

Puzzle Fonts About Puzzles

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Abstract

We present five recent puzzle fonts — where reading glyphs in the font require solving a puzzle — that illustrate five different puzzles/puzzle games, each with a corresponding mathematical result. Each font is an open-source interactive web application that lets the user write messages in the font, and then solve the resulting puzzles (or send them to a friend to solve), revealing the message.

1 Introduction

We have been developing a growing series of mathematical and puzzle fonts — which recently reached 30 different typefaces¹ — that you can interact with in web apps.² Every one of these fonts is *mathematical* in the sense that it illustrates a mathematical theorem or open problem. Most of our typefaces also offer one or more *puzzle* fonts, where reading the text requires solving a mathematical puzzle. These fonts were recently featured in *The New York Times* [Rob21].

In this paper, we describe five recent typefaces that share the theme of both having puzzle fonts and being *about* puzzles. Figure 1 gives a visual overview. The first typeface is about a puzzle video game, Tetris, while the other four typefaces are about pencil-and-paper puzzles: Sudoku, Yin-Yang, Path Puzzles, and Tatamibari.

The presentation corresponding to this paper is available on YouTube.³ All fonts presented here, including the slides and code that generates all shown figures, are free and open source, with code available on GitHub.⁴

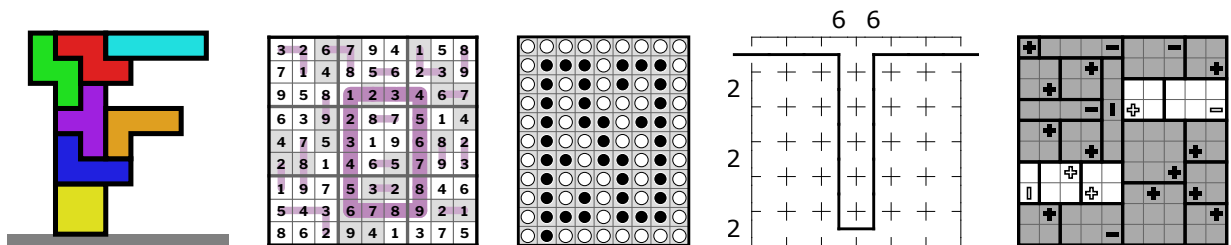


Figure 1: FONTS written in the five puzzle typefaces described here: Tetris (Section 2), Sudoku (Section 3), Yin-Yang (Section 4), Path Puzzles (Section 5), and Tatamibari (Section 6).

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¹A *typeface* is a collection of multiple related *fonts*.

²<https://erikdemaine.org/fonts/>

³<https://youtu.be/K6M3ELHr5Ls>

⁴<https://github.com/edemaine/talk-puzzle-fonts-about-puzzles/>

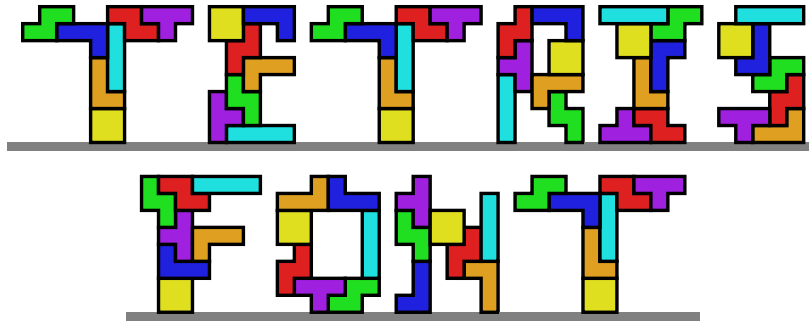


Figure 2: TETRIS FONT written in the Tetris solved font.

2 Tetris

The *Tetris typeface*⁵ represents each letter by a stacking of a complete set of oriented tetrominoes, that is, one of each of the possible pieces in the Tetris video game. Figure 3 shows the stackings for the entire alphabet. Each stacking is designed to be executable in Tetris physics, with pieces stacked in order and stopping when they hit a previously placed piece. Figure 4 shows how the pieces can be ordered so that their falling in sequence produces the letters in Figure 3. The web app offers an animated font that simulates Tetris gameplay.

Displaying all the pieces splayed out in fall order, as in Figure 4, is one way to make puzzles with a Tetris puzzle font. For example, can you read the secret messages in Figures 5 and 6?

Another way to make puzzles with the Tetris font is to hide the individual pieces, and ask the viewer to figure out how the Tetris pieces exactly tile the letter-shaped regions. These packing/tiling problems can be quite challenging; when developing the font, we made extensive use of the BurrTools software⁶ which can solve such puzzles by brute force.

We presented this font in a paper that proved a new mathematical result about Tetris: the perfect-information game is NP-complete even with just 8 columns or 4 rows [ACD⁺20].

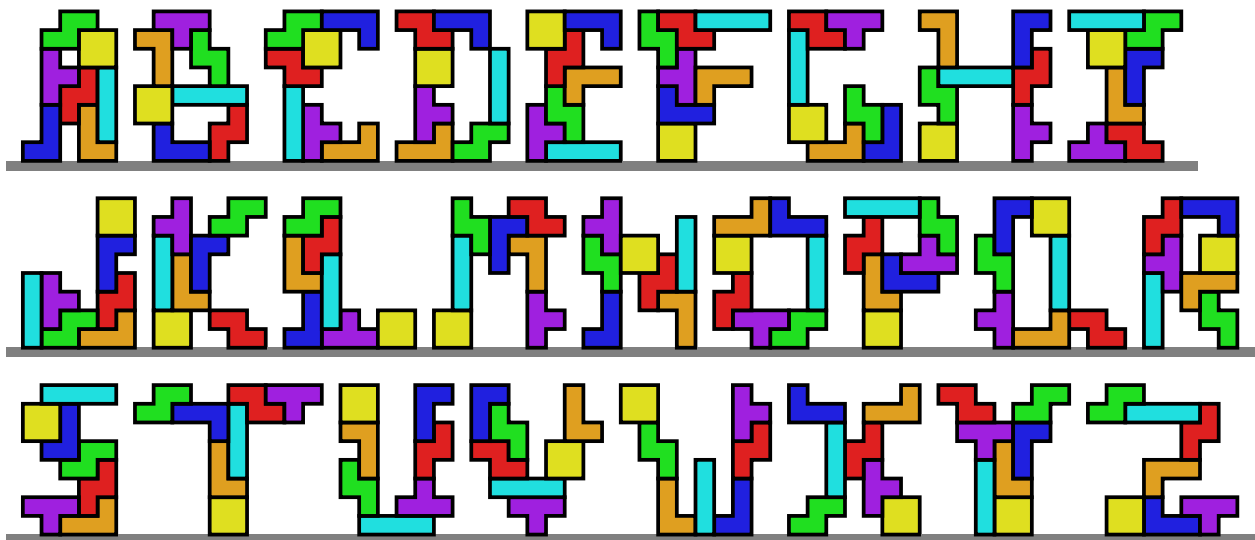


Figure 3: The entire alphabet in the Tetris solved font.

⁵<https://erikdemaine.org/fonts/tetris/>

⁶<http://burrtools.sourceforge.net/>

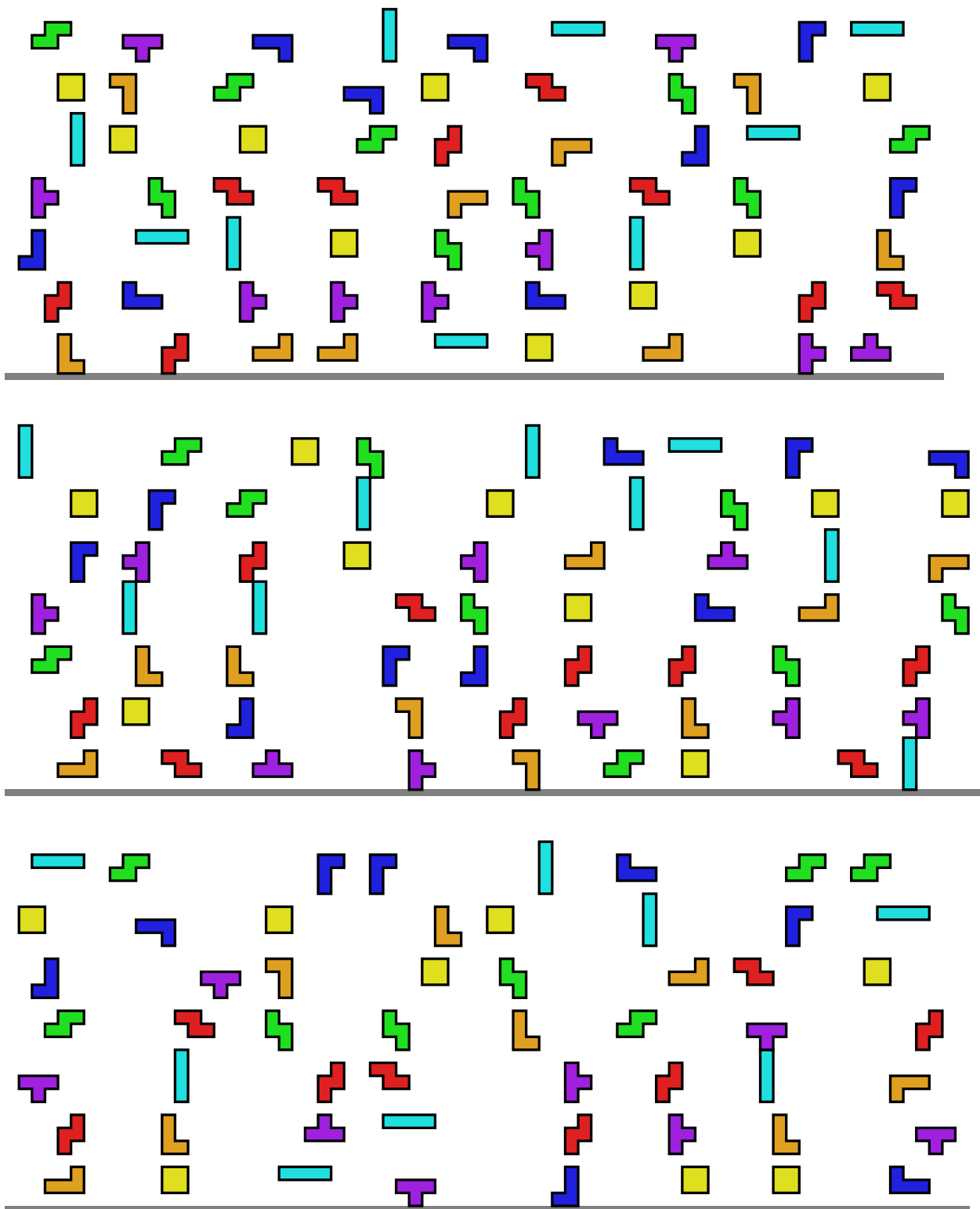


Figure 4: The entire alphabet in the Tetris falling-puzzle font.

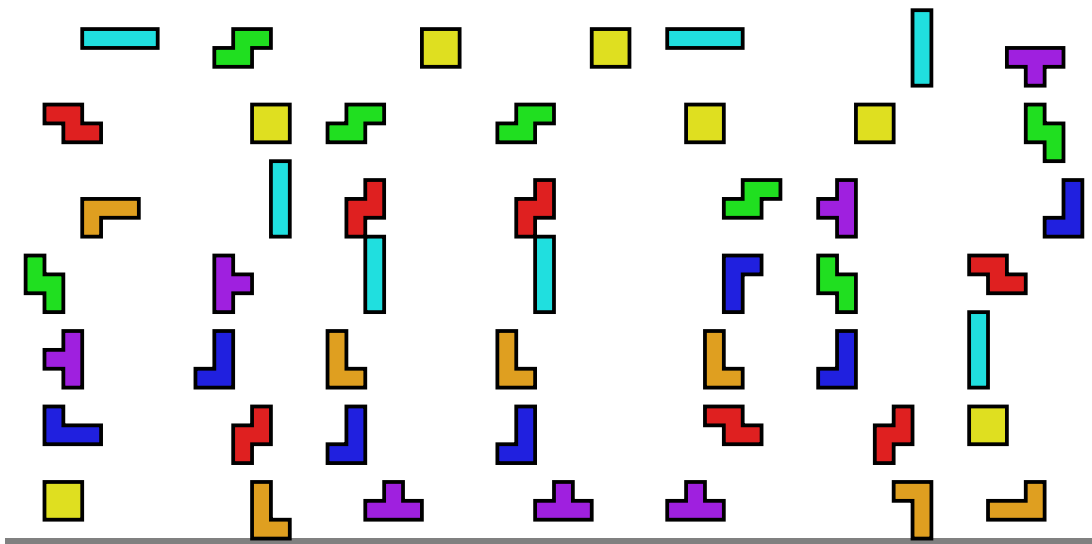


Figure 5: What message do you get if each piece falls straight down until it hits one of the other pieces (or the floor)? The solution is in Figure 25.

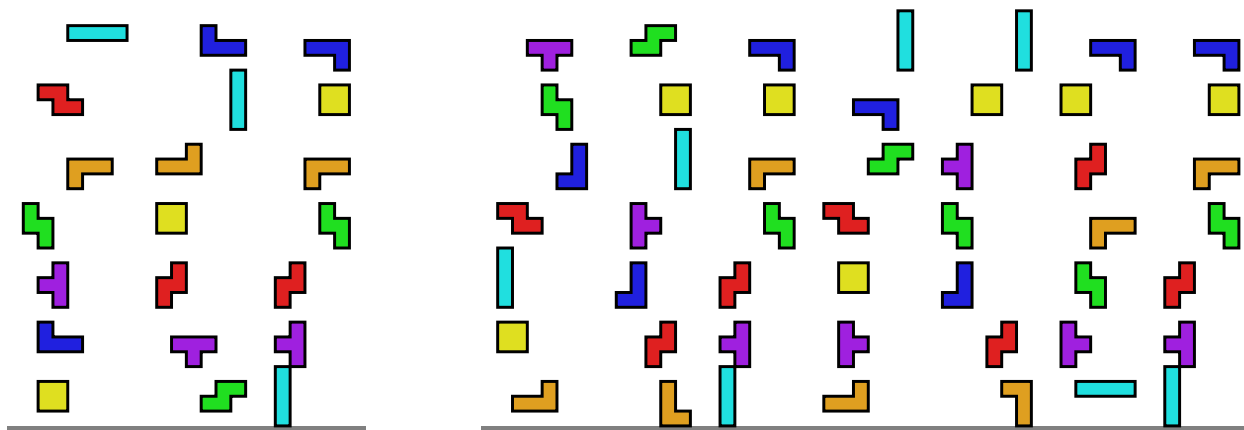
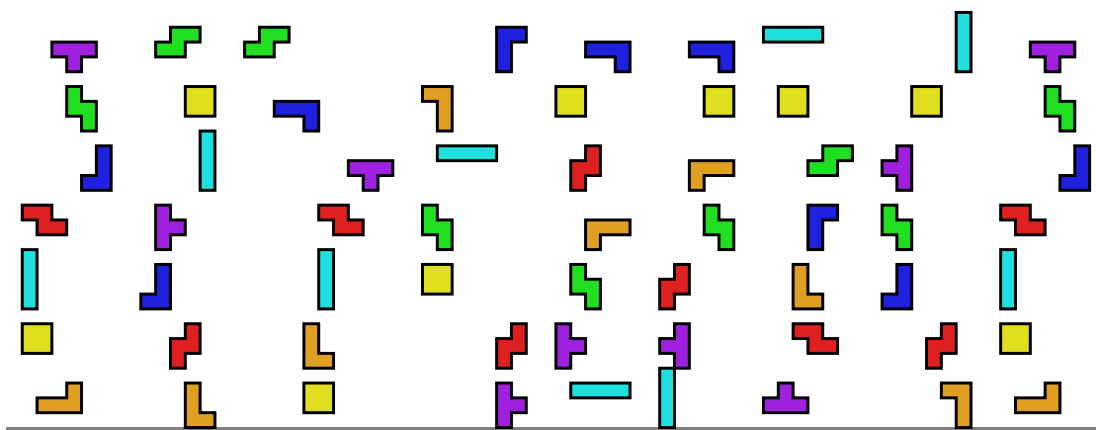


Figure 6: What message do you get if each piece falls straight down until it hits one of the other pieces (or the floor)? The solution is in Figure 26.



Figure 7: Can you tile each letter with exactly the pieces on the left? Solutions are in Figure 27.



Figure 8: Can you tile each letter with exactly the pieces on the left? Solutions are in Figure 28.

3 Sudoku

The *Sudoku typeface*⁷ draws a letter of the alphabet in a Sudoku puzzle, by connecting consecutive numbers in the solution (connecting all 1s to 2s, all 2s to 3s, etc.), and drawing the longest path among these connections. Figure 9 shows examples of puzzles and their solutions, and Figure 10 shows the full alphabet. We designed the intended paths by hand, and used our own brute-force computer search to find $81 = 9 \cdot 9$ compatible solutions. Then we reduced each solution to a corresponding minimal puzzle that can be uniquely solved by a human (without lookahead) by randomly removing locally derivable clues.

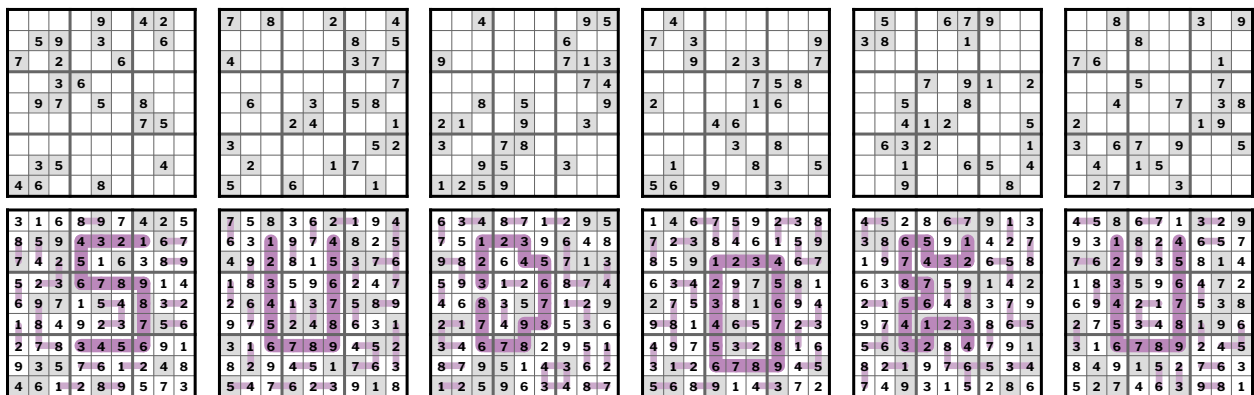


Figure 9: SUDOKU written in one of the 81 Sudoku puzzle fonts (top) and the corresponding solved font (bottom). The bottom figure highlights connections between consecutive numbers, with thick lines denoting the longest path of such connections.

⁷<https://erikdemaine.org/fonts/sudoku/>

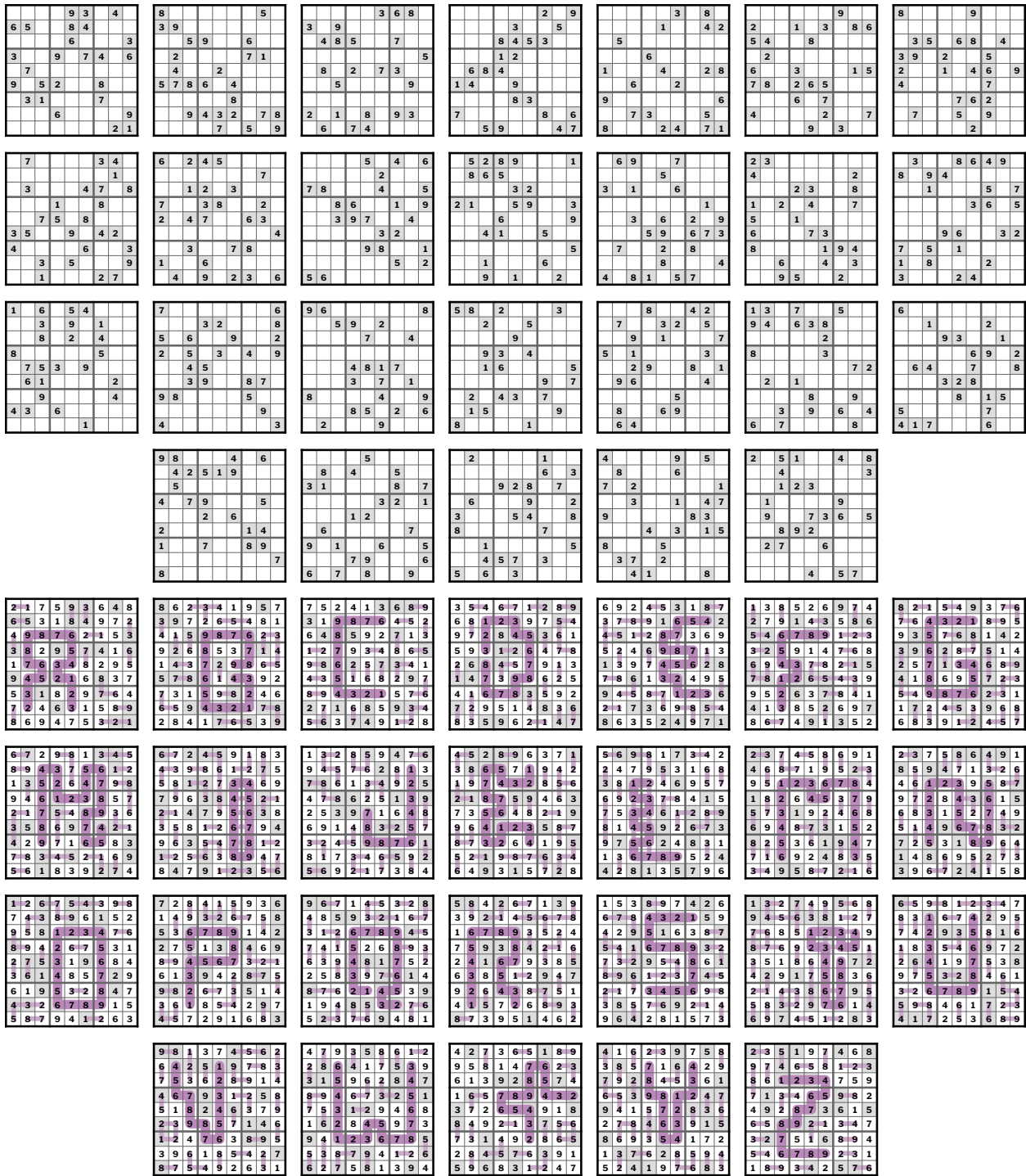


Figure 10: The entire alphabet written in one of the 81 Sudoku puzzle fonts (top) and the corresponding solved font (bottom).

The 81 Sudoku puzzle fonts let us hide messages in Sudoku puzzles. Figures 11 and 12 give two puzzles for you to try. Alternatively, try solving some puzzles on the interactive web app.

Sudoku also has a corresponding complexity result: it is NP-complete and, even stronger, ASP-complete [YS03]. In fact, finding the longest path among a square grid of connections is also NP-complete [IPS82].

6	2			1			9	
	4		7					
9						7		4
			6	4		9		
5		3					6	
							5	1
			3					9
2		6			8	5	3	
							1	2

9					1	5		
3	2		7			6	9	
		6		8				
2	9	7						
	4				5		2	
						3		2
8		2	4				7	
				5			1	

Figure 11: Can you solve the Sudoku puzzles, connect consecutive numbers, and find the longest paths, to reveal the hidden message? Or solve interactively on the web. The solution is in Figure 29.

			5				8	
	8				1	5		
	9		7			1		6
		6	2	8		4	5	
								9
4			6			7		
5	4					2	1	
			1				7	
	7	2						8

		1	7					6
					3		9	1
4			7		1			
	8			5		4	6	2
							5	
			2	1	6	3		
				2	9		8	
	2			3				
8	5	9						

	8			5				2	
			3						8
	6	7							1
	5		2	8					9
	4			7				1	
6					9	2	7	4	
5									7
			9					5	6
		9							

					6		8	9
					4			2
	8			3		5	7	
	7		8		3	6		
			9	5				4
	9				7			
6				2				
7			4		5			
			6		1	3		

			1			9	4		
			4	3					9
						6		7	
			1		6	8			
	2							3	
	9	5	1		3		4	6	
			3		5				
	5		6						
	7	8					5	9	

Figure 12: Can you solve the Sudoku puzzles, connect consecutive numbers, and find the longest paths, to reveal the hidden message? Or solve interactively on the web. The solution is in Figure 30.

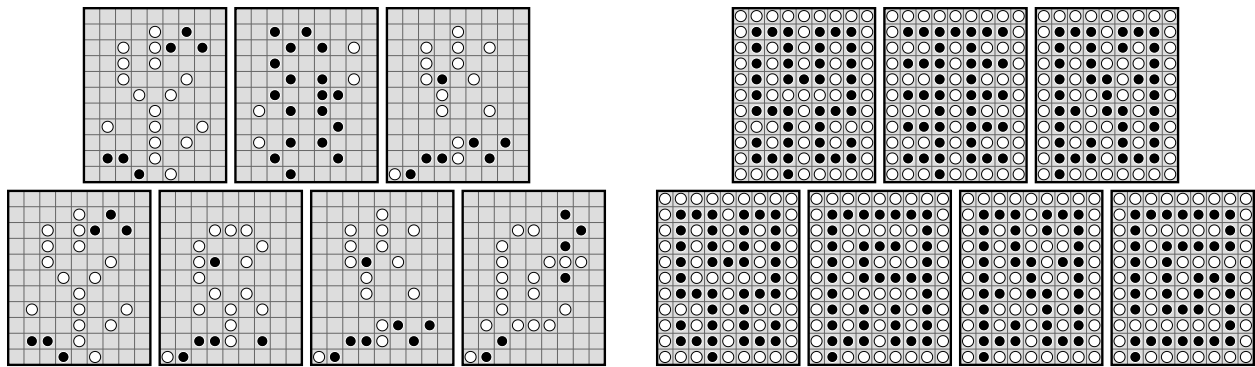


Figure 13: YIN YANG written in the Yin-Yang puzzle font (left) and solved font (right).

4 Yin-Yang

The *Yin-Yang typeface*⁸ represents each letter of the alphabet by another type of pencil-and-paper puzzle called *Yin-Yang*. The puzzle is on a square grid, with some of the squares prefilled with a black or white circle. The goal is to fill in the remaining squares with black and white circles so that (1) the black circles are connected by horizontal and vertical connections, (2) the white circles are similarly connected, and (3) there are no 2×2 squares with circles of the same color. Figure 13 shows examples of puzzles and their solutions, and Figure 15 shows the full alphabet. In each case, the black circles outline the letter.

We designed the solutions by hand, then used our own brute-force computer search to repeatedly remove clues that preserved unique solvability, resulting in a minimal puzzle with the intended solution. After hundreds of such trials, we hand-picked what seemed to be the most challenging puzzle for each letter. Nonetheless, some letters (such as V) are relatively difficult, while others (such as E, J, and M) are relatively easy.

As usual, the puzzle font lets us hide messages in the puzzle. Figures 14 and 16 give two such messages for you to try. The font is also designed to make it possible to combine multiple letters into a single puzzle (while still satisfying the constraints), though the resulting minimal puzzles seem to be substantially easier to solve. Figure 17 shows an example.

Along with this font, we proved that Yin-Yang puzzles are NP-complete [DLRU21].

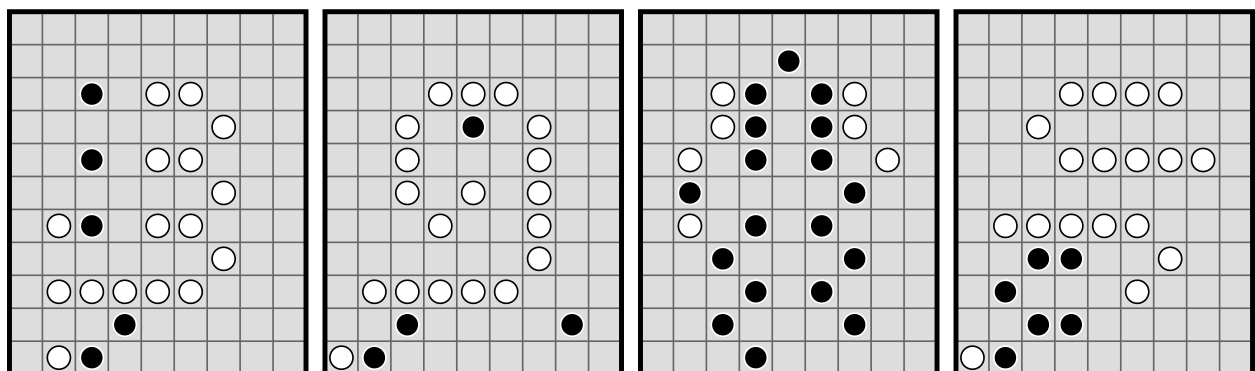


Figure 14: Can you connect together the black and white dots without a monochromatic 2×2 square? Or solve interactively on the web. The solution is in Figure 31.

⁸<https://erikdemaine.org/fonts/yinyang/>

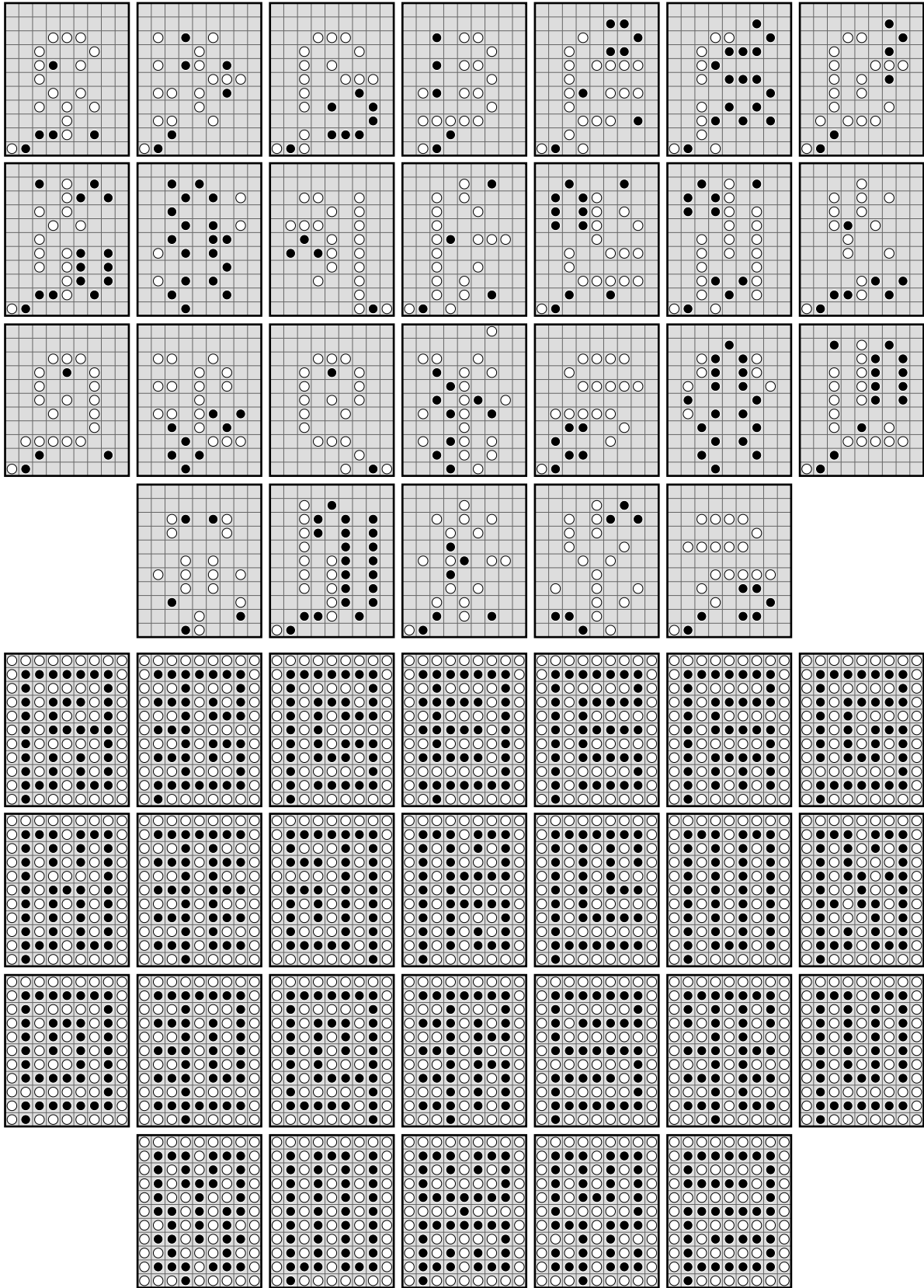


Figure 15: The entire alphabet written in the Yin-Yang puzzle font (top) and solved font (bottom).

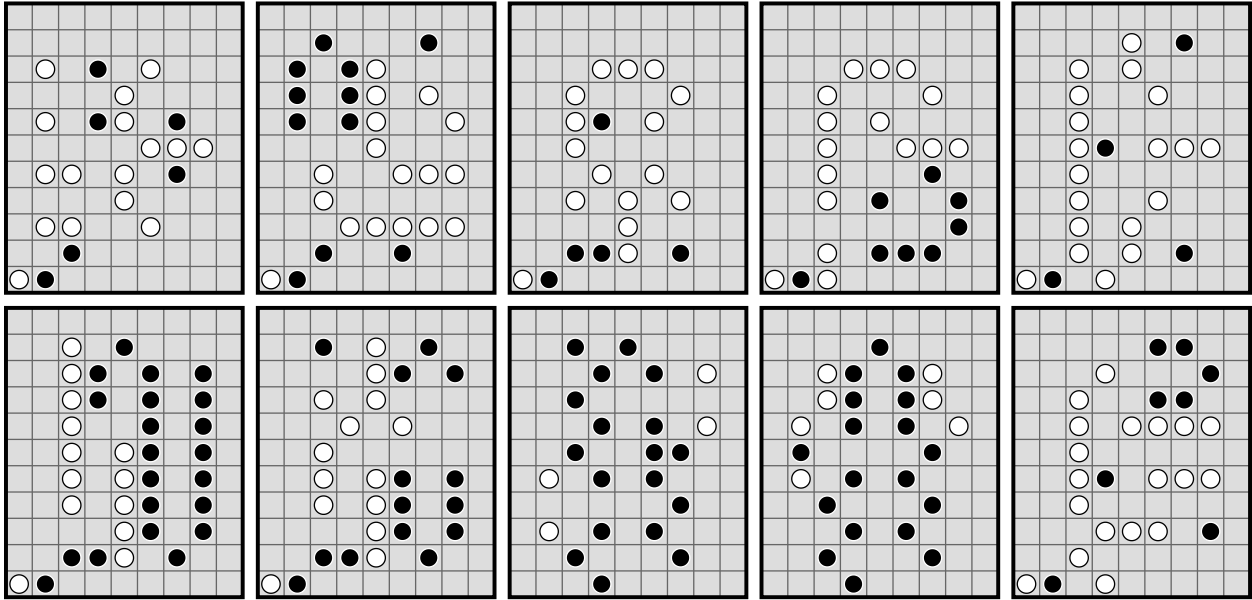


Figure 16: Can you connect together the black and white dots without a monochromatic 2×2 square? Or solve interactively on the web. The solution is in Figure 32.

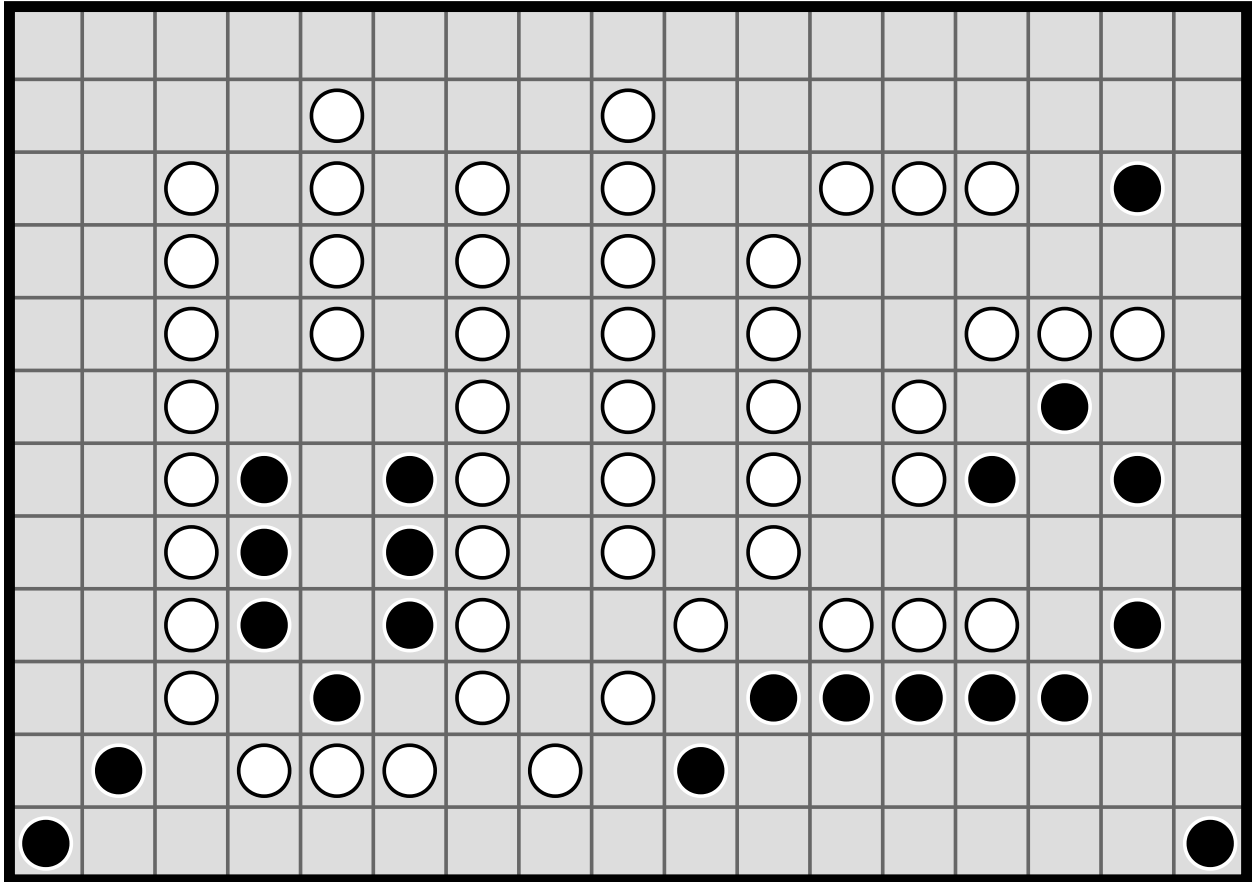


Figure 17: Can you connect together the black and white dots without a monochromatic 2×2 square? Or solve interactively on the web: <https://erikdemaine.org/fonts/yinyang/g4g.html>. The solution is in Figure 33.

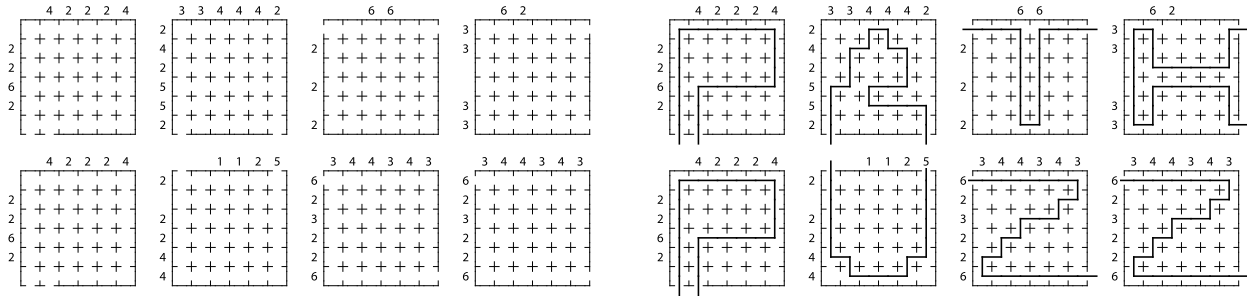


Figure 18: PATH PUZZ written in the path-puzzles puzzle font (left) and solved font (right).

5 Path Puzzles

The *path-puzzles typeface*⁹ represents each letter of the alphabet by another type of pencil-and-paper puzzle called *path puzzles*. The puzzle is on a square grid with two gaps on the boundary, where some rows and some columns are marked with an integer. The goal is to draw a single path between the two gaps such that the number of filled squares in each row and column matches the marked integer (if given). Figure 18 shows examples of puzzles and their solutions, and Figure 20 shows the full alphabet. In each case, the path draws the letter.

These puzzles were designed by hand to have unique solutions, and verified to have unique solutions by our own brute-force computer search, in a larger team. We presented this font in a paper that proved NP-completeness of path puzzles [BDD⁺20].

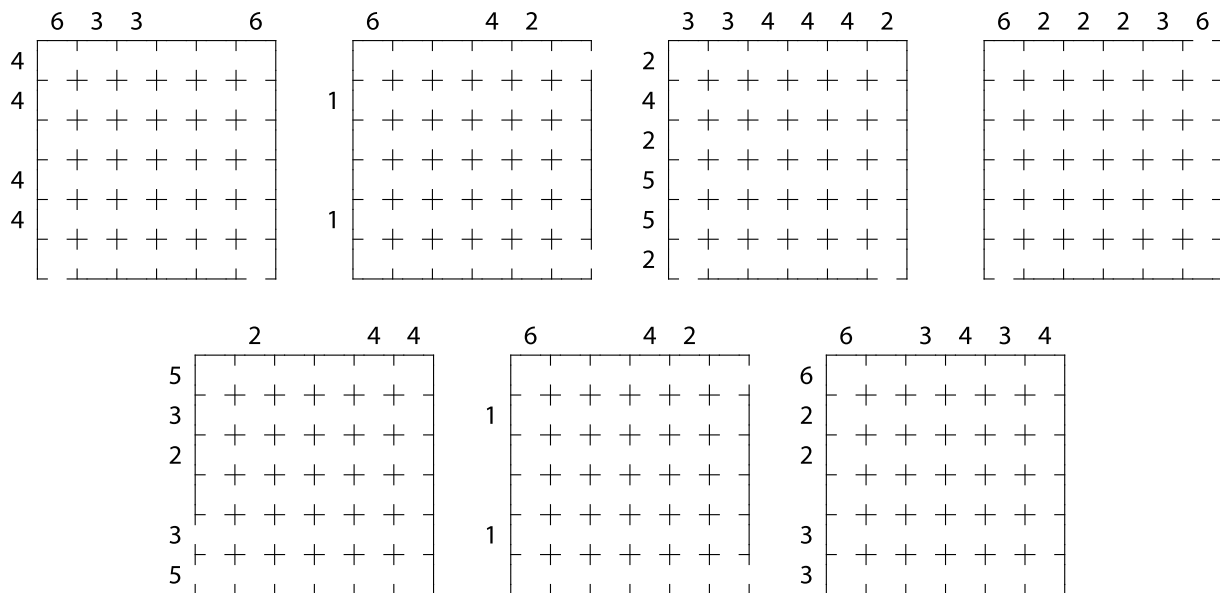


Figure 19: Can you draw a path between the two boundary gaps that has the specified numbers of filled squares in indicated rows and columns? Or solve interactively on the web. The solution is in Figure 34.

⁹<https://erikdemaine.org/fonts/pathpuzzles/>

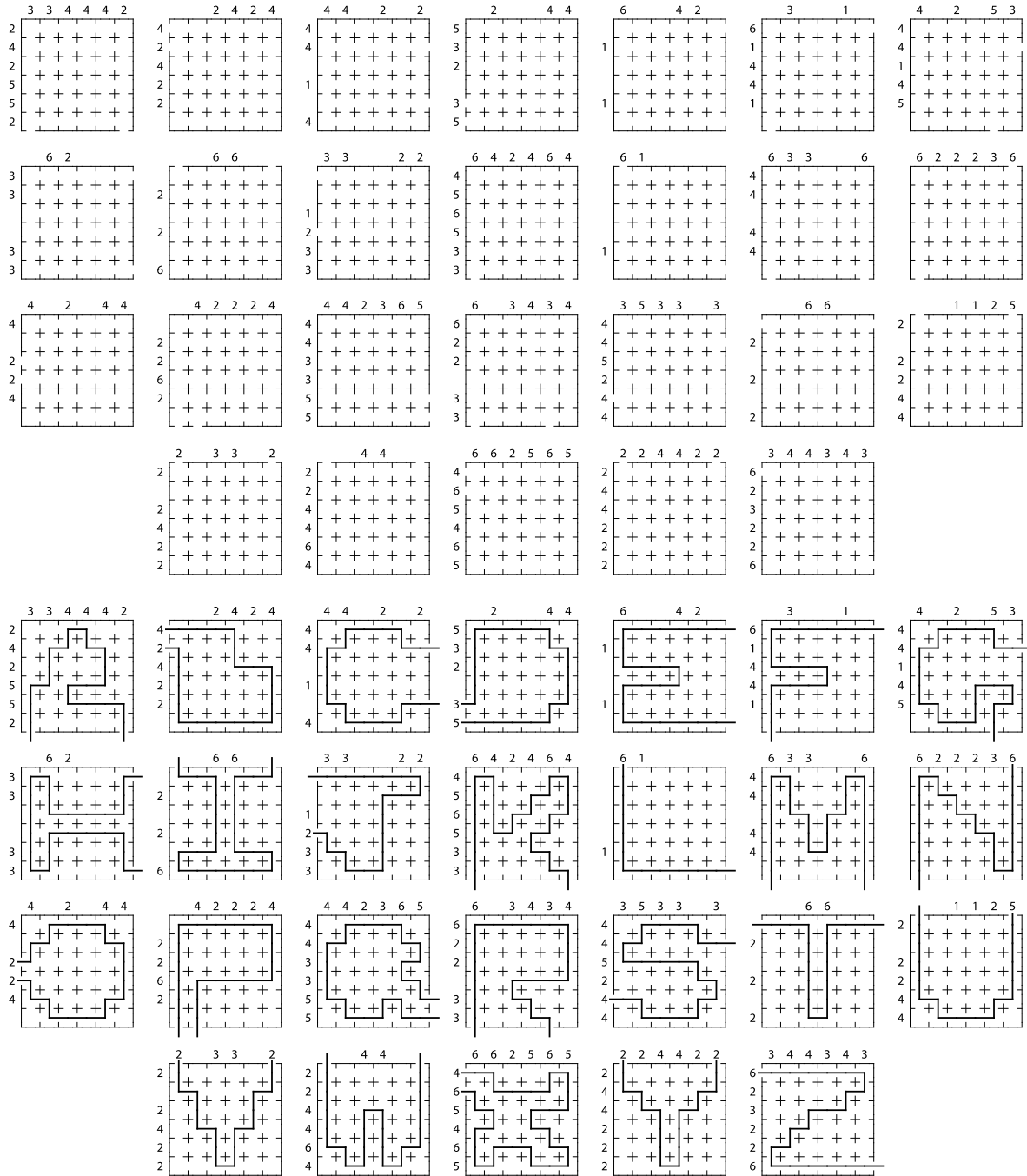


Figure 20: The entire alphabet written in the path-puzzles puzzle font (top) and solved font (bottom).

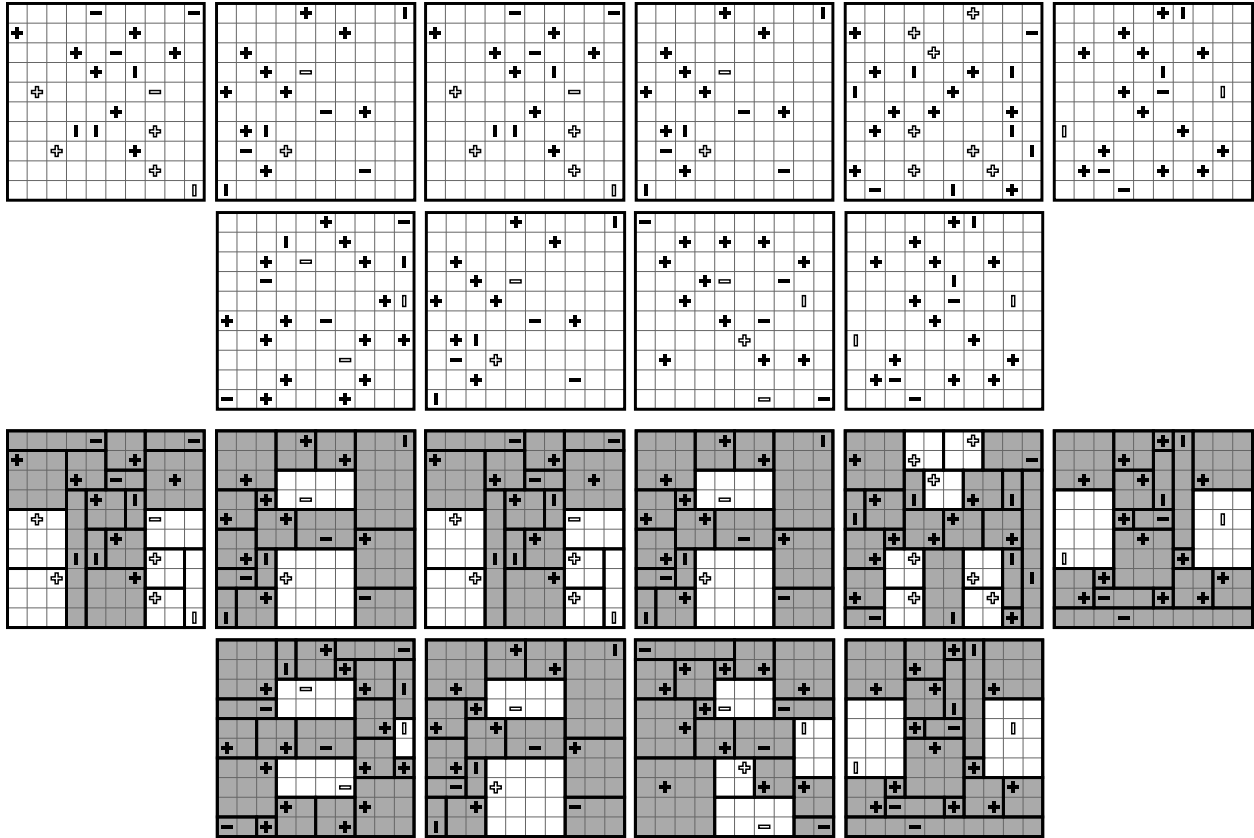


Figure 21: TATAMIBARI written in the Tatamibari puzzle font (top) and solved font (bottom).

6 Tatamibari

The *Tatamibari typeface*¹⁰ represents each letter of the alphabet by a final type of pencil-and-paper puzzle called *Tatamibari puzzles*, published by the famous Japanese puzzle publisher Nikoli. The puzzle is on a square grid, with some cells marked with a clue in the shape of a plus sign, horizontal bar, or vertical bar. The goal is to decompose the grid into exactly one rectangle per clue such that (1) each plus clue is in a square; (2) each horizontal clue is in a nonsquare rectangle that is wider (more horizontal) than it is tall; and (3) each vertical clue is in a nonsquare rectangle that is taller (more vertical) than it is wide. Figure 21 shows examples of puzzles and their solutions, and Figure 22 shows the full alphabet. In each case, coloring the rectangles the same as the (black and white) clues reveals the letter in black.

These puzzles were designed by hand to have unique solutions, while extensively aided by our own brute-force computer search to verify solvability and uniqueness, in a larger team. We presented this font in a paper that proved NP-completeness of Tatamibari puzzles [ABD⁺20].

¹⁰<https://erikdemaine.org/fonts/tatamibari/>

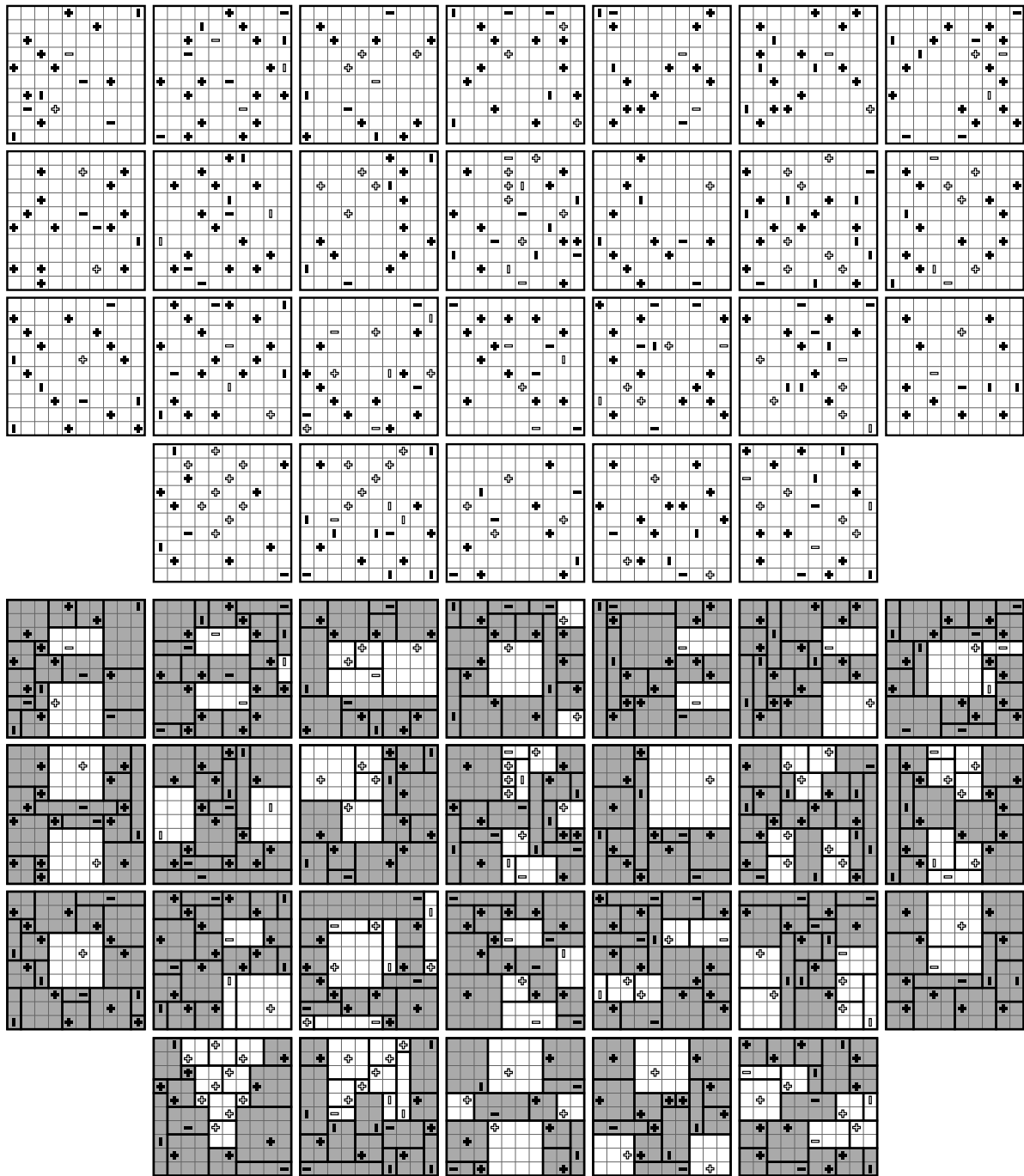


Figure 22: The entire alphabet written in the Tatamibari puzzle font (top) and solved font (bottom).

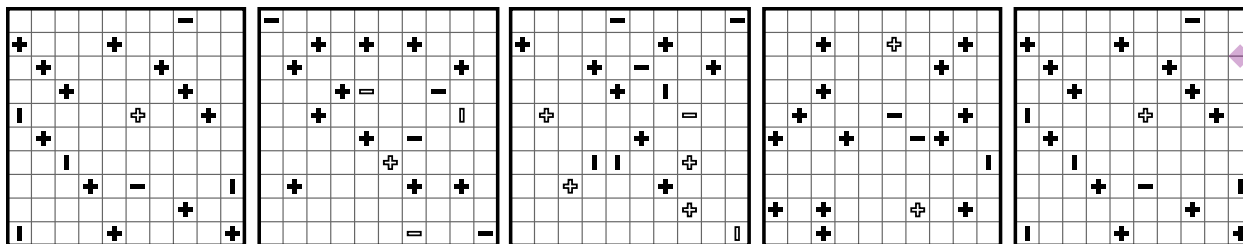


Figure 23: Can you draw one rectangle per clue so that plus clues are in squares, horizontal clues are in wider-than-square rectangles, and vertical clues are in taller-than-square rectangles? Or solve interactively on the web. The solution is in Figure 35.

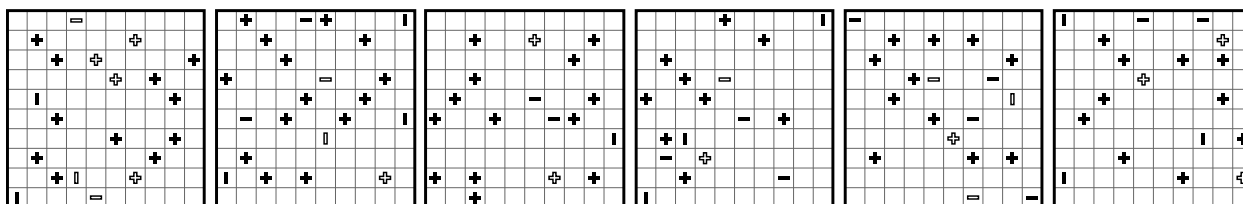


Figure 24: Can you draw one rectangle per clue so that plus clues are in squares, horizontal clues are in wider-than-square rectangles, and vertical clues are in taller-than-square rectangles? Or solve interactively on the web. The solution is in Figure 36.

References

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- [Rob21] Siobhan Roberts. Is this some kind of code? You can solve the *The New York Times*, 25 June 2021. <https://www.nytimes.com/2021/06/25/science/puzzles-fonts-math-demaine.html>.

[YS03] Takayuki Yato and Takahiro Seta. Complexity and completeness of finding another solution and its application to puzzles. *IEICE Transactions on Fundamentals of Electronics, Communications, and Computer Sciences*, E86-A(5):1052–1060, 2003. Also IPSJ SIG Notes 2002-AL-87-2, 2002.

A Puzzle Solutions

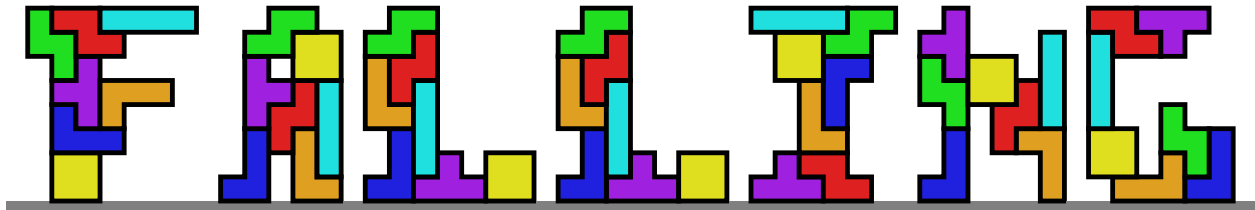


Figure 25: The self-referential message hidden in Figure 5.

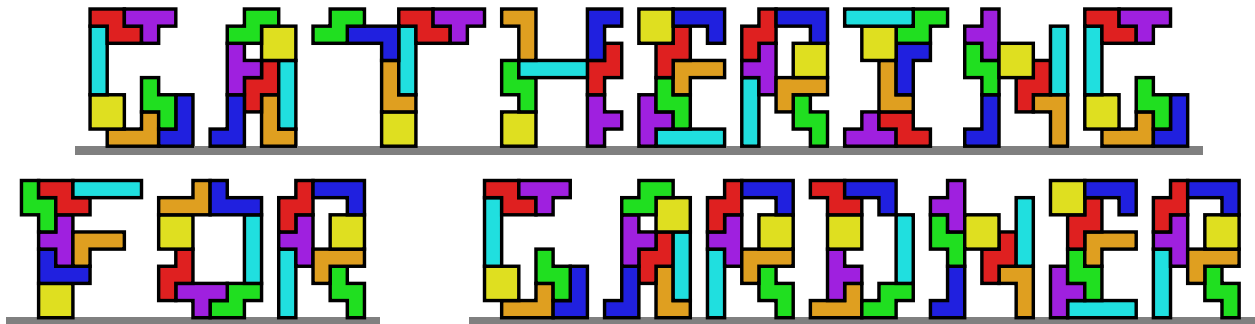


Figure 26: The message hidden in Figure 6.

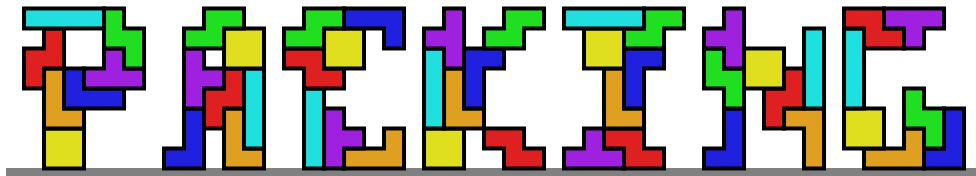


Figure 27: Tilings for the PACKING letter shapes in Figure 7.

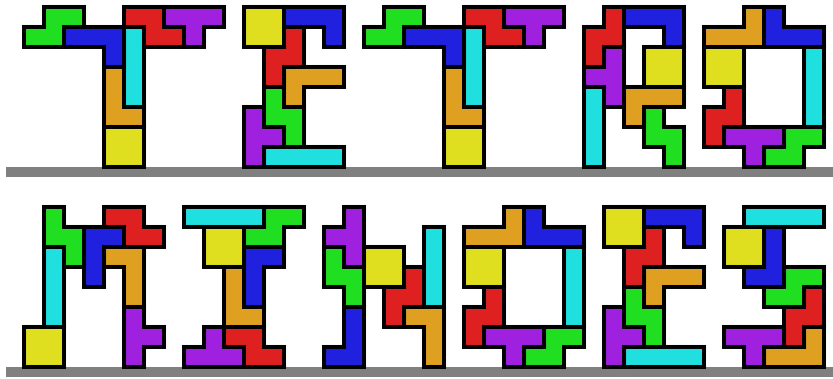


Figure 28: Tilings for the TETROMINOES letter shapes in Figure 8.

6	2	7	8	1	4	3	9	5
3	4	8	7	5	9	1	2	6
9	5	1	2	3	6	7	8	4
1	8	2	6	4	5	9	7	3
5	9	3	1	7	2	4	6	8
7	6	4	9	8	3	2	5	1
8	7	5	3	2	1	6	4	9
2	1	6	4	9	8	5	3	7
4	3	9	5	6	7	8	1	2

7	8	1	5	6	9	2	3	4
9	6	4	3	2	1	5	8	7
3	2	5	7	4	8	6	9	1
5	3	6	2	8	7	1	4	9
2	9	7	1	3	4	8	6	5
1	4	8	6	9	5	7	2	3
4	1	9	8	7	6	3	5	2
8	5	2	4	1	3	9	7	6
6	7	3	9	5	2	4	1	8

Figure 29: Solved Sudoku puzzles from Figure 11, in honor of Martin Gardner.

7	2	1	5	6	9	3	8	4
6	8	4	3	2	1	5	9	7
3	9	5	7	4	8	1	2	6
9	3	6	2	8	7	4	5	1
2	5	7	1	3	4	8	6	9
4	1	8	6	9	5	7	3	2
5	4	9	8	7	6	2	1	3
8	6	3	4	1	2	9	7	5
1	7	2	9	5	3	6	4	8

3	1	7	5	9	2	8	4	6
5	6	2	4	8	3	7	9	1
4	9	8	7	6	1	2	3	5
1	8	3	9	5	7	4	6	2
2	7	6	3	4	8	1	5	9
9	4	5	2	1	6	3	7	8
7	3	1	6	2	9	5	8	4
6	2	4	8	3	5	9	1	7
8	5	9	1	7	4	6	2	3

1	8	4	7	5	6	9	2	3
2	9	5	3	4	1	7	6	8
3	6	7	8	9	2	5	4	1
7	5	1	2	8	4	6	3	9
9	4	2	6	7	3	8	1	5
6	3	8	5	1	9	2	7	4
5	2	6	4	3	8	1	9	7
8	1	3	9	2	7	4	5	6
4	7	9	1	6	5	3	8	2

3	2	7	5	1	6	4	8	9
9	5	6	7	8	4	1	3	2
4	8	1	2	3	9	5	7	6
1	7	2	8	4	3	6	9	5
8	6	3	9	5	2	7	1	4
5	9	4	1	6	7	8	2	3
6	1	5	3	2	8	9	4	7
7	3	8	4	9	5	2	6	1
2	4	9	6	7	1	3	5	8

3	5	1	7	8	9	4	6	2
6	7	8	4	3	2	1	5	9
9	4	2	5	1	6	3	7	8
4	1	3	6	7	8	9	2	5
2	6	7	9	5	4	8	3	1
8	9	5	1	2	3	7	4	6
1	2	9	3	4	5	6	8	7
5	3	6	8	9	7	2	1	4
7	8	4	2	6	1	5	9	3

Figure 30: Solved Sudoku puzzles from Figure 12, in honor of Howard Garns who invented Sudoku puzzles (under the name "Number Place"), first published in May 1979.

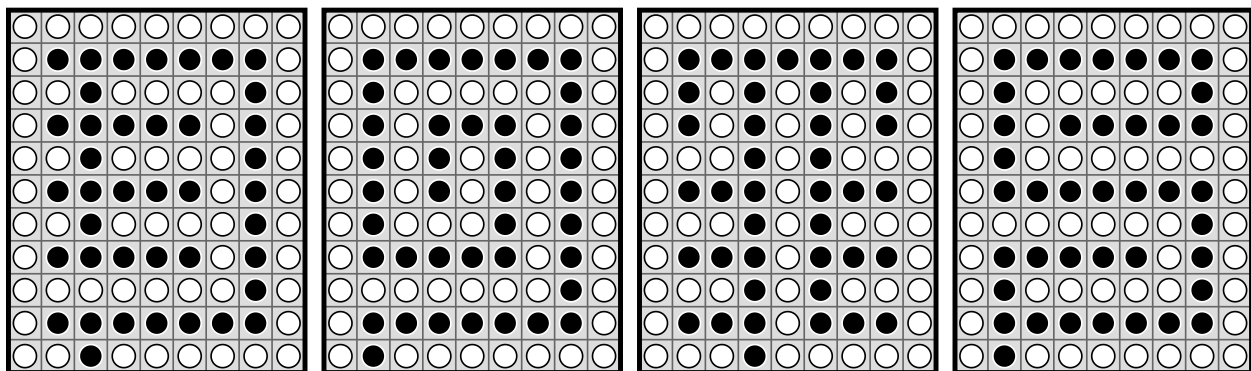


Figure 31: Solved Yin-Yang puzzles from Figure 14, revealing the self-referential message DOTS.

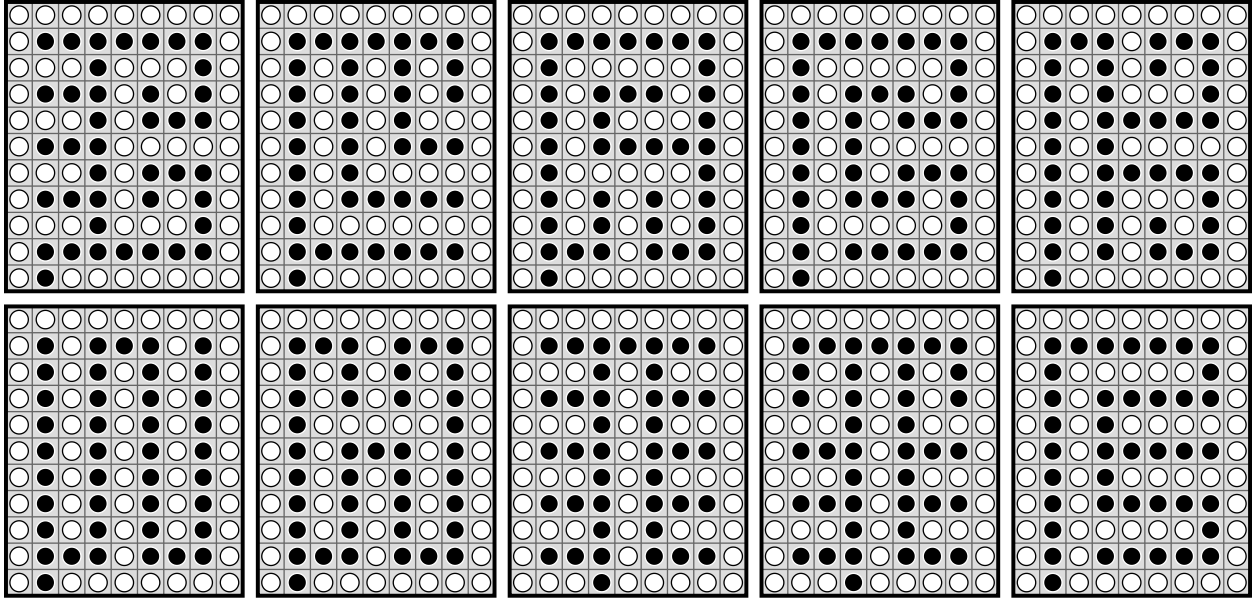


Figure 32: Solved Yin-Yang puzzles from Figure 16, revealing the self-referential message BLACK/WHITE.

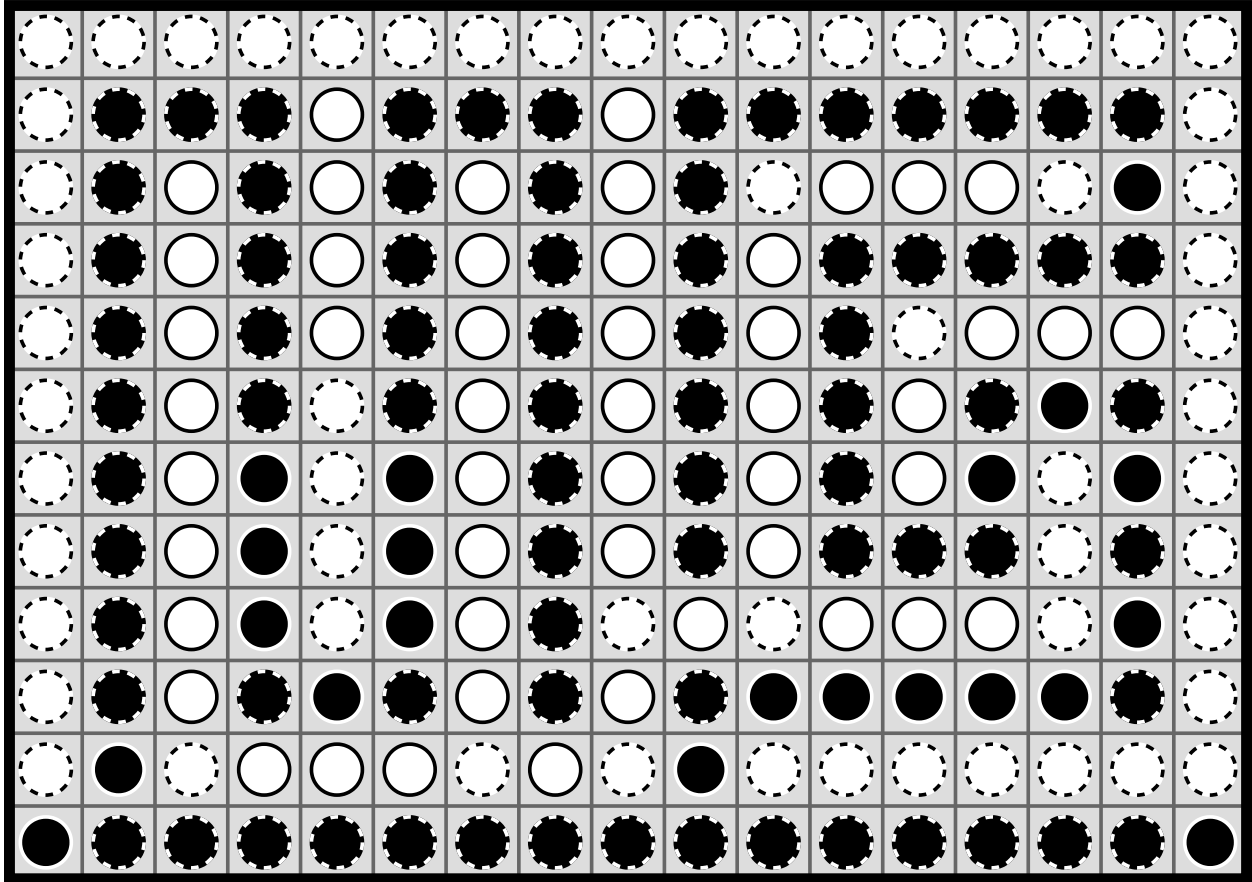


Figure 33: Solved Yin-Yang puzzle from Figure 14, in honor of Martin Gardner.

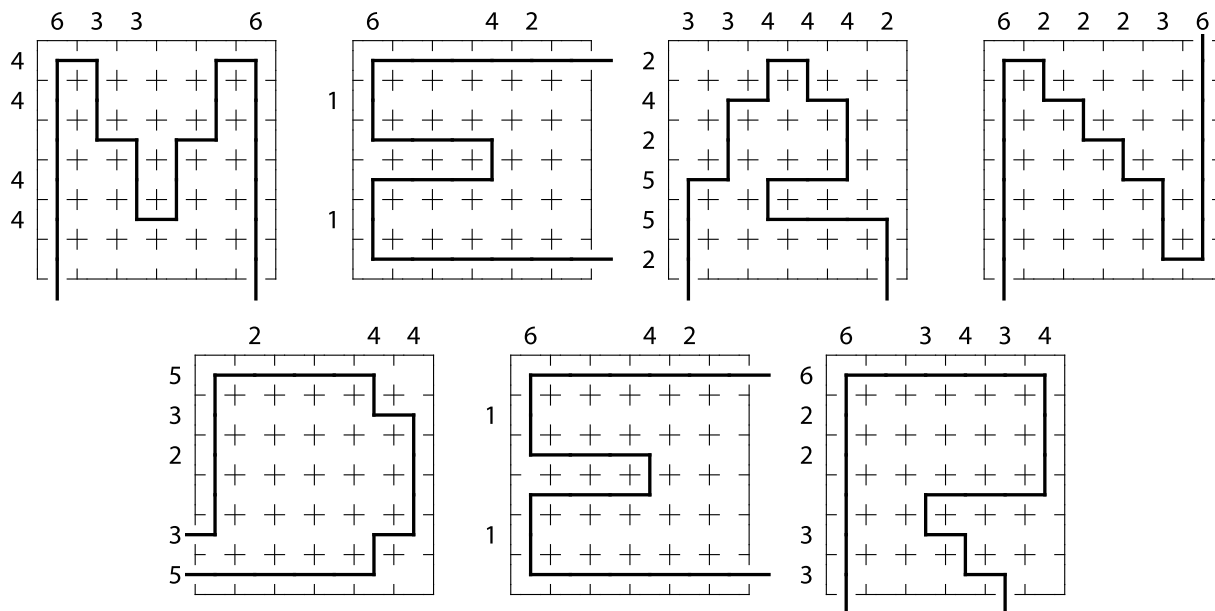


Figure 34: Solved path puzzles from Figure 19, revealing the self-referential message MEANDER.

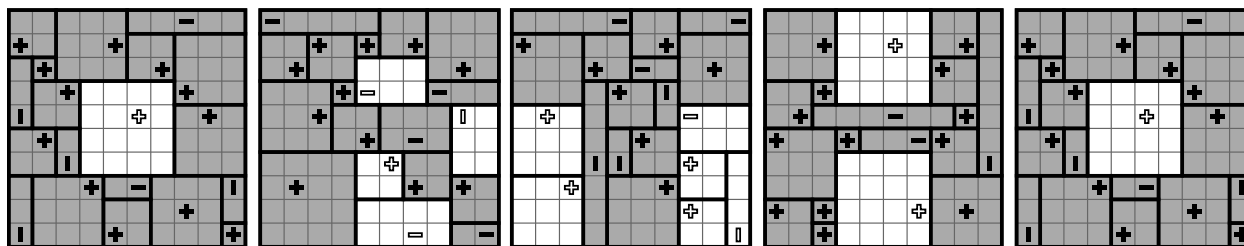


Figure 35: Solved Tatamibari puzzles from Figure 23, revealing the message ORTHO (referring to the orthogonal nature of rectangles).

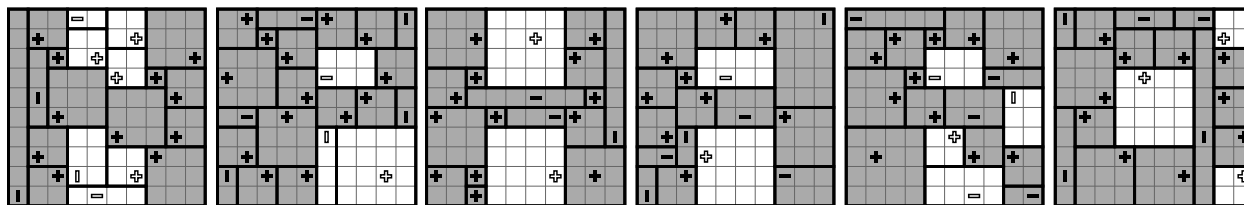


Figure 36: Solved Tatamibari puzzles from Figure 24, revealing the message NP HARD.