

## Conned Again, Watson! – Gardner's Two Child Problem Re-visited

Anais Acree

April 2018

Atlanta, GA

In the book *Conned Again, Watson*, author Colin Bruce takes the reader on adventurous journeys, explaining probability theory through interesting stories. Overall it is insightful and helps explain math by relating it through episodes, but there seems to be some ambiguity in one of his examples. In Chapter 5 titled, “The Case of the Unmarked Graves” he describes a problem closely resembling Gardner’s famous *Two Child Problem*.

The situation (given at the end of this paper) is that a son must know with a better than 50 percent chance that a specific grave is a woman’s burial place before his father will let him dig it open. There are two competing legends, but as the father says, “Now by either legend, the number of male and female skeletons buried here will be equal.” By this statement, there is of course a 50 percent chance that any randomly selected grave will contain the remains of a woman.

To make things more interesting, a shiny woman’s ring is found at a location equally spaced between two graves. All agree that this means that a woman must be buried in at least one of the two graves. The son states, “Father, we know that one of these graves definitely contains a woman’s remains. The other has an even chance of being a man or woman. So, if we dig up one grave, the chances that it contains a female are three in four.”

This assertion argues for an event table similar to Table A where all four events could occur with equal likelihood. Events 1 and 2 are the cases where Grave 1 contains a Female and there is an equal chance that Grave 2 contains a Male or Female. Events 3 and 4 are the cases where Grave 2 contains a Female and there is an equal chance that Grave 1 contains a Male or Female. The probability that Grave 1 is Female given that at least one grave is female is three out of the four events, or  $3/4$ .

Table A

	Grave 1	Grave 2
Event 1	Female	Female
Event 2	Female	Male
Event 3	Female	Female
Event 4	Male	Female

There is, however, another way of looking at this problem as shown in Table B. This event table is populated giving each individual grave an equal chance of being a Male or Female.

Table B

	Grave 1	Grave 2
Event 1	Female	Female
Event 2	Female	Male
Event 3	Male	Female
Event 4	Male	Male

There are three equally likely events with at least one female, (Events 1, 2, and 3). Of these three events, Grave 1 will contain a Female only in Events 1 and 2. This represents a chance of two out of three (not three in four as the son states).

As the story continues, the first grave is opened and contains the remains of a female. They now need to calculate the probability that the second grave also contains a female given that the first contains a female which is analogous to Gardner's "Two Child Problem."

From Table A, this would be two events out of three (Events 1&3 out of Events 1,2,& 3) as described in the book for a probability of 2/3. Table B, however, would give a different answer. Specifically, it would only be one out of two events (Event 1 out of Events 1&2) for a probability of 1/2.

So, which is the right answer? It depends on what can be assumed about the problem and which Event table is correct. Since it is stated that "the number of male and female skeletons buried here will be equal" it could be argued that Table B is correct and the probabilities assigned in the book are incorrect. Another way of thinking about this problem is whether the chance of a ring being present is double if it is between two Female graves rather than if only one of the two graves is Female. In this case, consider Table C showing the Table B possible outcomes with weights assigned for the likelihood of a ring being present.

Table C

	Grave 1	Grave 2	Weight
Event 1 2X	Female	Female	2X
Event 2 1X	Female	Male	1X
Event 3 1X	Male	Female	1X
Event 4 0X	Male	Male	0X

Since Event 1 has two females, it has double the chance, or a weight of 2, for a ring to be present compared to Event 2 or Event 3. Of course, Event 4 has no chance of having a ring since both graves contain Males. Reconstructing Table C to make two separate equally likely events from the doubly likely Event 1, and removing Event 4 (since it would have no rings) essentially reverts back to Table A which gives the results in the book.

To achieve the solution in the book it seems that one must effectively assume that there is a double chance of a ring being present if it is found between two Female graves compared to between a Male and Female grave. However, this is never explicitly stated in the book, where the finding of the ring is posed as being rather incidental. Specifically, no mention was given to the probability of a ring being present and ironically in the end both graves contained the remains of females but still only one ring was found!

Just like the "Two Child Problem" different people can reach different conclusions depending on how they view this. In my opinion, Table B best describes the equally possible events for this problem. Therefore, instead of the answers of 3/4 and 2/3 as published in the book, I take the position that the answers should be 2/3 and 1/2.

What do you think?! Email me at [anaisacree@gmail.com](mailto:anaisacree@gmail.com) because I would like to know!

The following pages contain copyrighted material from "Conned Again, Watson" which have been reproduced here for academic purposes.

"This is it!" said Prendergast excitedly. "This is the ancient burial ground that was used in earlier times. Each of these stones marks the resting place of a king or queen of Arthur's ancient lineage. In a few minutes, we shall know which legend is true. You see, there are two differing stories about how these ancient royals lived and died. Both versions agree that a man must marry to become king, so as to rule with the benefit of both male and female insight, and that king and queen were in due course buried here."

Prendergast eyed us intently as he continued. "Now, one version has it that their marriage was for life, in the modern Christian tradition, and that each was buried when they died of old age or natural causes. But a rival story is darker. It maintains that the king would every seven years take a new young wife, divorcing the old. When the king eventually died, his current wife was immediately killed by beheading, to leave the way clear for the succession. To resolve the matter, we have but to dig up a grave containing a female body and verify that its spine is intact."

I bent down and peered at one or two of the white stones which were the grave-markers. But none seemed to have any writing or other indication upon it. "How can you tell which type of grave is which?" I asked. From behind me there came a harsh laugh: I turned and saw that the Mage had managed, despite his age, to follow us into the bowl.

"Quite so!" he said triumphantly. "I gave my son permission to dig up a grave here—one grave only—on condition that there would be a greater than even chance of finding out the truth. Now by either legend, the number of male and female skeletons buried here will be equal. If you dig up a grave at random, there is an even chance it will turn out to contain a male skeleton, which will tell you nothing. By my edict, you are not allowed to proceed."

Prendergast grimaced, but I could tell by the expression on his face that he saw no way to dispute his father's argument. At his suggestion, we spread out and started to comb the surface of the bowl, though what we hoped to discover was not clear to me. I found myself drawn to a corner where a huge oak tree stood. Its roots had disturbed the ground about it, one great shaft running deep between two of the marker stones. My eye caught something bright in the grass. I bent down and picked up a gold ring, its surface miraculously untarnished, made in the form of the Norse Midgard serpent that lies circling the world with its tail in its mouth. My shout brought the others running.

"It is a woman's ring, a queen's ring," Prendergast shouted. "Show me exactly where you found it."

Alas, I was forced to point to a spot exactly halfway between the two nearest stones.

"You cannot possibly be sure which grave it came from. You do not know which is the female one," said the Mage firmly. Dodgson was about to speak, but Prendergast raised his hand.

"Thank you, Reverend, but I can solve this one for myself. Father, we know that one of these graves definitely contains a woman's remains. The other has an even chance of being man or woman. So if we dig up one grave, the chances that it contains a female skeleton are three in four. By your edict, we may proceed."

The dig took some time, for we proceeded with both caution and reverence. We unearthed the legs, then the pelvis. I was able to identify the pelvis as definitely female, and Prendergast gave a cry of triumph. But as we excavated toward the head, we fell victim to an extraordinary piece of bad luck. A root of the tree had pushed just past the top of the rib cage, and the ground became very wet at that point. Beyond the root there was no further sign of bones. The neck bones and skull, intact or otherwise, were gone.

"I am afraid that the combined work of the tree roots and an underground stream has long since carried that part of the skeleton away, to be scattered and destroyed," I said when it was clear there was no further hope.

Prendergast flung his trowel on the ground in fury. "What an incredible mischance. Really, the gods themselves seem to be against me," he shouted blasphemously. "Father, in the circumstances, may I open the second grave?"

The Mage smiled maliciously. "Of course not," he said. "The ring could obviously have come from this female skeleton, so the chance that the remaining grave is a woman's is again only one in two. I cannot allow you to proceed."

We resumed our seemingly futile search of the bowl. I was devastated that my potentially useful discovery should have led nowhere, and Prendergast must have been feeling far worse. But suddenly there came a cry from the Reverend.

"I have it. Really, I am almost tempted to shout Eureka! The chance that the second grave by the tree root contains a woman is not one-half. It is two-thirds."

The Mage looked at him scornfully. "One-half to two-thirds," he said savagely. "That seems to be your theme song, Reverend, but I am afraid I will not take your word for it. Surely we know nothing about the sex of the second grave."

Dodgson made no reply, but bending to the ground, he picked up a white pebble and a black one. Then he turned to me with a smile.

"Doctor, would you be so good as to lend me your top hat?"

Although somewhat baffled, I gave it to him.

"Let me once again demonstrate the point as a children's game," he said. "I will shuffle these pebbles in my hand and place one at random into the hat. The other I discard without looking at it. So the hat contains a white or a black pebble, with equal probability." We nodded.

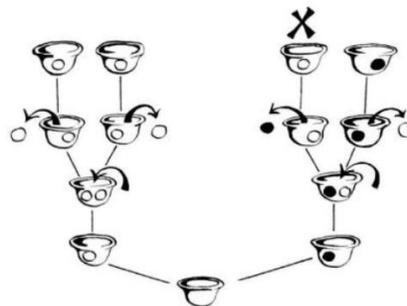
"Now I pick up a second white pebble"—he did so—"and place it in the hat. I toss the pebbles around so I cannot tell which is which.

"If a white pebble denotes a female skeleton and a black a male, I have created a puzzle equivalent to that of the graves by the tree. One is definitely white—that is female. The other is black or white—male or female—with equal probability. Now I take out a stone. I am in luck—it is white." He held up the pebble. "But unfortunately, the stone is mute as to how it met its end." He flung it down. "Now, given that the first stone was white, what is the chance that the second stone is also white?"

He paused. I felt there was something oddly elusive about the problem but was unable to put my finger on it. The others looked equally baffled. At one moment I convinced myself that the probability was only one-third, because we had already used up one female stone, so to speak. Or was it one-half after all?

Dodgson produced a sheet of paper. He pointed to the tree above us.

"The best way to illustrate the possibilities is by drawing a branching tree. I call it my many-worlds tree." He began to draw.



*The Many-Worlds Tree*

"At the start, we have one version of the world, with a hat that is empty. Now I place in it a stone that may be either white or black, and so we have two versions of the world—two potential worlds in which subsequent events will unfold differently. I add my second white stone, the same in each world, so there are still only two potential realities. Then I take out a stone at random. This may be either the original stone or the second one, so our two worlds fork into four realities."

He counted them across from the top. "In the first reality, I discard the original stone. The second was white, so

the remaining stone is white. In the second reality, I discard the second stone, but the original was white. So the remaining stone is again white. In the third reality, I discard the black stone, so that remaining is white. In the fourth reality, I discard the white, and the remaining stone is black.”

“Then the chances are really three in four that the second stone is white,” I exclaimed.

“No, Doctor, because one of the four realities must be crossed out. The first stone I took out was *not* black, so we are definitely not in the third reality. There are three realities we may be inhabiting, each equally likely, and in two of the three the last stone is white. The chances are two in three that the remaining grave contains a female.”