Quick Review of Remote Sensing Basic Theory

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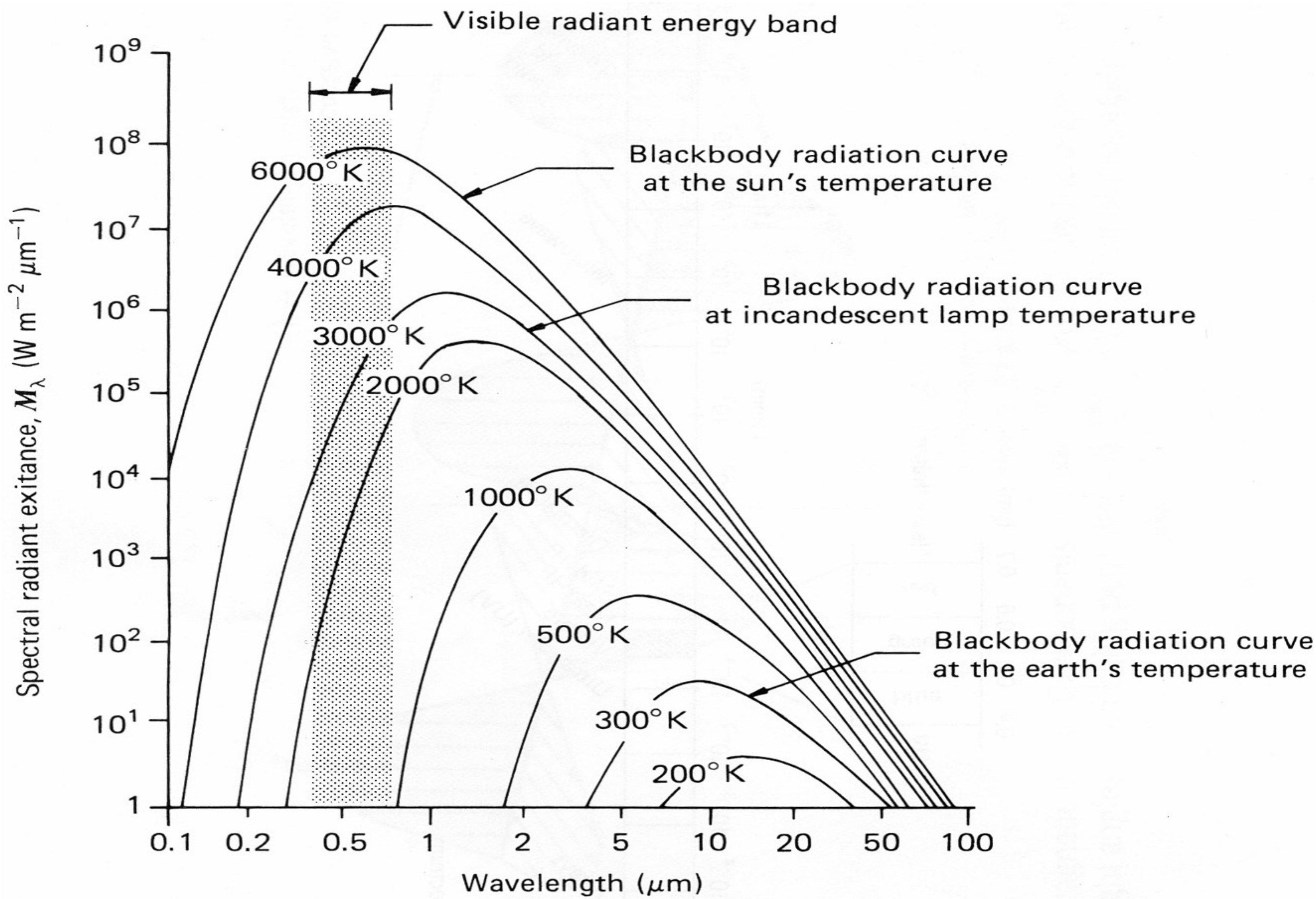
Outline

- Planck Function
- Infrared: Thermal Sensitivity
- Bit Depth

Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures



Radiation is governed by Planck's Law

In wavelenght:

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c2/\lambda T} - 1] \} \quad (mW/m^2/ster/cm)$$

where λ = wavelength (cm)

T = temperature of emitting surface (deg K)

c_1 = 1.191044 x 10-8 ($W/m^2/ster/cm^{-4}$)

c_2 = 1.438769 (cm deg K)

In wavenumber:

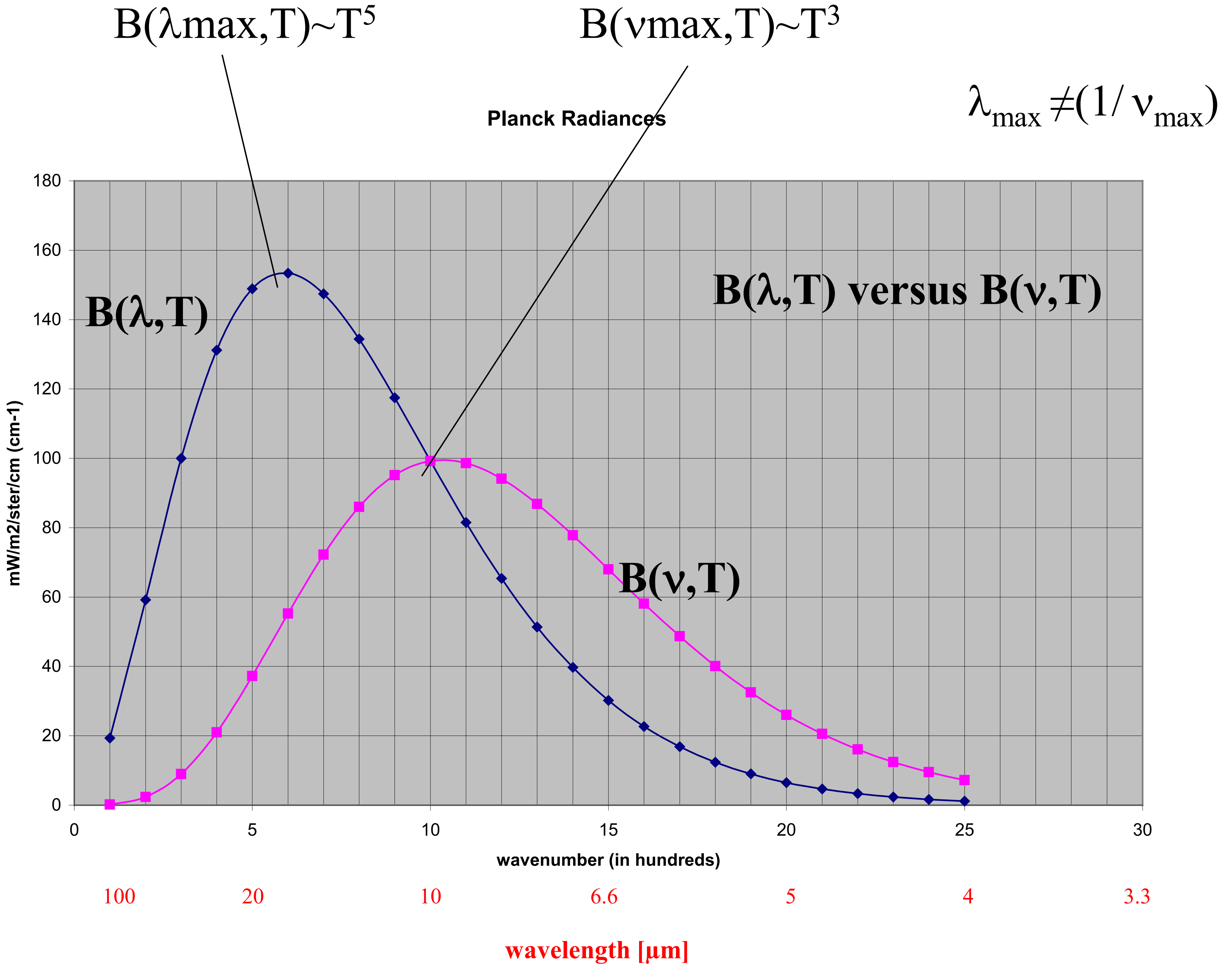
$$B(\nu,T) = c_1 \nu^3 / [e^{c2\nu/T} - 1] \quad (mW/m^2/ster/cm^{-1})$$

where ν = # wavelengths in one centimeter (cm-1)

T = temperature of emitting surface (deg K)

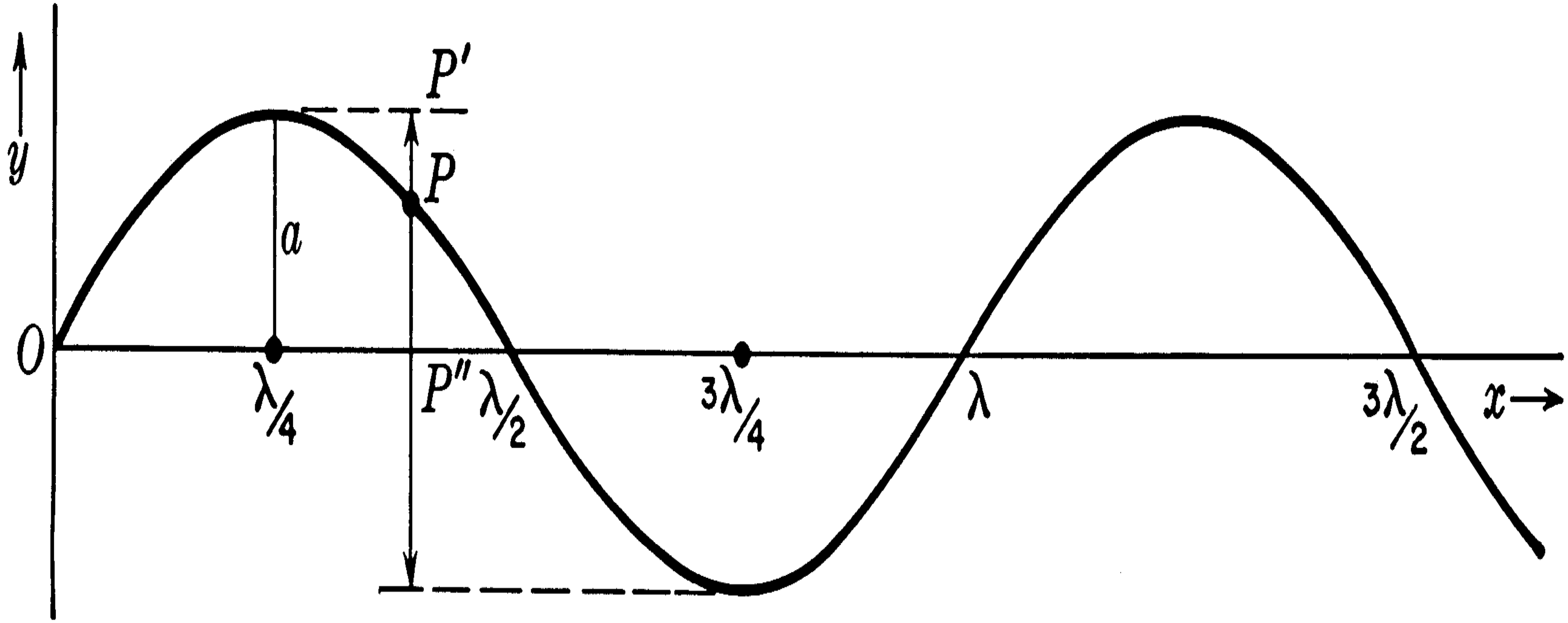
c_1 = 1.191044 x 10-5 ($mW/m^2/ster/cm^{-4}$)

c_2 = 1.438769 (cm deg K)



B(λmax,T)~T5
B(νmax,T)~T3
λmax ≠(1/ νmax)
Planck Radiances
B(λ,T) versus B(ν,T)
B(λ,T)
B(ν,T)
mW/m2/ster/cm (cm-1)
180
160
140
120
100
80
60
40
20
0
0
5
10
15
20
25
30
wavenumber (in hundreds)
100
20
10
6.6
5
4
3.3
wavelength [µm]

wavelength λ : distance between peaks (μm)



wavenumber ν : number of waves per unit distance (cm)

$$\lambda = 1/\nu$$

$$d\lambda = -1/\nu^2\, d\nu$$

Using wavenumbers

Wien's Law $dB(\nu_{max},T) / dT = 0$ where $\nu(max) = 1.95T$
indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\nu_{max},T) \sim T^{**}3$.

Stefan-Boltzmann Law

$$E = \pi \int_{o}^{\infty} B(\nu,T)\, d\nu = \sigma T^4$$

where $\sigma = 5.67 \times 10\text{-}8\ W/m^2/deg^4$.

states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

Brightness Temperature

$$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$

is determined by inverting Planck function

Brightness temperature is uniquely related to radiance for a given wavelength by the Planck function.

Using wavenumbers

$$B(\nu,T) = c_1\nu^3 / [e^{c_2\nu/T} - 1]$$

$(mW/m^2/ster/cm^{-1})$

ν(max in cm-1) = 1.95T

$B(\nu_{max},T) \sim T^{**}3.$

$$E = \pi \int_o^\infty B(\nu,T)\, d\nu = \sigma T^4,$$

$$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$

Using wavelengths

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

$(mW/m^2/ster/cm)$

λ(max in cm)T = 0.2897

$B(\lambda_{max},T) \sim T^{**}5.$

$$E = \pi \int_o^\infty B(\lambda,T)\, d\lambda = \sigma T^4,$$

$$T = c_2/[\lambda \ln(\frac{c_1}{\lambda^5 B_\lambda} + 1)]$$

Temperature sensitivity

$$dB/B = \alpha \, dT/T$$

The Temperature Sensitivity α is the percentage change in radiance corresponding to a percentage change in temperature

Substituting the Planck Expression, the equation can be solved in α:

$$\alpha = c_2 \nu / T$$

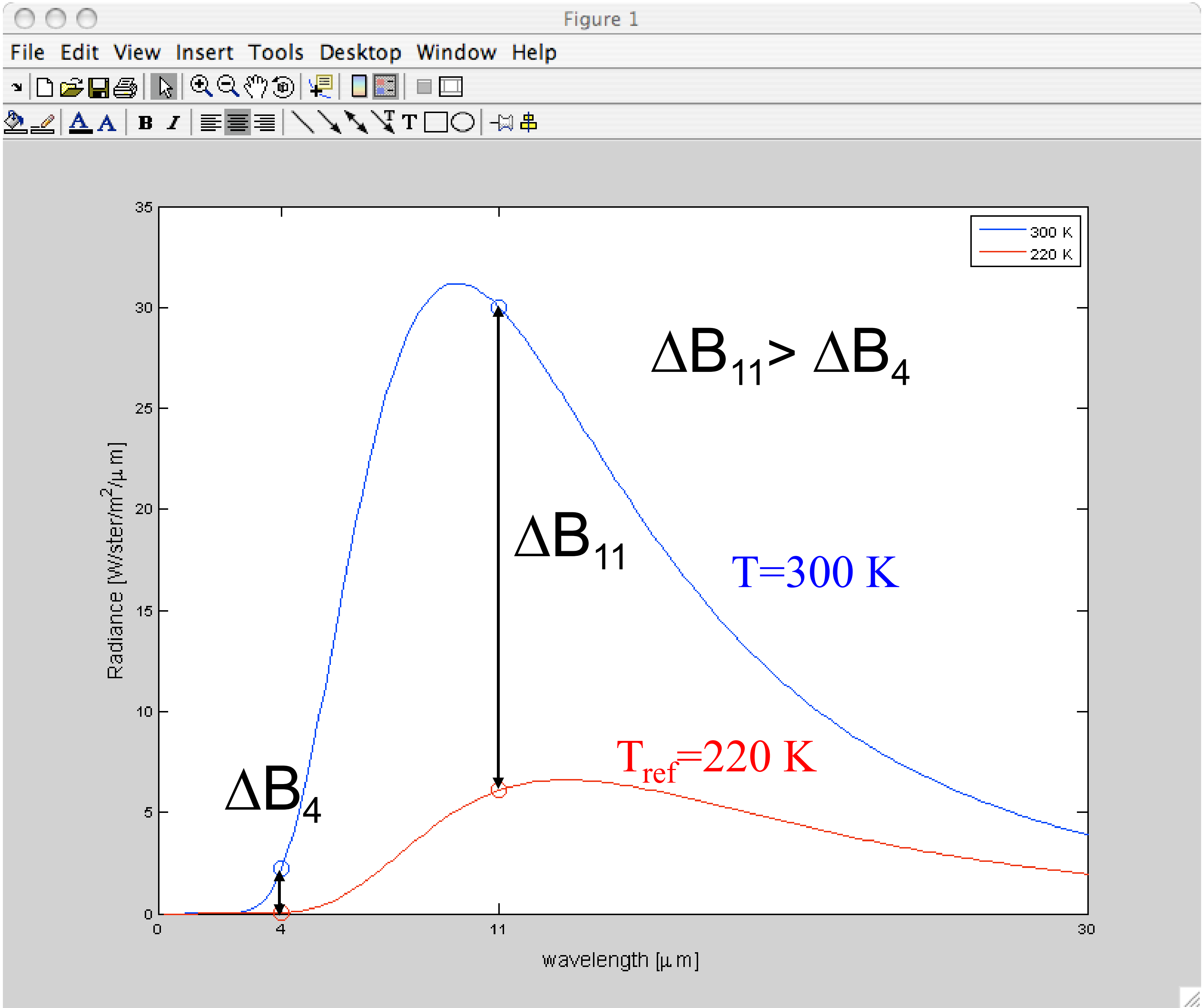
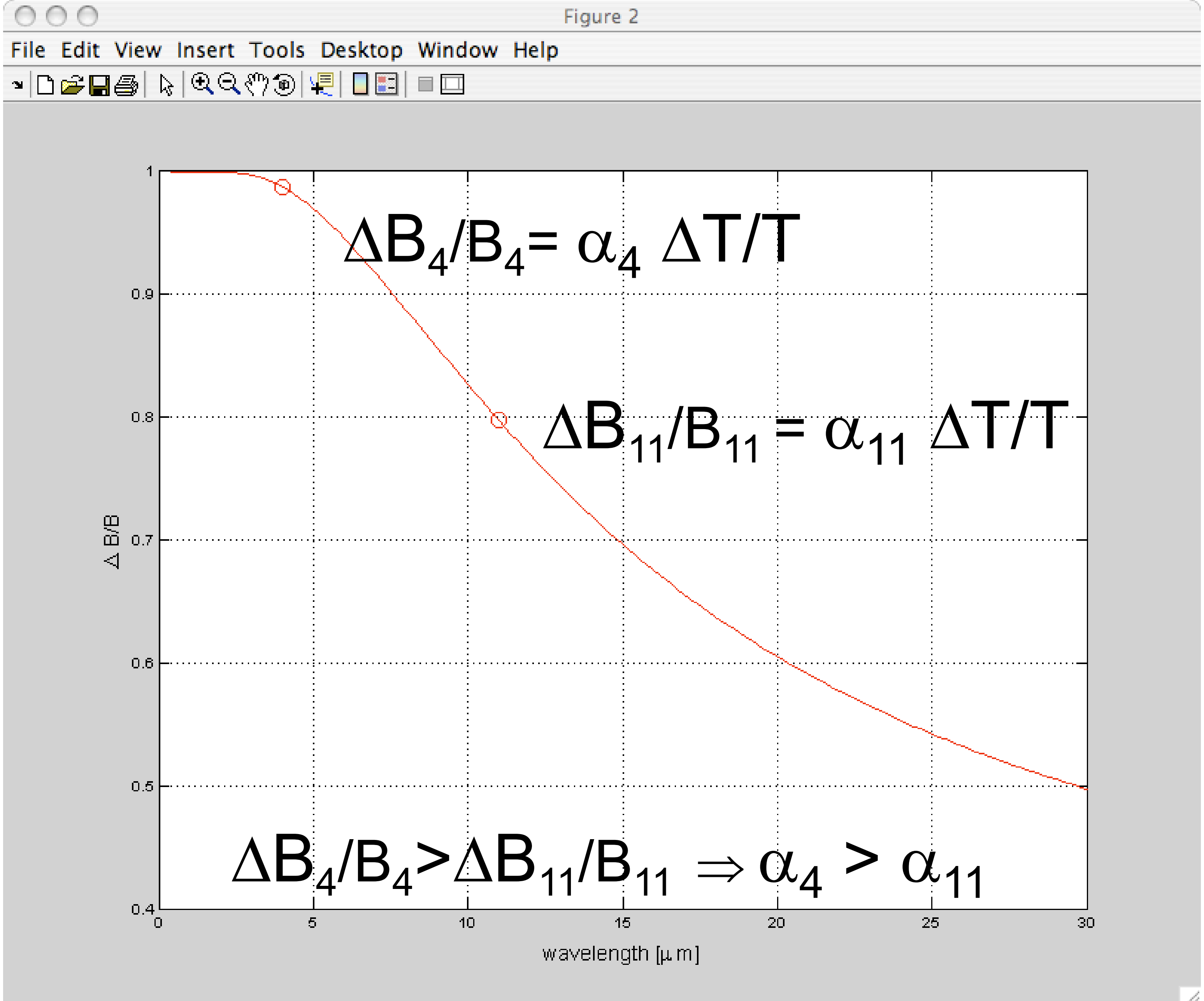


Figure 1
File Edit View Insert Tools Desktop Window Help
300 K
220 K
ΔB11> ΔB4
ΔB11
ΔB4
T=300 K
Tref=220 K
Radiance [W/ster/m2/µm]
wavelength [µm]
0
4
11
30
35
30
25
20
15
10
5
0



(values in plot are referred to wavelength)

(Approximation of) B as function of α and T

$\Delta B/B=\alpha \, \Delta T/T$

Integrating the Temperature Sensitivity Equation Between T_{ref} and T (B_{ref} and B):

$B=B_{ref}(T/T_{ref})^{\alpha}$

Where $\alpha=c_2\nu/T_{ref}$ (in wavenumber space)

$$B = B_{ref}(T/T_{ref})^{\alpha}$$

$$\Downarrow$$

$$B = (B_{ref}/\ T_{ref}^{\ \alpha})\ T^{\ \alpha}$$

$$\Downarrow$$

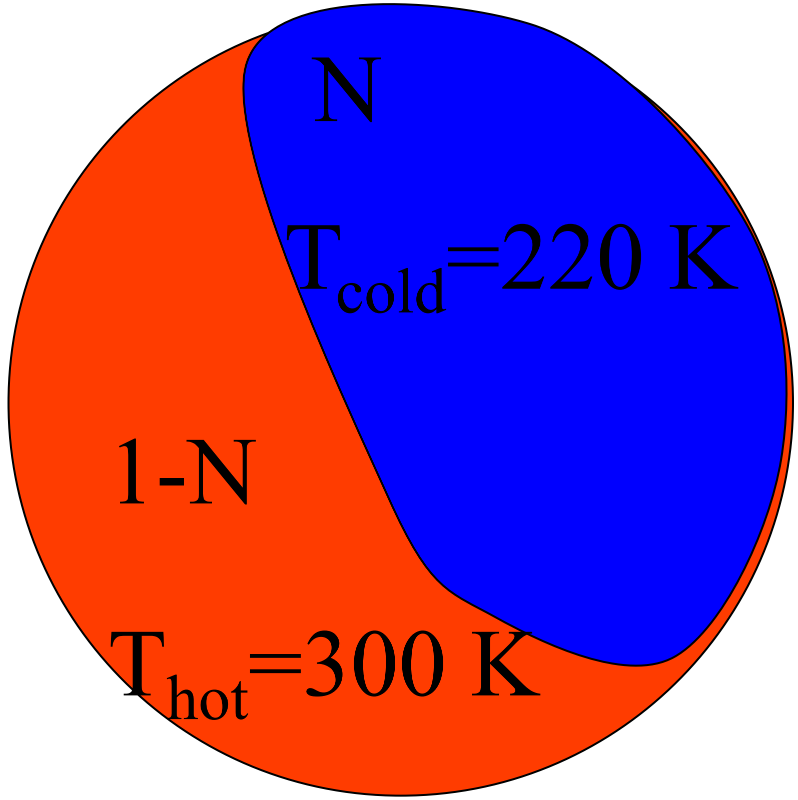
$$B \propto T^{\ \alpha}$$

The temperature sensitivity indicates the power to which the Planck radiance depends on temperature, since B proportional to T^{α} satisfies the equation. For infrared wavelengths,

$$\alpha = c_2\nu/T = c_2/\lambda T.$$

Wavenumber	Typical Scene Temperature	Temperature Sensitivity
900	300	4.32
2500	300	11.99

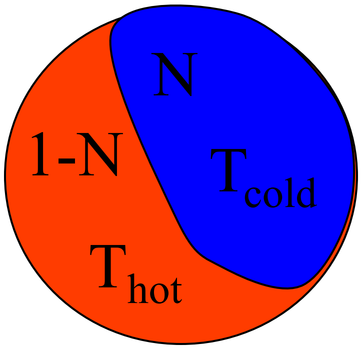
Non-Homogeneous FOV



$$B = N B(T_{cold}) + (1-N) B(T_{hot})$$

$$BT = N BT_{hot} + (1-N) BT_{cold}$$

For NON-UNIFORM FOVs:



$B_{obs}=NB_{cold}+(1\text{-}N)B_{hot}$

$B_{obs}=N\ B_{ref}(T_{cold}/T_{ref})^{\alpha}+(1\text{-}N)\ B_{ref}(T_{hot}/T_{ref})^{\alpha}$

$B_{obs}=B_{ref}(1/T_{ref})^{\alpha}\ (N\ T_{cold}{}^{\alpha}+(1\text{-}N)T_{hot}{}^{\alpha})$

For N=.5

$B_{obs}/B_{ref}=.5\ (1/T_{ref})^{\alpha}\ (\ T_{cold}{}^{\alpha}+T_{hot}{}^{\alpha})$

$B_{obs}/B_{ref}=.5\ (1/T_{ref}T_{cold})^{\alpha}\ (1+(T_{hot}/\ T_{cold})^{\alpha})$

The greater α the more predominant the hot term

At 4 μm (α=12) the hot term more dominating than at 11 μm (α=4)

Consequences

- At 4 μm (α=12) clouds look smaller than at 11 μm (α=4)
- In presence of fires the difference BT_4-BT_{11} is larger than the solar contribution
- The different response in these 2 windows allow for cloud detection and for fire detection

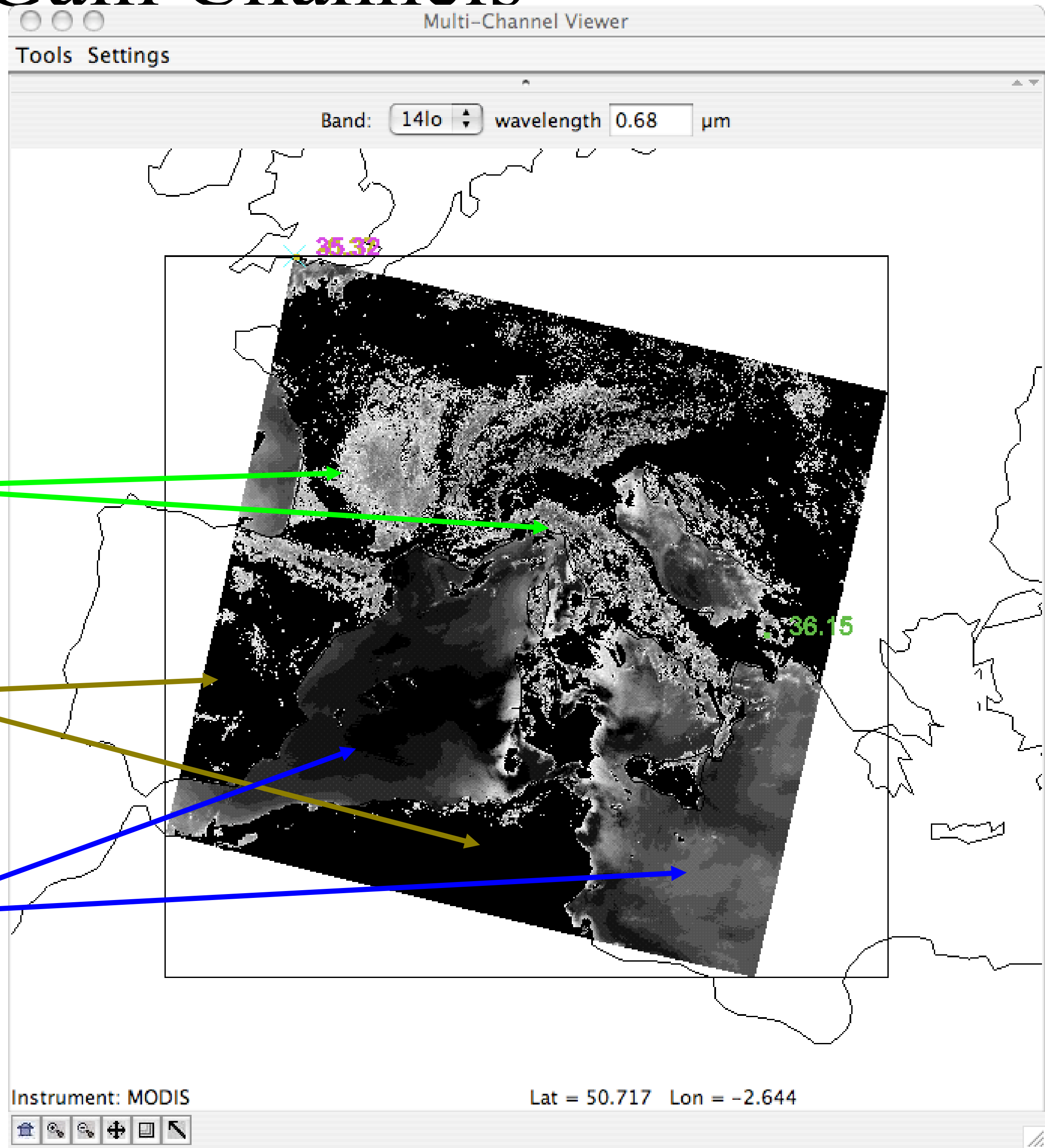
Low Gain Channels

Band 14 low
0.68 µm

Vegetated areas
Are visible

Saturation over
Barren Soil

Visible details
over water



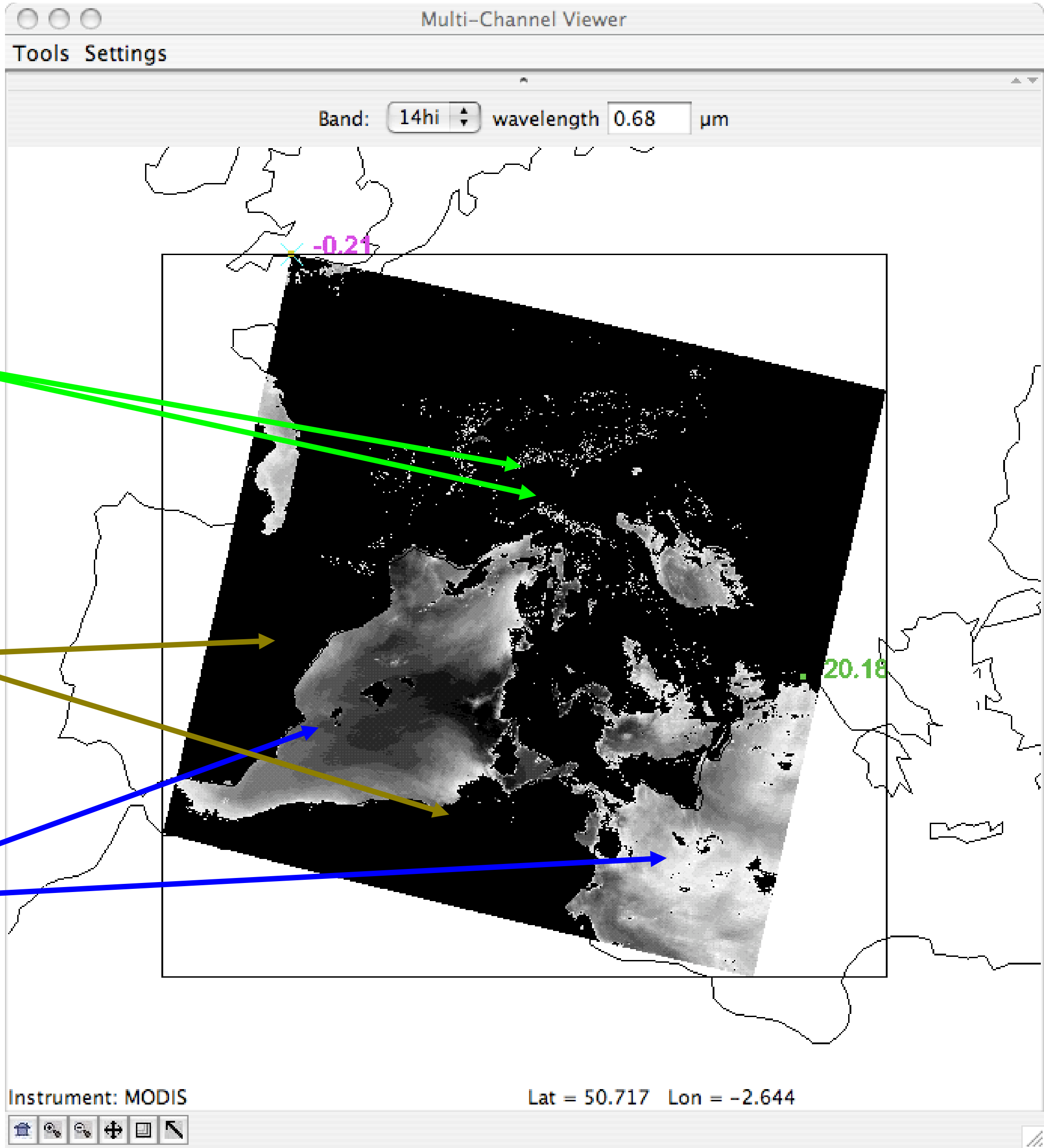
High Gain Channels

Band 14 hi
0.68 µm

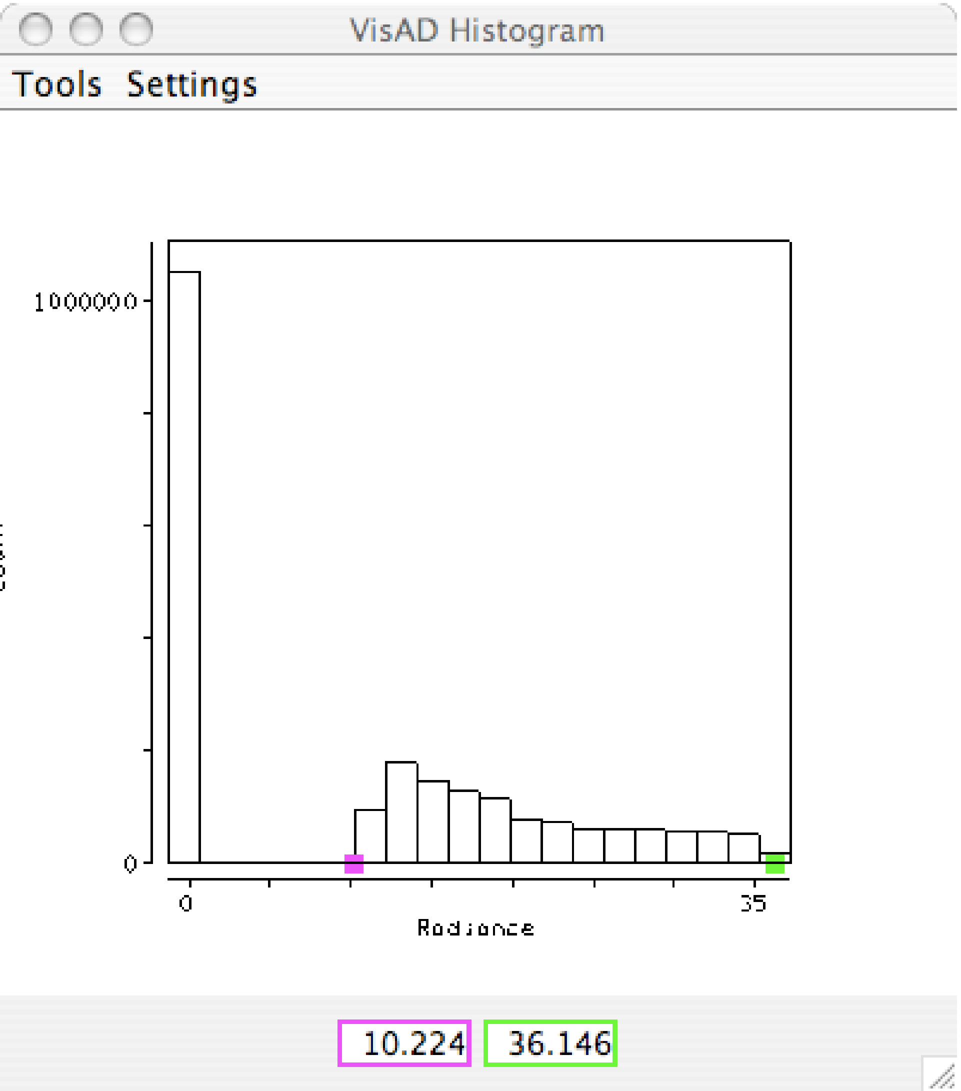
Saturation over
Vegetated areas
little barely visible

Saturation over
Barren Soil

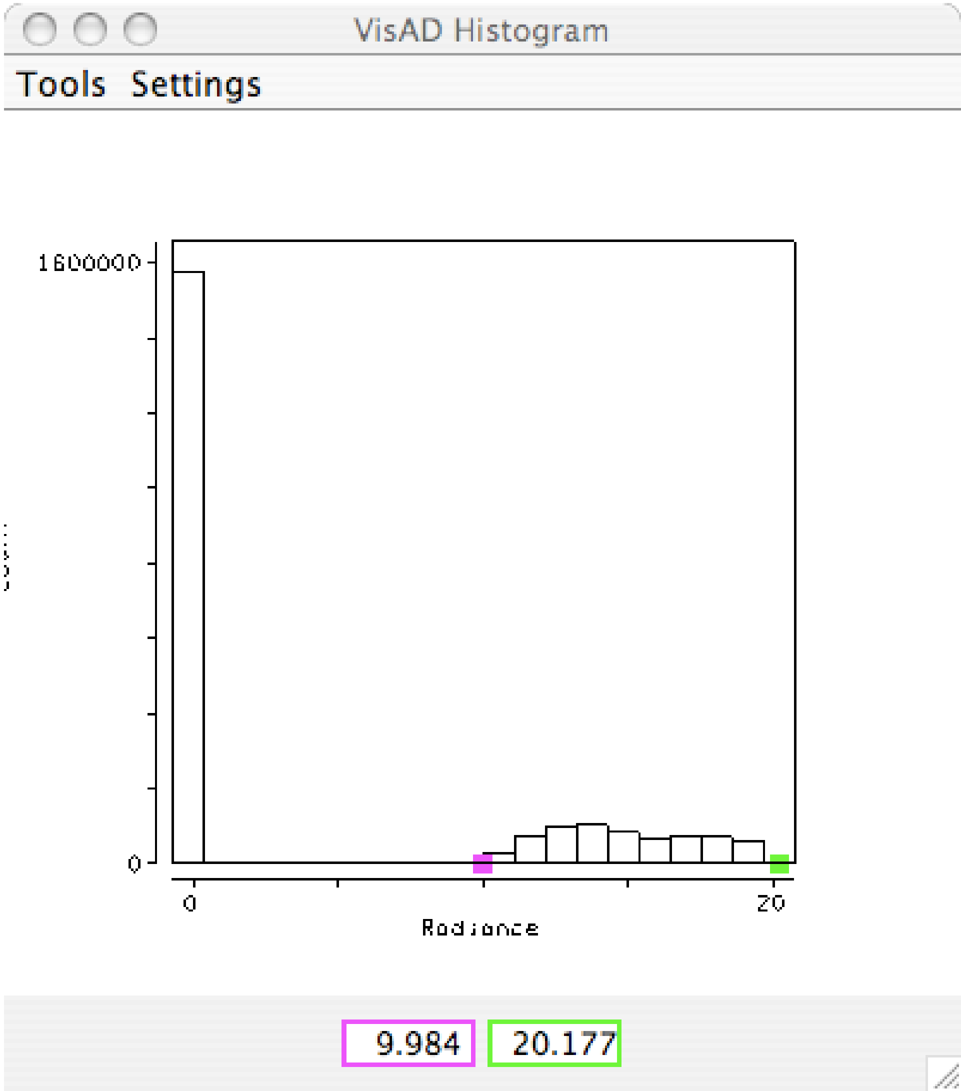
Visible details
over water



Range for Band 14 low 0.68 µm



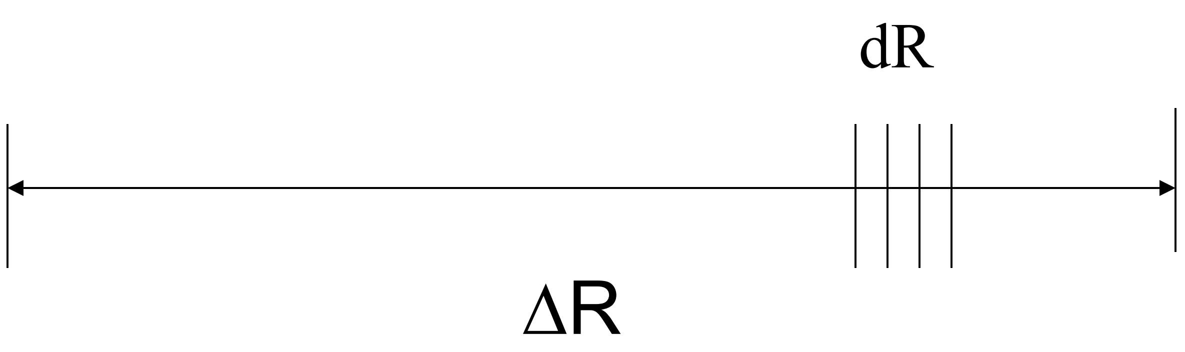
Range for Band 14 high 0.68 µm



Bit Depth and Value Range

- With 12 bits 2^{12} integer numbers can be represented
  - Given ΔR, the range of radiances we want to observe, the smallest observable variation is $\Delta R / 2^{12}$
  - Given dR smallest observable variation, the range of observable radiances is $dR * 2^{12}$

  For this reason Band 14low (larger range) is used for cloud detection and Band 14hi (smaller range) is used for ocean products



Conclusions

- Planck Function: at any wavenumber/wavelength relates the temperature of the observed target to its radiance (for Blackbodies);
- Thermal Sensitivity: different emissive channels respond differently to target temperature variations. Thermal Sensitivity helps in explaining why, and allows for cloud and fire detection;
- Bit Depth: key concepts are spectral range and number of bits.