G4G13 Exchange Gift
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4 x 13

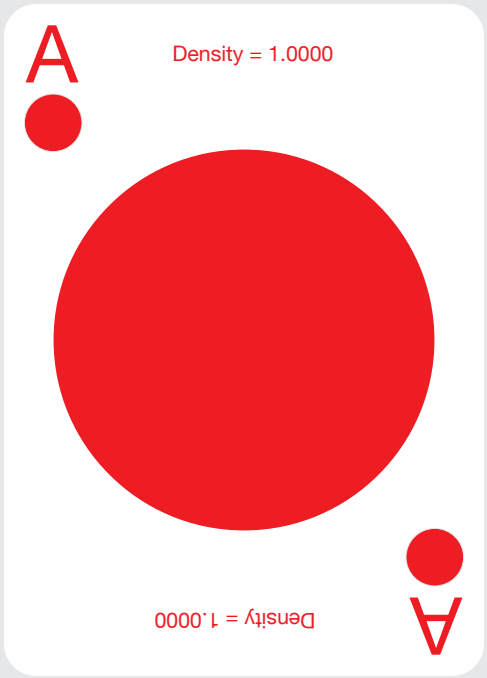
## 4 x 13

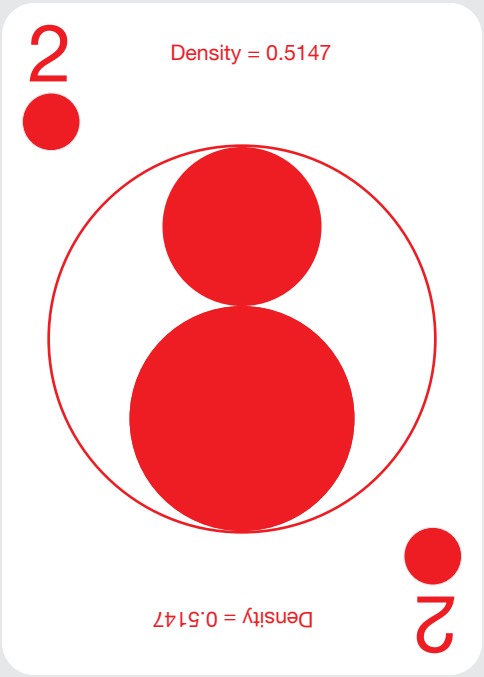
This is a progress report on a project to design a deck of playing cards using mathematical ideas. Each of the **4** suits in the deck is based on a different mathematical concept that can be extended logically throughout the **13** cards in the suit. The **4** concepts that are used include knots, circle packing, dissections, and tiling. The challenge is to make images that are both graphically appealing and mathematically meaningful. The pattern on the back of the cards is generated using a recursive substitution rule with **13** triangles.

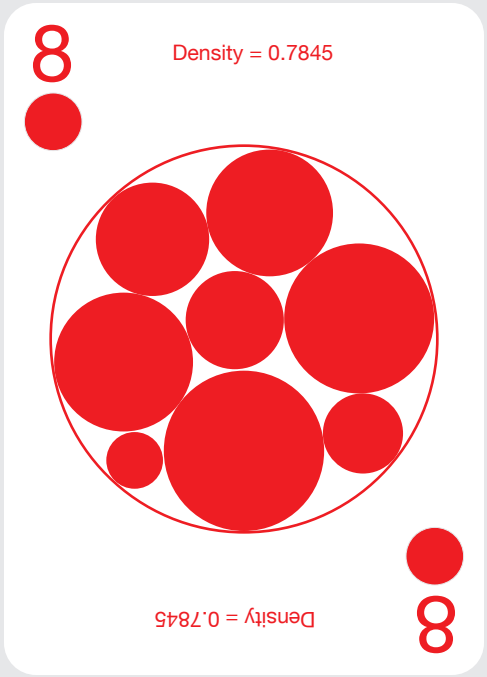
| | |
|---|---|
| Knots |  Knots classified by crossing number |
| Circles |  Circles packed into a larger circle |
| Triangles |  Triangles dissected to form other shapes |
| Tiles |  Tiles arranged to make polyforms |

## Circles

This suit features circles packed inside an outer enveloping circle of constant size. For the One of Circles (the Ace), a single circle is “packed” inside the outer circle. On the Two card, two circles are packed inside; however, these circles are scaled so that if the smaller one is assigned an area of 1, the area of the larger one is 2. In a similar manner, the card for the Eight of Circles shows eight circles packed inside the outer circle, with their relative areas ranging from 1 to 8.

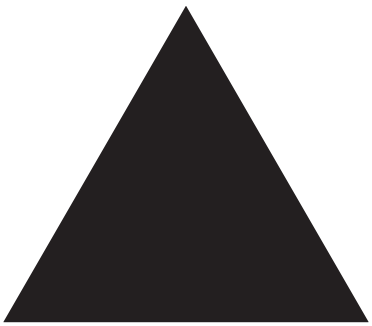


A

One Piece - Equilateral Triangle



A

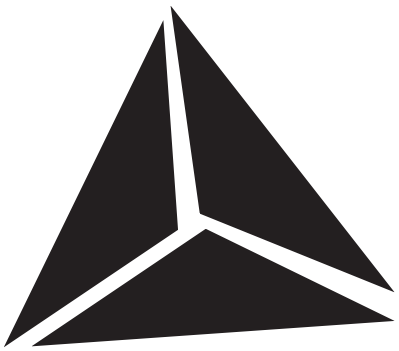
One Piece - Equilateral Triangle

## Triangles

Geometric dissections provide the imagery for this suit. On each number card N, a large equilateral triangle is dissected into N pieces. These same pieces can be rearranged to make a different geometric shape. For example, the Six of Triangles card shows a (slightly exploded) triangle that has been broken into six pieces; these pieces can also be used to form a regular pentagon. For the Eight card, the large triangle is cut up into eight pieces that can be rearranged into an octogram.

3

Three Pieces - Trapezoid?



3

Three Pieces - Trapezoid?

6

Six Pieces - Pentagon?

6

Six Pieces - Pentagon?

## Tiles

The images depicted in this suit are polyforms – shapes based on various building blocks, or tiles. A square tile is used for the overall suit symbol, and several cards have images of polyominoes, shapes that are generated from square tiles. For example, the Five of Tiles card shows five pentominoes. On several cards, polyforms composed of triangular or hexagonal tiles are used. The Four of Tiles card displays four tetraboloes, while the Six card has six hexahexes.

A

Monomino - 1 / 1

Monomino - 1 / 1

A

4

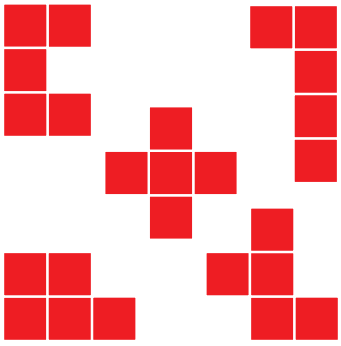
Tetraboloes - 4 / 14

Tetraboloes - 4 / 14

4

5

Pentominoes - 5 / 12

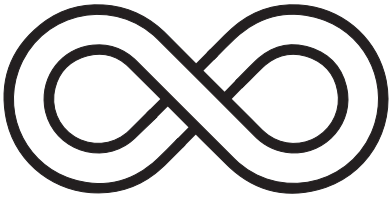


Pentominoes - 5 / 12

5

A

One Crossing - Unknot



A

One Crossing - Unknot

## Knots

This suit uses diagrams of knots, classified by their number of crossings. Some artistic license is employed for the Ace and Two cards, since no true knots exist with fewer than three crossings; for these, the “unknot” is depicted, twisted to show one and two crossings. On the Three of Knots card, the single 3-crossing knot is shown three times. For the Seven of Knots, all seven of the 7-crossing knots are displayed. A subset of the possible knots is used for higher number cards.

3

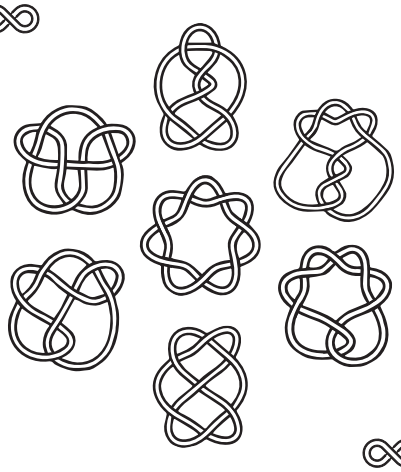
Three Crossings - 3 / 1

3

Three Crossings - 3 / 1

7

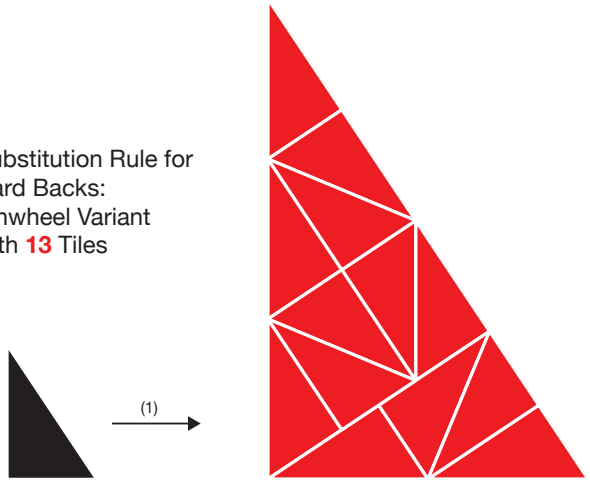
Seven Crossings - 7 / 7

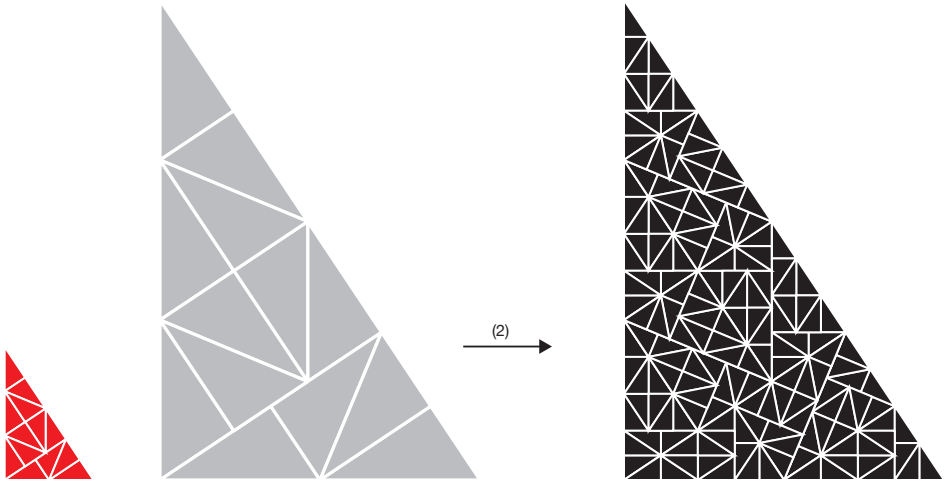


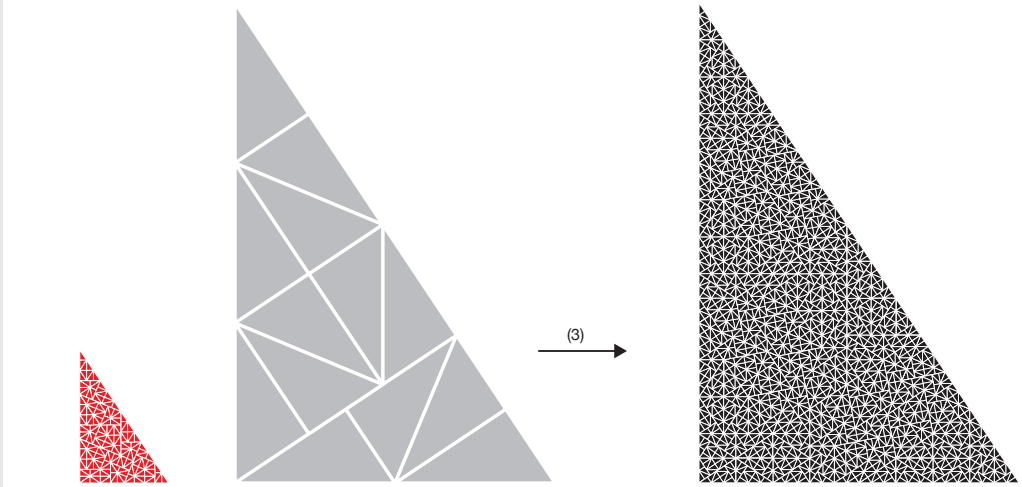
7

Seven Crossings - 7 / 7

Substitution Rule for
Card Backs:
Pinwheel Variant
with **13** Tiles

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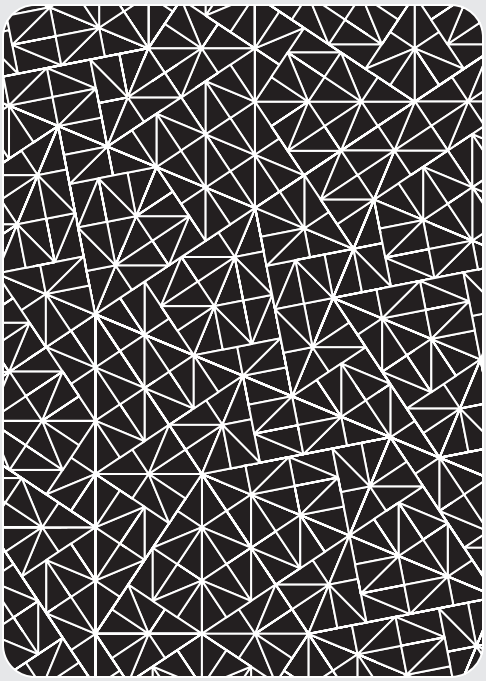
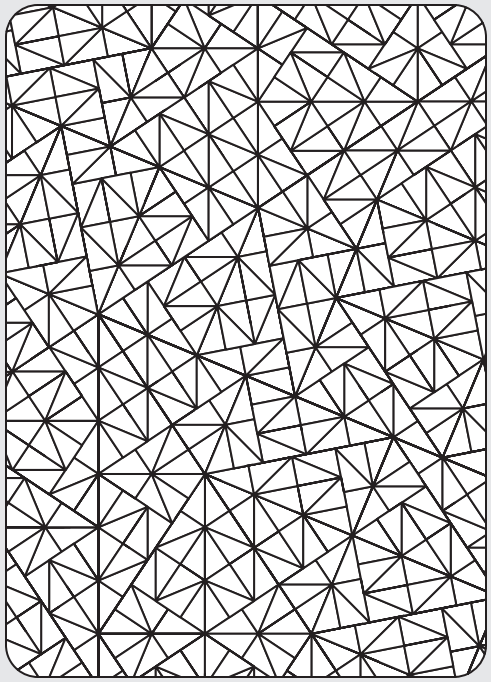
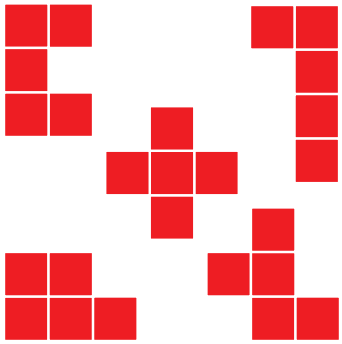




5

Pentominoes - 5 / 12



5

Pentominoes - 5 / 12

6

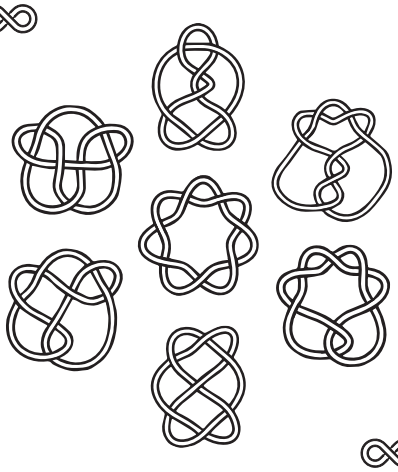
Six Pieces - Pentagon?

6

Six Pieces - Pentagon?

7

Seven Crossings - 7 / 7

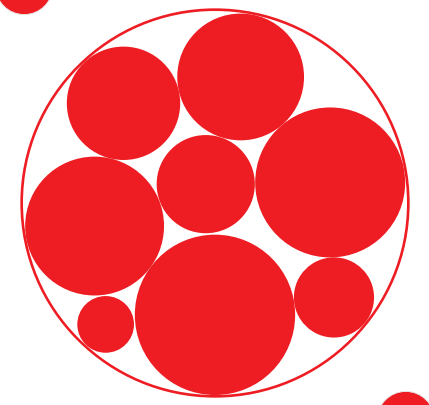


7

Seven Crossings - 7 / 7

8

Density = 0.7845



8

Density = 0.7845

References

*Wikipedia and Mathworld*: Articles on Packing Problems, Dissection Puzzles, Polyforms, Knot Theory, and Aperiodic Tilings

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