	4	1	5	2	6	3
4	1	5	2	6	3	0
1	5	2	6	3	0	4
5	2	6	3	0	4	1
2	6	3	0	4	1	5
6	3	0	4	1	5	2
3	0	4	1	5	2	6

The 1531 ed. of De Occulta Philosophia by Heinrich Cornelius Agrippa has this 7x7 magic square as planet Venus (astrology). Presented here, 11, 2017, as a base 7 space number, magic constant =168. Ref. [https://en.wikipedia.org/wiki/Magic\\_square](https://en.wikipedia.org/wiki/Magic_square)

21	46	15	40	9	34	3
4	22	47	16	41	10	28
29	5	23	48	17	35	11
12	30	6	24	42	18	36
37	13	31	0	25	43	19
20	38	7	32	1	26	44
45	14	39	8	33	2	27

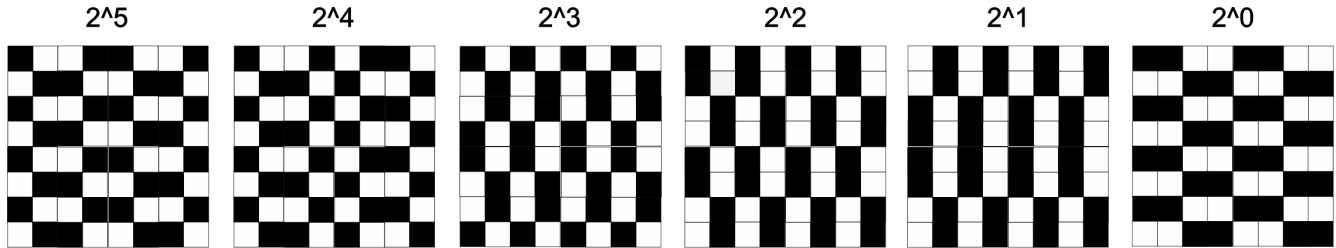
Positions for new magic and semi magic squares: 7 permutation per square =49, 2 180 rotate per square =4, 2 reflection per square =4, 2 shuffle. This gives 49x4x4x2 =1,568(est.) magic and mostly semi magic squares. For semi magic squares at least the columns and rows all add up to the magic constant.

The 8x8 magic square below is presented as a binary space number with 6 power patterns. You can see that the columns and rows all have 4 black cells and 4 white cells. For each column and row the black cells must intersect equal numbers of black and white cells and this is also true for the white cells. The same is true for the two main diagonals. This is the equal fractional intersection rule for a space number to uniquely number all the cells and be magic when combined. By considering the power pattern symmetry operations of rotations, shuffles(exchanges), reverses of black and white we get 92,160 magic squares from this space number.

Pandiagonal 8x8 magic square, sum 252, presented here as a binary space number. By Willem Barink, 4,2007.  
 Ref. [https://en.wikipedia.org/wiki/Magic\\_square](https://en.wikipedia.org/wiki/Magic_square)

61	3	12	50	45	19	28	34
4	58	53	11	20	42	37	27
51	13	2	60	35	29	18	44
10	52	59	5	26	36	43	21
63	1	14	48	47	17	30	32
6	56	55	9	22	40	39	25
49	15	0	62	33	31	16	46
8	54	57	7	24	38	41	23

Looking at the binary space number patterns you can see immediately why it is so magical. Every row, column, main diagonal, pandiagonal, 2x2 subsquare, linear group of 4, 4x4 subsquare contains equal numbers of black and white squares. The space number generates 6! possible shufflings, and 2^6 black/w color reverses which equals 46,080(est. min.) order 8 magic squares.



The same 8x8 magic square above can be analyzed with a base 8 space number. Shown below you can see it is really four 4x4 magic squares. On some permutations it probably produces many semi magic squares while the binary 8x8 space number above produces only fully magic squares. The base 8 patterns each combine 3 of the binary power patterns into one power pattern. Thus less freedom exists to move the power patterns around by shuffling and color permutation. This is partly made up for since more permutations are allowed with the base 8 pattern.

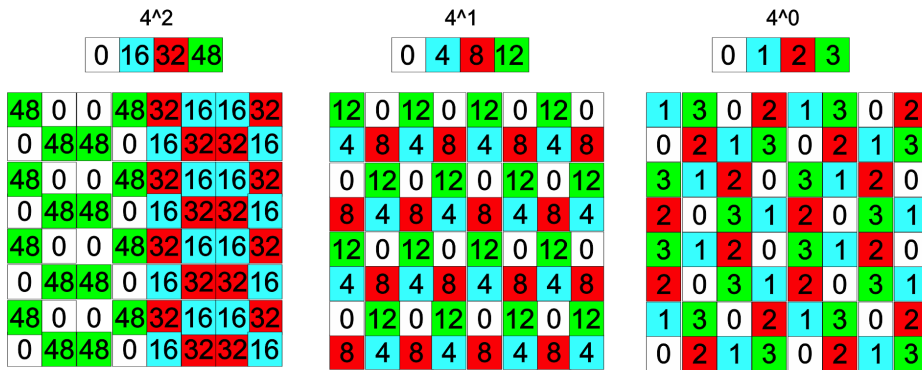
8^1								8^0							
0	8	16	24	32	40	48	56	0	1	2	3	4	5	6	7
56	0	8	48	40	16	24	32	5	3	4	2	5	3	4	2
0	56	48	0	16	40	32	24	4	2	5	3	4	2	5	3
48	8	0	56	32	24	16	40	3	5	2	4	3	5	2	4
8	48	56	0	24	32	40	16	2	4	3	5	2	4	3	5
56	0	8	48	40	16	24	32	7	1	6	0	7	1	6	0
0	56	48	0	16	40	32	24	6	0	7	1	6	0	7	1
48	8	0	56	32	24	16	40	1	7	0	6	1	7	0	6
8	48	56	0	24	32	40	16	0	6	1	7	0	6	1	7

The Barink Pandiagonal 8x8 magic square, as a base 8 space number. (See the 8x8 magic square used for the binary space #) [https://en.wikipedia.org/wiki/Magic\\_square](https://en.wikipedia.org/wiki/Magic_square)

61	3	12	50	45	19	28	34
4	58	53	11	20	42	37	27
51	13	2	60	35	29	18	44
10	52	59	5	26	36	43	21
63	1	14	48	47	17	30	32
6	56	55	9	22	40	39	25
49	15	0	62	33	31	16	46
8	54	57	7	24	38	41	23

Color permutation = 4x8x4x8, rotation = 2(180 deg only),  
 vert. reflection = 2 and shuffling = 2 this base 8 space #  
 generates 4,608(est.) magic and mostly semi magic squares (does not count rotations and mirror images).

We can also use a base 4 space number to analyze the Barink 8x8 magic square. This results in the figure below showing three 4x4 power patterns that add together to equal the 8x8 magic square. Using a different but compatible base produces a different set of magic squares where the space number symmetry operations of power shuffling, rotation, reflection and permutation are performed. It is interesting that the 4x4 pattern produces the greatest number of new magic squares.



The Barink Pandiagonal 8x8 magic square, as a base 4 space number. See [https://en.wikipedia.org/wiki/Magic\\_square](https://en.wikipedia.org/wiki/Magic_square) Operations (tries to eliminate mirror images and rotations) 3!=6 shuffling, 16x4x32 circular permutation, 2x2 180 deg rotation, 2 reflection 98,304(est.) magic & mostly semi magic sq's.

61	3	12	50	45	19	28	34
4	58	53	11	20	42	37	27
51	13	2	60	35	29	18	44
10	52	59	5	26	36	43	21
63	1	14	48	47	17	30	32
6	56	55	9	22	40	39	25
49	15	0	62	33	31	16	46
8	54	57	7	24	38	41	23

This is but a tiny fraction of the total number of possible 8x8 magic squares which is a large number. Using space numbers this huge number can be reduced by about 10^5 since each space number representation could be taken to represent all the magic squares that can be generated from it.

**Introduction to the concept of Space Numbers**

*Space numbers are a democracy of numbers where all numbers and dimensions have equal importance. While appearing complex Space Numbers are wonders of simplicity and symmetry.*

Historical: The idea for space numbers came to me one morning around 1968, waking up at a Mining camp in the Colorado mountains, altitude over 12000 feet. I was learning to work with an IBM 360. It filled a whole room. Now you have many times the capacity of a 360 in a smart phone.

Looking at the walls there appeared different checkerboard patterns and the idea that they could combine to create numbers when overlapped. Several years later this was published as “*Number Patterns in More than One Dimension*” in the Journal of Recreational Mathematics, edited by my friend Harry L. Nelson, working at Lawrence Livermore labs in California.

**Space Numbers**

I have continued to work with this idea, now calling them space numbers Sn, and Sn() which is the set of all possible space numbers. I have devised some entertaining space number computer games. These games use rules of symmetry to recombine the space number patterns in many interesting ways to create different symmetrical and random number lists.

**A Simple Description of Space Numbers and their symmetries**

A space number consists of a set of power patterns each occupying an identical cellular geometry with one base position number, b^p, in each corresponding cell of each separate cellular geometry. The power patterns are added together to make a combined identical shaped cellular geometry. This is done by adding the number in identical cells of each power pattern. The cells of the combined pattern can be a linear list, like a line of square cells, or a square grid of cells or a cubic grid of cells, or a four or larger dimension grid of cells, or any symmetrical cell geometry such as the surface of a dodecahedron with each face divided into a pentagonal grid of cells. The power patterns might be rotated, reflected, permuted, and shuffled(where b^1 becomes b^3 and b^3 becomes b^1 for instance before being recombined. These operations are only permitted, assuming we always want a complete numbering of the cells, depending on the design of the power patterns and the symmetry of the cellular geometry.

According to this description a group of cells could be randomly scattered but would still have translation (shuffle, swap, or exchange) symmetry since it could be superimposed over itself, and permutation symmetry if some path or linear set of paths connect the cells in a way that respects their power pattern symmetry. All sorts of other detailed dimensional operations within a given power pattern. are possible depending on design.

### Mathematical Description

Given positive integer base  $b$ , a power pattern is a list of  $n$  numbers 0 thru  $n-1$  filling the  $n$  cells (1 number per cell) of a symmetric geometric pattern, where you take all the ones positions in the base  $b$  number list as the  $b^0$  power pattern, all the  $b^1$  positions as the next power pattern up to  $b^{(k-1)}$ , as the final power pattern. For a single base number  $b$  and a simple geometry, line, square, cube, etc.,  $b^k = n$  is the total number of cells in the pattern. The power patterns consist of positive base integers and zeroes so that the integers and zeroes can be given colors where the zeroes are usually white.

A 4x4 square grid of cells contains 16 cells. Using a binary base the numbers 0 thru 15 can be gotten using four base 2 power patterns,  $2^3$ ,  $2^2$ ,  $2^1$ ,  $2^0$  of 4x4 cells in each pattern. The  $2^3$  pattern will have eight 8's and eight 0's in its 16 cells, while the  $2^2$  pattern will have eight 4's and eight 0's, and the  $2^1$  pattern will have eight 2's and eight 0's and the  $2^0$  pattern will have eight 1's and eight 0's. If the power patterns have symmetry then rotation, reflection, reverse of 0's and 1's (2 color permutation) are all possible. In addition exchange (shuffle, or swap) of powers is always possible since this is translation symmetry. For instance the  $2^0$  power pattern could exchange values with the  $2^3$  power pattern so that 1's and 8's in the two patterns would change places. If the power pattern numbers do not have rotation or reflection symmetry then the only allowed symmetrical operations are translation, and usually permutation of colors.

The base  $b$  could consist of a combined set of bases. For instance with a 6x6 square of 36 cells the numbers 0 thru 35 can be gotten with four power patterns by using two base 2 power patterns and two base 3 power patterns. This is done by multiplying rightmost base position number with leftmost base power numbers as required. This definition leaves open all sorts of other ways to form the number list and thus the base positions, such as complex numbers, negative numbers functions and so forth.

### Linear Example

The simplest  $S_n$  examples are linear power patterns or just a list of numbers in a line. The simplest linear example employs the binary base using zeroes and ones. Write down eight binary numbers in a column. This is an element of  $S_n(2 \times 2 \times 2)$  shown here with decimal equivalents. A second column is shown with the decimal values of each binary pattern. Adding these decimal values for each triplet produces the column of decimal numbers. The last three columns show the power patterns separated.

Binary	Dec. equiv.	Power patterns	$2^2$	$2^1$	$2^0$
000 = 0	000 = 0		0	0	0
001 = 1	001 = 1		0	0	1
010 = 2	020 = 2		0	1	0
011 = 3	021 = 3		0	1	1
100 = 4	400 = 4		1	0	0
101 = 5	401 = 5		1	0	1
110 = 6	420 = 6		1	1	0



dimension. A space number with at least a rotation symmetry must have Ds. Ds is the principle that allows us to mix different base numbers to produce space numbers for any number that can be factored.

**Intersection rule:** *For binary power patterns any pair of power patterns must overlap to produce an equal number of the four paired numbers 00,01,10,11.* Example call the four 4x4 square power patterns A, B, C and D. Each of these is divided into a group of 8 white and 8 colored cells. The white cells are zeros and the colored cells are  $2^0, 2^1, 2^3$ , or  $2^4$ . Thus  $A \wedge B$  must produce the following pairs of overlapped cells (where  $\wedge$  means intersection), four each of 00, 01, 10 and 11.

Then  $A \wedge B \wedge C$  must produce two groups of 000, 001, 010, 011, 100, 101, 110, 111 to satisfy the pair rule for  $A \wedge B$ ,  $A \wedge C$  and  $B \wedge C$ . Then  $A \wedge B \wedge C \wedge D$  has to produce the full 0000 thru 1111 sequential binary numbering of all 16 cells. Any exchanging or shuffling of this intersection such as  $D \wedge A \wedge C \wedge B$  does the same but moves the numbers around in the solution pattern. If the patterns all have the correct intersections for the operations rotation, mirror, permutation the result is a balanced set.

At this point I must end. Much more is known about space numbers. Time being available some of this mathematical and practical application knowledge will be published. Anyone wishing to contribute ideas or their own discoveries can contact me by email at [dengel99@aol.com](mailto:dengel99@aol.com).