Some Paper Puzzles[1]

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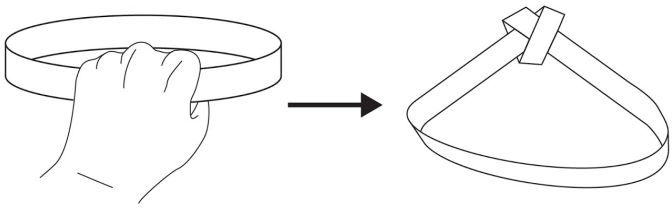
Paper Knot Puzzle

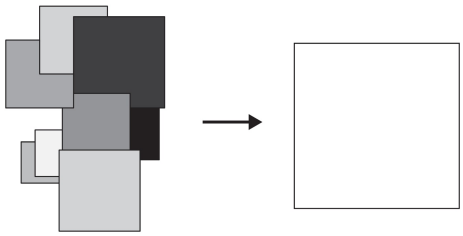
Make a band out of a strip of paper. Tie a knot in the band without cutting the band open (that is, without cutting the band along its width)!

Hint: What kind of a band is needed to begin with?



Overlapping Papers Puzzle

Arrange square sheets of paper one on top of the other to form a square. What is the smallest number of sheets needed to ensure that no sheet is fully visible? There are no other limits to this puzzle.



The following figure shows a counter-example using four different colored square sheets of paper. You can see that there is one sheet which is totally visible, which does not fulfil the requirement of the solution.

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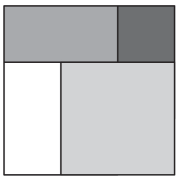
[1] Adapted with permission from:

Ilan Garibi, David Goodman and Yossi Elran, *"The Paper Puzzle Book: All You Need is Paper"*, World Scientific Press, New Jersey, 2017



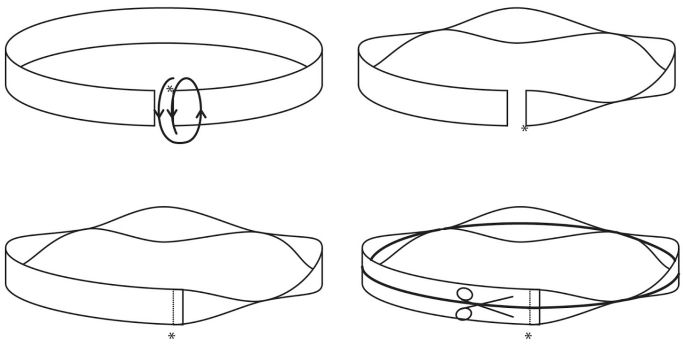
Follow up challenge: Arrange square sheets of paper, all the same size, one on top of the others to form a square. What is the smallest number of sheets needed if it is required that no sheet is fully visible?

Quadrisecting Rectangles into Triangles Puzzle

Find at least six different triangles that you can fold from a sheet of printer paper, where each triangles area is a quarter of the area of the whole sheet. You are allowed to use only two fold lines. A 'pinch' made to mark a certain point on the paper is not considered a fold line.

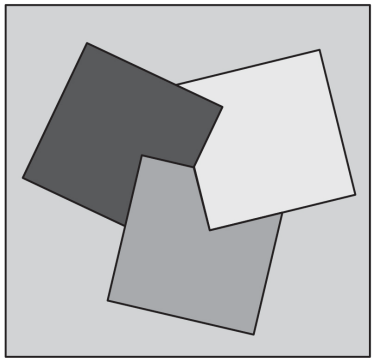
Paper Knot Puzzle Solution

The trick lies in the preparation of the paper band before you start cutting. The paper band has to be half-twisted three times. When cutting along the center line and opening up, a band with a knot in it is created. This is a less-known property of Möbius bands. Generally speaking, making *n* half-twists in a strip of paper and taping its ends will form either a one-sided or two-sided Möbius band, depending on the parity (odd *n* generate single-sided bands while even *n* generate two-sided bands). When cutting along the center line of these bands, either one (for odd *n*) or two (for even *n*) bands are created, with $\frac{1}{2}(n-1)$, for odd n, or $\frac{1}{2}(n-2)$, for even *n,* knots in them.

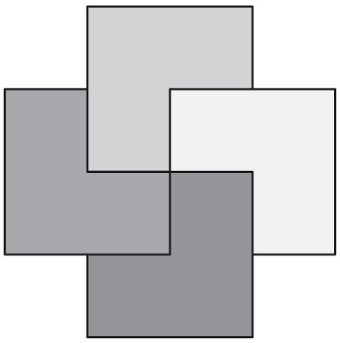


Overlapping Papers Puzzle Solution

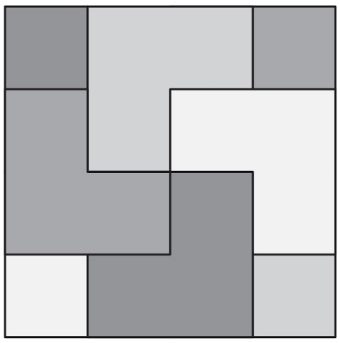
Four sheets is the minimal amount. Three that cover each other and the last sheet, large enough to encompass this assemble and placed behind them, solve the puzzle.



When the sheets have to be the same size, the minimum number of identical sheets you need is eight. The building block is the mutually overlapping 'plus sign' shape, shown below, made out of four sheets of paper. Add four more sheets for the corners.



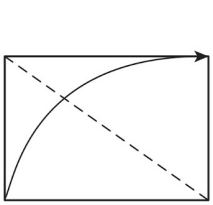
Four sheets to cover each other.



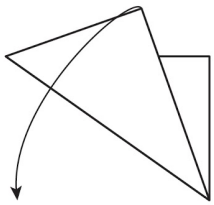
Four more underneath assemble the background square.

Quadrisecting Rectangles into Triangles Puzzle Solution

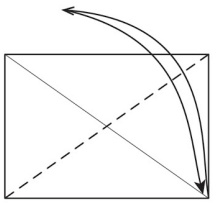
There are six different triangles. The first two emerge when you fold the two diagonals:



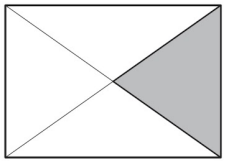
Fold along the diagonal.

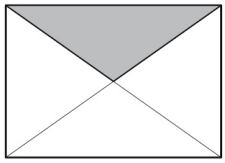


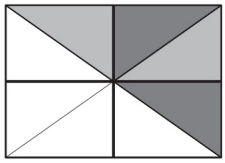
Unfold.



Fold the other diagonal. Unfold.

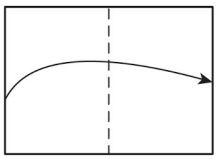




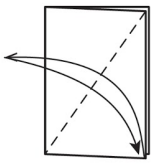


These two triangles are equal in area, and together they are half the area of the rectangle. A simple way to show this is to divide the rectangle into four smaller ones. You can see that each triangle is divided into two triangles and all are half of the quarter rectangle.

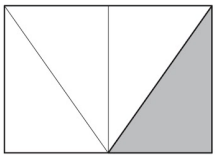
By folding the paper in half, you get two more triangles:



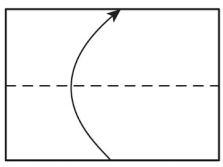
Fold edge to edge.



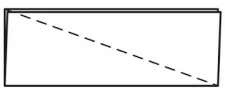
Fold along the diagonal of the smaller rectangle. Unfold all folds.



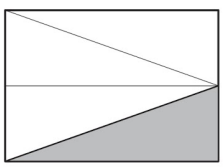
All triangle are half the area of half the rectangle.



Fold edge to edge.

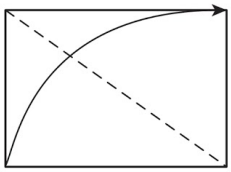


Fold along the diagonal of the smaller rectangle. Unfold all folds.

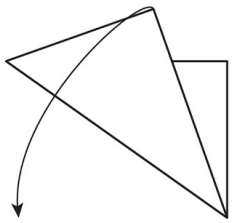


All triangles are half the area of half the rectangle.

The last two triangles are shown below:

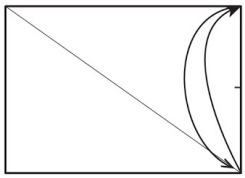


Fold along the diagonal.

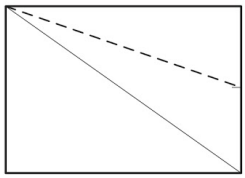


Unfold.

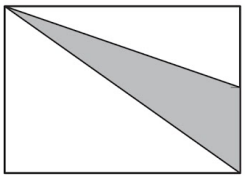
Solution A:



Mark with a pinch at the center of the right edge.

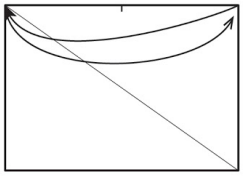


Fold along the line that connects the top left corner to the pinch.

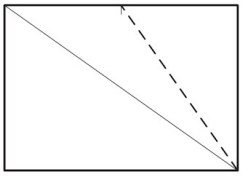


This triangle has the area of a quarter rectangle.

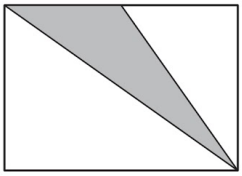
Solution B:



Mark with a pinch at the center of the top edge.



Fold along the line that connects the bottom right corner to the pinch.



This triangle has the area of a quarter rectangle.

The explanation is based on the way you calculate the area of an triangle. You can see that both triangles in the half-rectangle triangle have the same base length and the same height, hence the same area.