

Cross sections of three dimensional hyperbolic tilings *by Vladimir Bulatov (bulatov.org)*

Three dimensional tiling of hyperbolic 3-space is difficult to visualize. In the standard models of hyperbolic space like Poincaré ball or upper half space the tiles become very small very fast near the boundary of the hyperbolic space. The complete tiling is actually quite boring – just a round ball or half space. Taking cross section of the tiling with hyperbolic planes gives picture similar to images of two dimensional hyperbolic tiling. However there is very special kind of surface in the hyperbolic space - horosphere. It is surface of zero curvature and it's intrinsic geometry is geometry of euclidean plane. All horospheres are equivalent to each other. The horospheres in the ball model are spheres tangent to the ball boundary. Horospheres in the upper half space model are spheres tangent to the boundary plane. Especially convenient are horospheres, which have tangent point at infinity. These are just planes parallel to the boundary plane. The cross section of some three dimensional hyperbolic tiling by such horosphere are illustrated here. Nice property of such cross section is the constant scale of the pattern. It is consistent with the euclidean nature of the horosphere geometry. Another nice property is that the pattern is not periodical and does not repeat itself exactly and also keeps the similar uniform appearance everywhere.

