Symmetrix Puzzles

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Symmetrix puzzles are a new craze where pieces are put together to form symmetric figures. They may be rotated or reflected, but may not overlap. In this article, we analyse a three-piece puzzle designed by Vladimir Krasnoukhov of Russia. It consists of a very large $30° - 60° - 90°$ triangles, a similar triangle which is much smaller, and a trapezoid with two right angles and two angles of measures $60°$ and $120°$ respectively. These are shown in Figure 1.
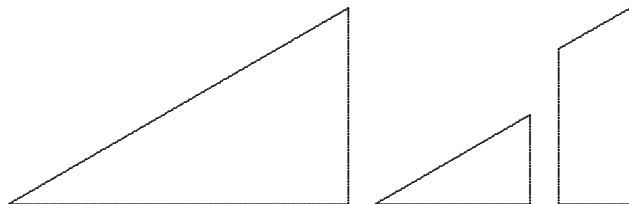

Figure 1

The most probable motivation for this puzzle is the equilateral triangle partitioned into six congruent triangles, as shown in Figure 2 on the left. Two of these triangles are discarded, while three of the remaining ones are combined into a large triangle, as shown in Figure 2 on the right.
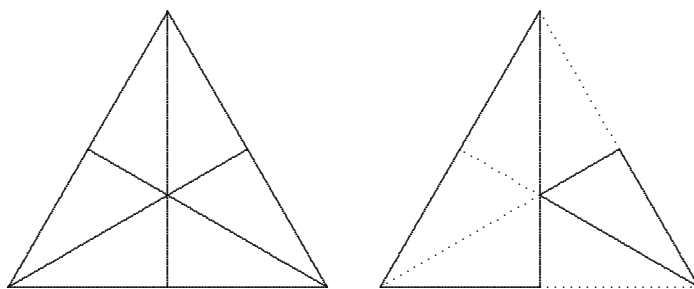

Figure 2

Despite their difference in size, these two pieces can be put together to form a symmetric figure, in two different ways, as shown in Figure 3.
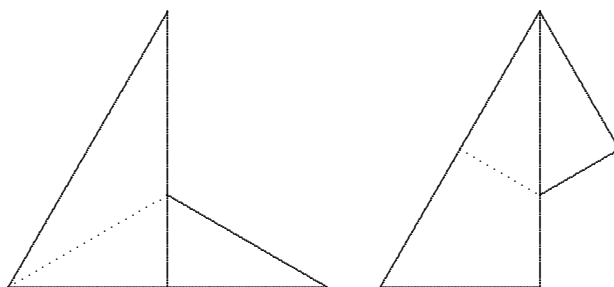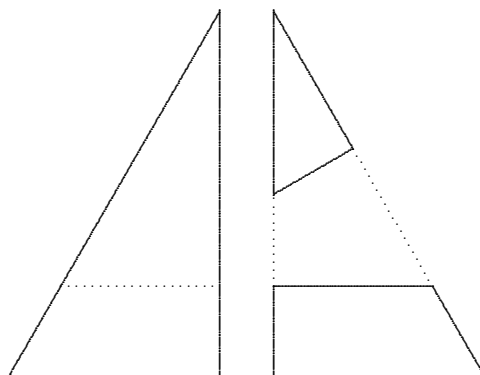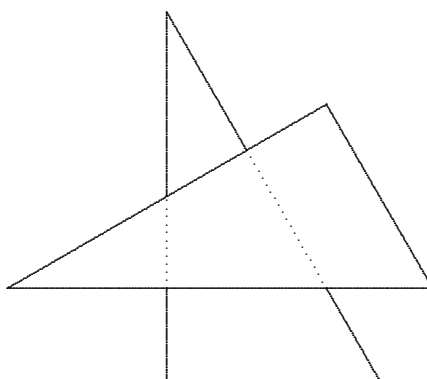

Figure 3

As such, this would not have been much of a puzzle. In a crafty move, the large triangle is further enlarged as shown in Figure 4, and a new trapezoidal piece congruent to the the enlargement is introduced.
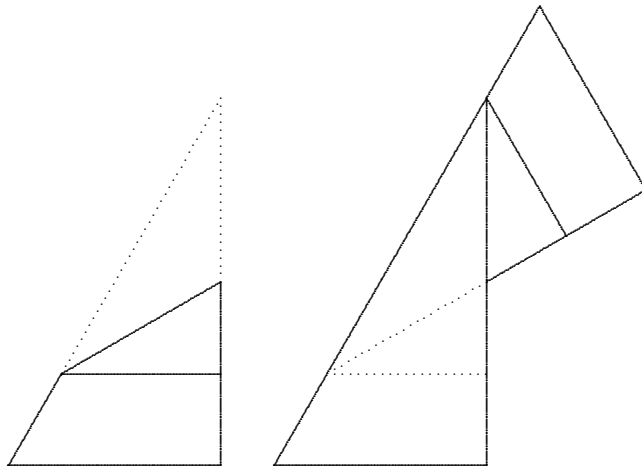

Figure 4

For the puzzle to work, the height of the trapezoid can be chosen arbitrarily. However, if it is too large, then we have two large pieces of more or less the same size, and the psychological impact of one very large piece versus two relatively small ones is lost. This height is chosen to be one-third of that of the original equilateral triangle.

In Figure 4 on the right, we subtract the two small pieces from the large piece, leaving behind a symmetric shape. This leads to the first of two symmetric figures that can be constructed with these three pieces, as shown in Figure 5.
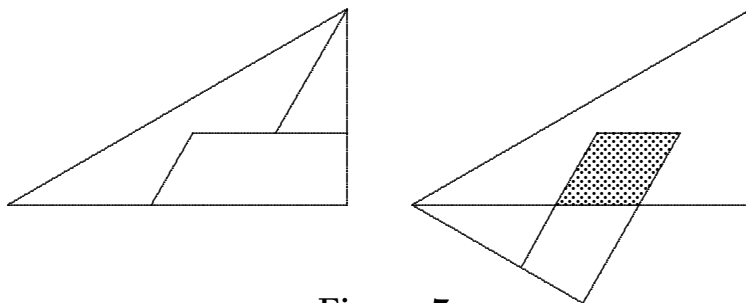

Figure 5

The two small pieces may be subtracted from the large piece in another way, as shown in Figure 6 on the left. This leads to the second solution of the puzzle, as shown in Figure 6 on the right. These two solutions are based on the same idea as those in Figure 3.
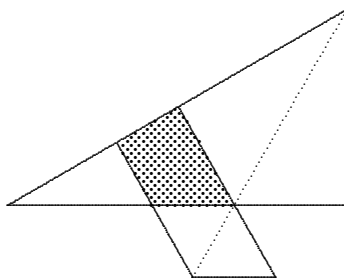
Figure 6

In another crafty move, Alan Tsay of Canada replaces the smaller triangle by an even smaller similar triangle, whose shortest edge is equal in length to the height of the trapezoid. This time, there is only one way in which the two smaller pieces may be subtracted from the large piece in order to leave behind a symmetric shape. This is shown in Figure 7 on the left.
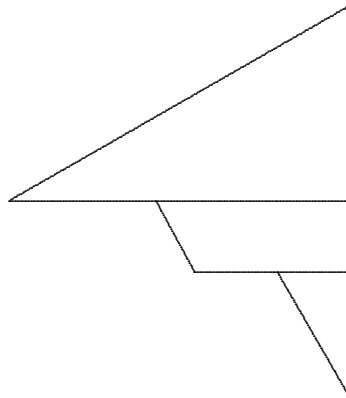

Figure 7

However, when we move the two smaller pieces to the other side, we discover that the trapezoid overlaps the large triangle in a rhombus. This is shaded in Figure 7 on the right.

Figure 8

In Figure 8, we subtract the smaller triangle from the large one. Then we take the *symmetric difference* between the trapezoid and what is left of the large triangle, by removing their *intersection*, which is shaded. The symmetric difference consists of a kite left over from the large triangle, and a rhombus from the trapezoid. They have a common axis of symmetry.

This time, we obtain the desired solution shown in Figure 9. Note that this could have emerged had we reflected the two smaller pieces in Figure 7 on the left across the longer leg of the large triangle.

Figure 9

The similarity between the components of these two puzzles may be exploited in many ways. In a small group presentation, one small triangle may be substituted surreptitiously for the other. In a large group presentation, the two puzzles may be handed out to participants seated in alternating columns.

It should be pointed out that subtraction is also a form of taking the symmetric difference. In this case, the intersection happens to be identical to the smaller piece. It may be argued that finding the symmetric differences is not any easier than finding the symmetric figures themselves. Nevertheless, it does give us some additional things to look for, and broadens the avenue of approach to the problem. A good starting point is forming the union of two pieces with some aspect of symmetry.