Radiative Transfer in the Atmosphere

Lectures in Benevento
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Paul Menzel
UW/CIMSS/AOS

Using wavelengths

Planck's Law $B(\lambda,T) = c_1 / \lambda^5 / [e^{c_2/\lambda T} - 1]$ (mW/m²/ster/cm)

where
λ = wavelengths in cm
T = temperature of emitting surface (deg K)
$c_1 = 1.191044 \times 10\text{-}5$ (mW/m²/ster/cm^{-4})
$c_2 = 1.438769$ (cm deg K)

Wien's Law $dB(\lambda_{max},T) / d\lambda = 0$ where $\lambda(max) = .2897/T$
indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\lambda_{max},T) \sim T^5$.

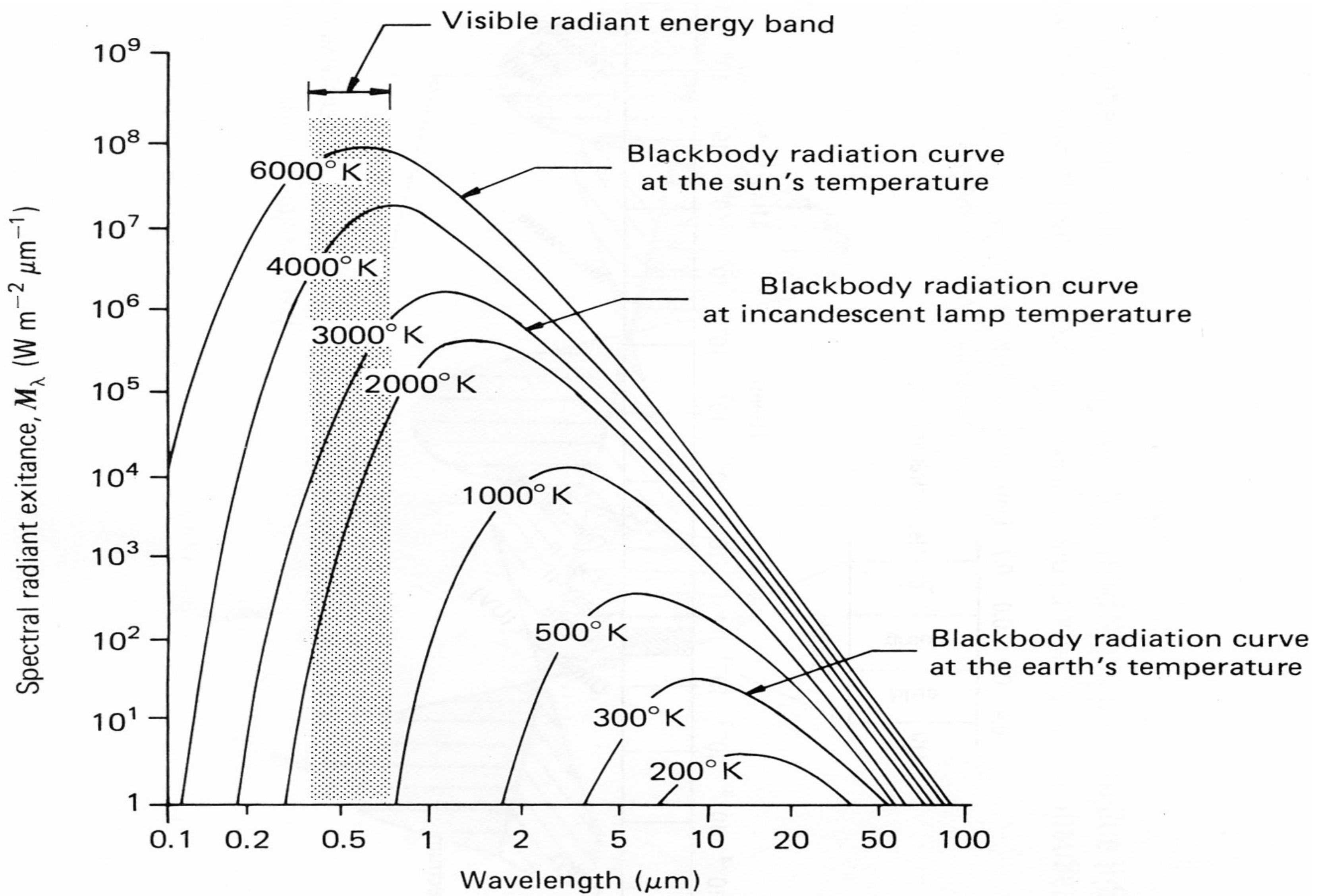
Stefan-Boltzmann Law $E = \pi \int_o^{\infty} B(\lambda,T)\, d\lambda = \sigma T^4$, where $\sigma = 5.67 \times 10\text{-}8$ W/m2/deg4.

states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

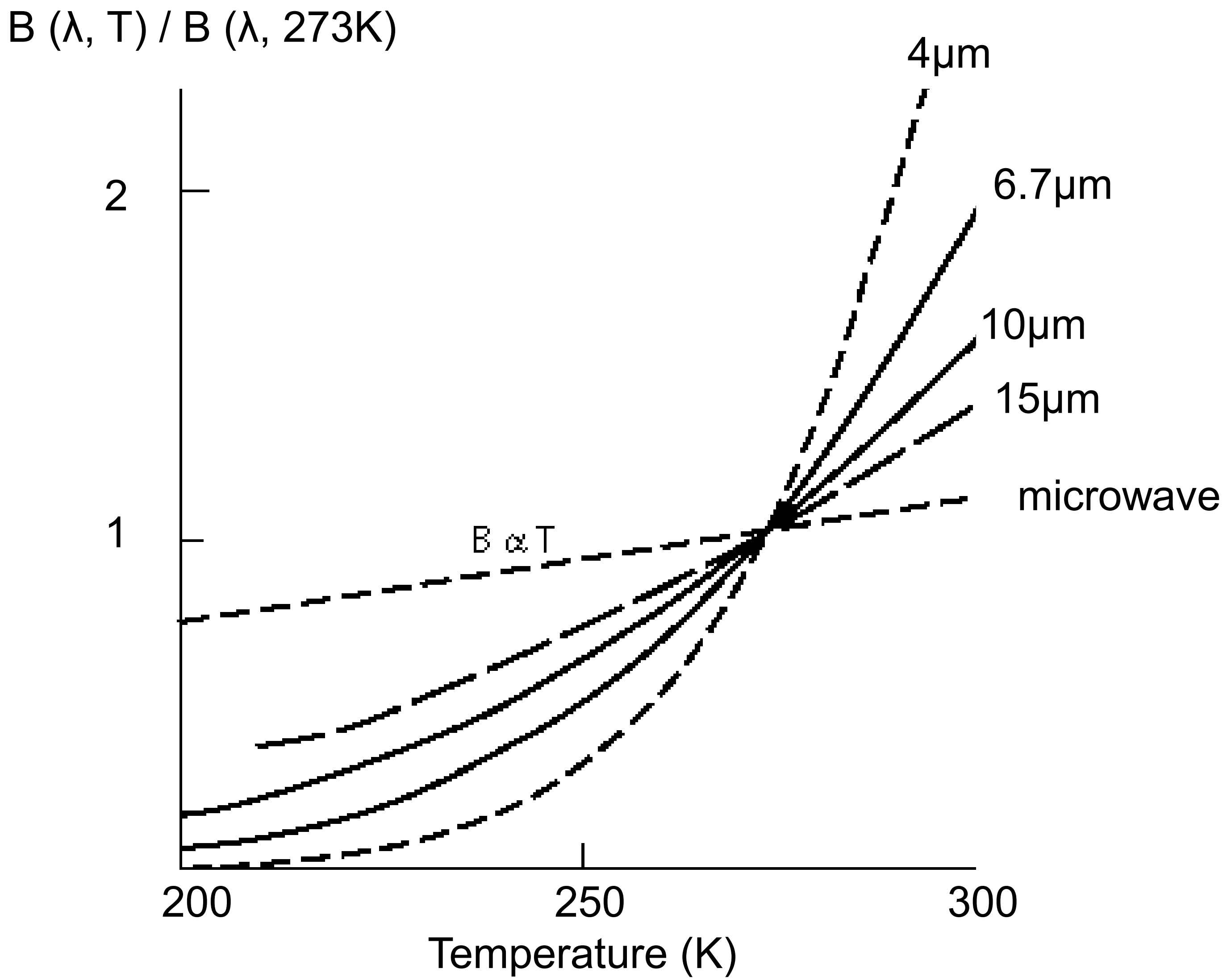
Brightness Temperature

$$T = c_2 / [\lambda \ln(\frac{c_1}{\lambda^5 B_\lambda} + 1)]$$ is determined by inverting Planck function

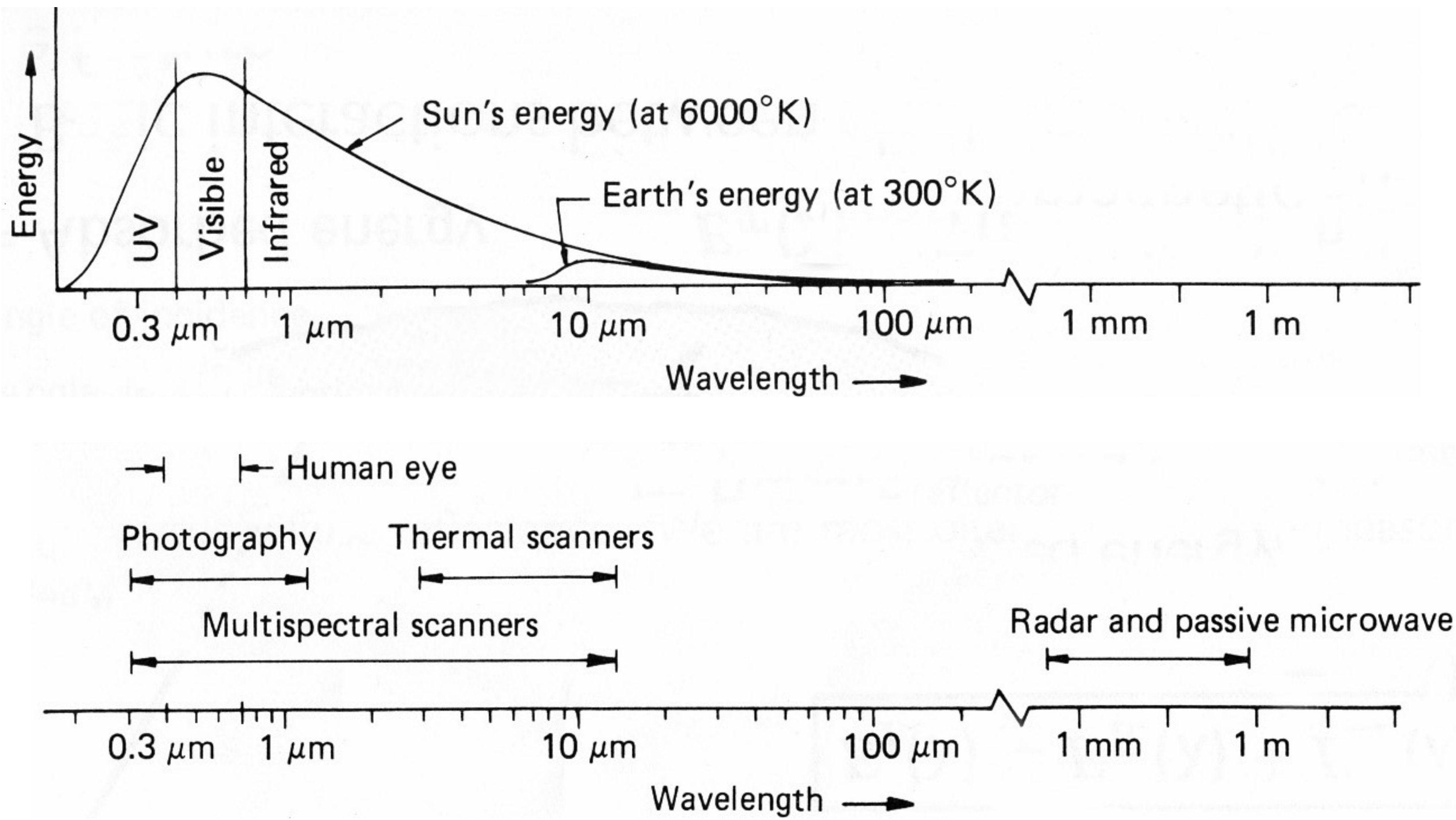
Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures

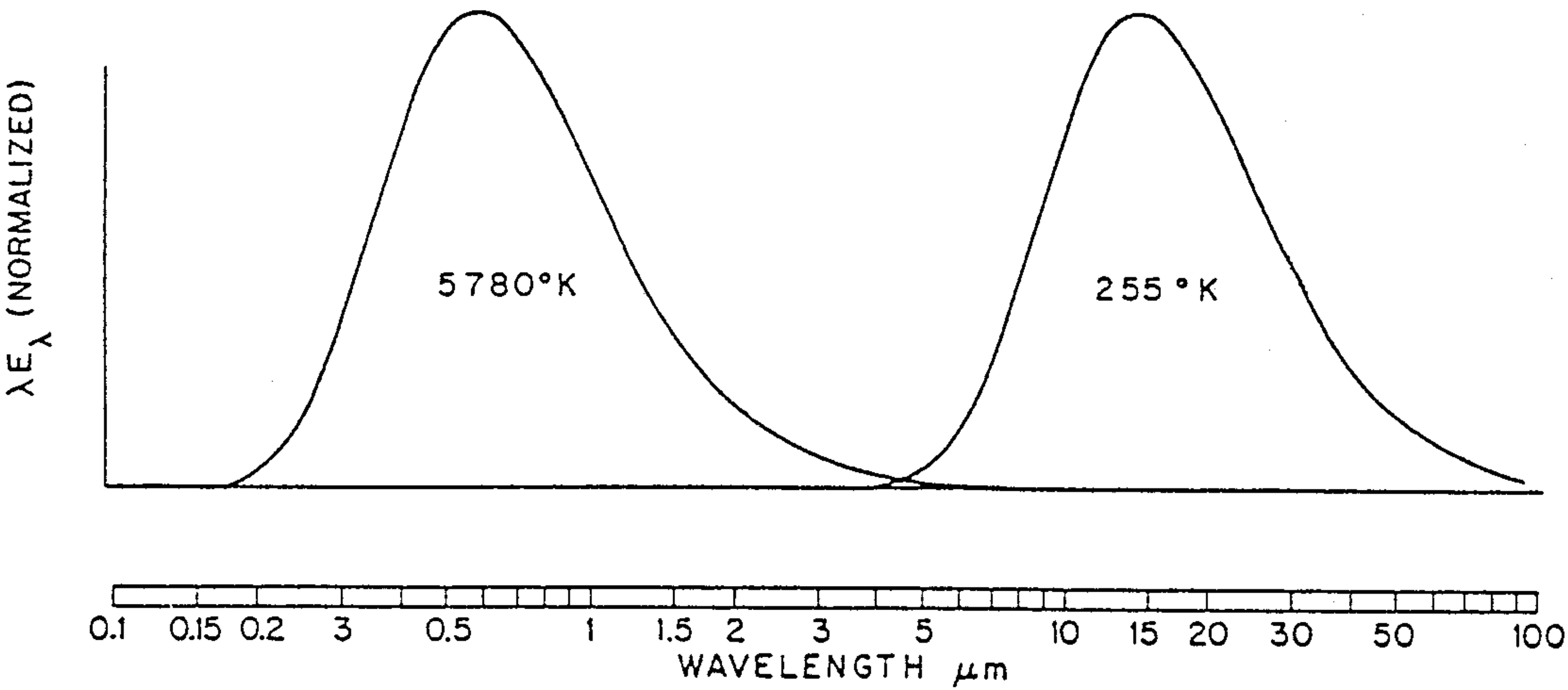


Temperature Sensitivity of B(λ,T) for typical earth scene temperatures



Spectral Characteristics of Energy Sources and Sensing Systems





Normalized black body spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength so that the area under the curves is proportional to irradiance.

Relevant Material in Applications of Meteorological Satellites

Emission, Absorption, Reflection, and Scattering

Blackbody radiation B_λ represents the upper limit to the amount of radiation that a real substance may emit at a given temperature for a given wavelength.

Emissivity ε_λ is defined as the fraction of emitted radiation R_λ to Blackbody radiation,

$$\varepsilon_\lambda = R_\lambda / B_\lambda \ .$$

In a medium at thermal equilibrium, what is absorbed is emitted (what goes in comes out) so

$$a_\lambda = \varepsilon_\lambda \ .$$

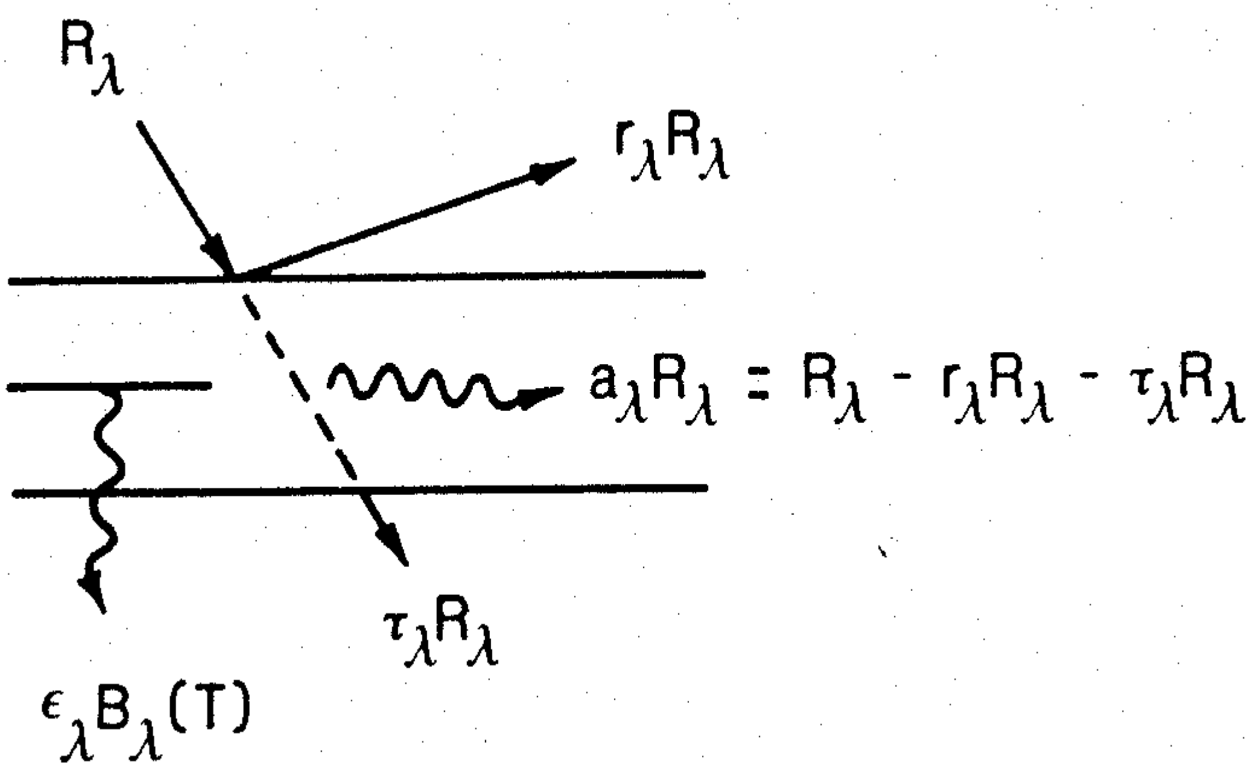
Thus, materials which are strong absorbers at a given wavelength are also strong emitters at that wavelength; similarly weak absorbers are weak emitters.

If a_λ, r_λ, and τ_λ represent the fractional absorption, reflectance, and transmittance, respectively, then conservation of energy says

$$a_\lambda + r_\lambda + \tau_\lambda = 1 \ .$$

For a blackbody $a_\lambda = 1$, it follows that $r_\lambda = 0$ and $\tau_\lambda = 0$ for blackbody radiation. Also, for a perfect window $\tau_\lambda = 1$, $a_\lambda = 0$ and $r_\lambda = 0$. For any opaque surface $\tau_\lambda = 0$, so radiation is either absorbed or reflected $a_\lambda + r_\lambda = 1$.

At any wavelength, strong reflectors are weak absorbers (i.e., snow at visible wavelengths), and weak reflectors are strong absorbers (i.e., asphalt at visible wavelengths).



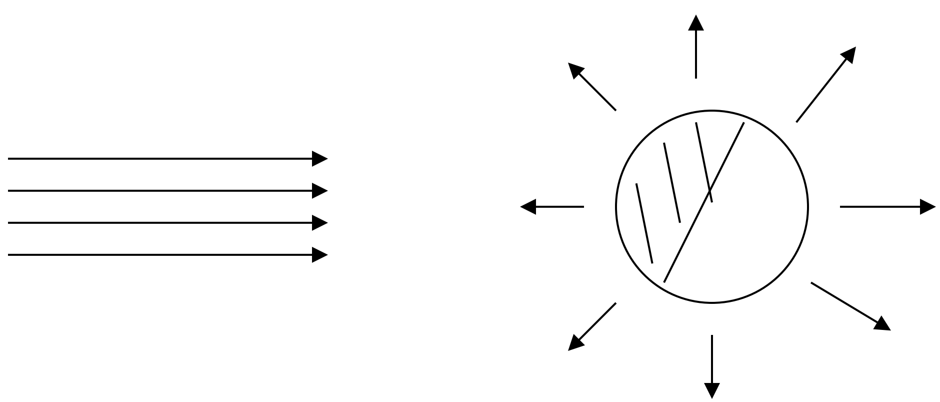
'ENERGY CONSERVATION'

Planetary Albedo

Planetary albedo is defined as the fraction of the total incident solar irradiance, S, that is reflected back into space. Radiation balance then requires that the absorbed solar irradiance is given by

$$E = (1 - A)\, S/4.$$

The factor of one-fourth arises because the cross sectional area of the earth disc to solar radiation, πr^2, is one-fourth the earth radiating surface, $4\pi r^2$. Thus recalling that $S = 1380\ Wm^{-2}$, if the earth albedo is 30 percent, then $E = 241\ Wm^{-2}$.



Selective Absorption and Transmission

Assume that the earth behaves like a blackbody and that the atmosphere has an absorptivity a_S for incoming solar radiation and a_L for outgoing longwave radiation. Let Y_a be the irradiance emitted by the atmosphere (both upward and downward); Y_s the irradiance emitted from the earth's surface; and E the solar irradiance absorbed by the earth-atmosphere system. Then, radiative equilibrium requires

$$E - (1-a_L)\, Y_s - Y_a = 0 \text{ , at the top of the atmosphere,}$$
$$(1-a_S)\, E - Y_s + Y_a = 0 \text{ , at the surface.}$$

Solving yields

$$Y_s = \frac{(2-a_S)}{(2-a_L)} E \text{ , and}$$

$$Y_a = \frac{(2-a_L) - (1-a_L)(2-a_S)}{(2-a_L)} E \, .$$

Since $a_L > a_S$, the irradiance and hence the radiative equilibrium temperature at the earth surface is increased by the presence of the atmosphere. With $a_L = .8$ and $a_S = .1$ and E = 241 Wm^{-2}, Stefans Law yields a blackbody temperature at the surface of 286 K, in contrast to the 255 K it would be if the atmospheric absorptance was independent of wavelength ($a_S = a_L$). The atmospheric gray body temperature in this example turns out to be 245 K.

Incoming solar

Outgoing IR

$\downarrow E$ $\qquad$ $\uparrow (1-a_l)\,Y_s$ $\quad$ $\uparrow Y_a$

top of the atmosphere

$\downarrow (1-a_s)\,E$ $\quad$ $\uparrow Y_s$ $\qquad$ $\downarrow Y_a$

earth surface.

$$Y_s = \frac{(2-a_S)}{(2-a_L)}\,E = \sigma T_s^4$$

Expanding on the previous example, let the atmosphere be represented by two layers and let us compute the vertical profile of radiative equilibrium temperature. For simplicity in our two layer atmosphere, let $a_S = 0$ and $a_L = a = .5$, u indicate upper layer, l indicate lower layer, and s denote the earth surface. Schematically we have:

$\downarrow E$	$\uparrow (1\text{-}a)^2 Y_s$	$\uparrow (1\text{-}a) Y_l$	$\uparrow Y_u$	top of the atmosphere
$\downarrow E$	$\uparrow (1\text{-}a) Y_s$	$\uparrow Y_l$	$\downarrow Y_u$	middle of the atmosphere
$\downarrow E$	$\uparrow Y_s$	$\downarrow Y_l$	$\downarrow (1\text{-}a) Y_u$	earth surface.

Radiative equilibrium at each surface requires

$$
\begin{aligned}
E &= .25\, Y_s + .5\, Y_l + Y_u\,, \\
E &= .5\, Y_s + Y_l - Y_u\,, \\
E &= Y_s - Y_l - .5\, Y_u\,.
\end{aligned}
$$

Solving yields $Y_s = 1.6$ E, $Y_l = .5$ E and $Y_u = .33$ E. The radiative equilibrium temperatures (blackbody at the surface and gray body in the atmosphere) are readily computed.

$$
\begin{aligned}
T_s &= [1.6E / \sigma]^{1/4} = 287\ \text{K}\,, \\
T_l &= [0.5E / 0.5\sigma]^{1/4} = 255\ \text{K}\,, \\
T_u &= [0.33E / 0.5\sigma]^{1/4} = 231\ \text{K}\,.
\end{aligned}
$$

Thus, a crude temperature profile emerges for this simple two-layer model of the atmosphere.

Transmittance

Transmission through an absorbing medium for a given wavelength is governed by the number of intervening absorbing molecules (path length u) and their absorbing power (k_λ) at that wavelength. Beer's law indicates that transmittance decays exponentially with increasing path length

$$\tau_\lambda (z \to \infty) = e^{-k_\lambda u(z)}$$

where the path length is given by $u(z) = \int_z^{\infty} \rho \, dz$.

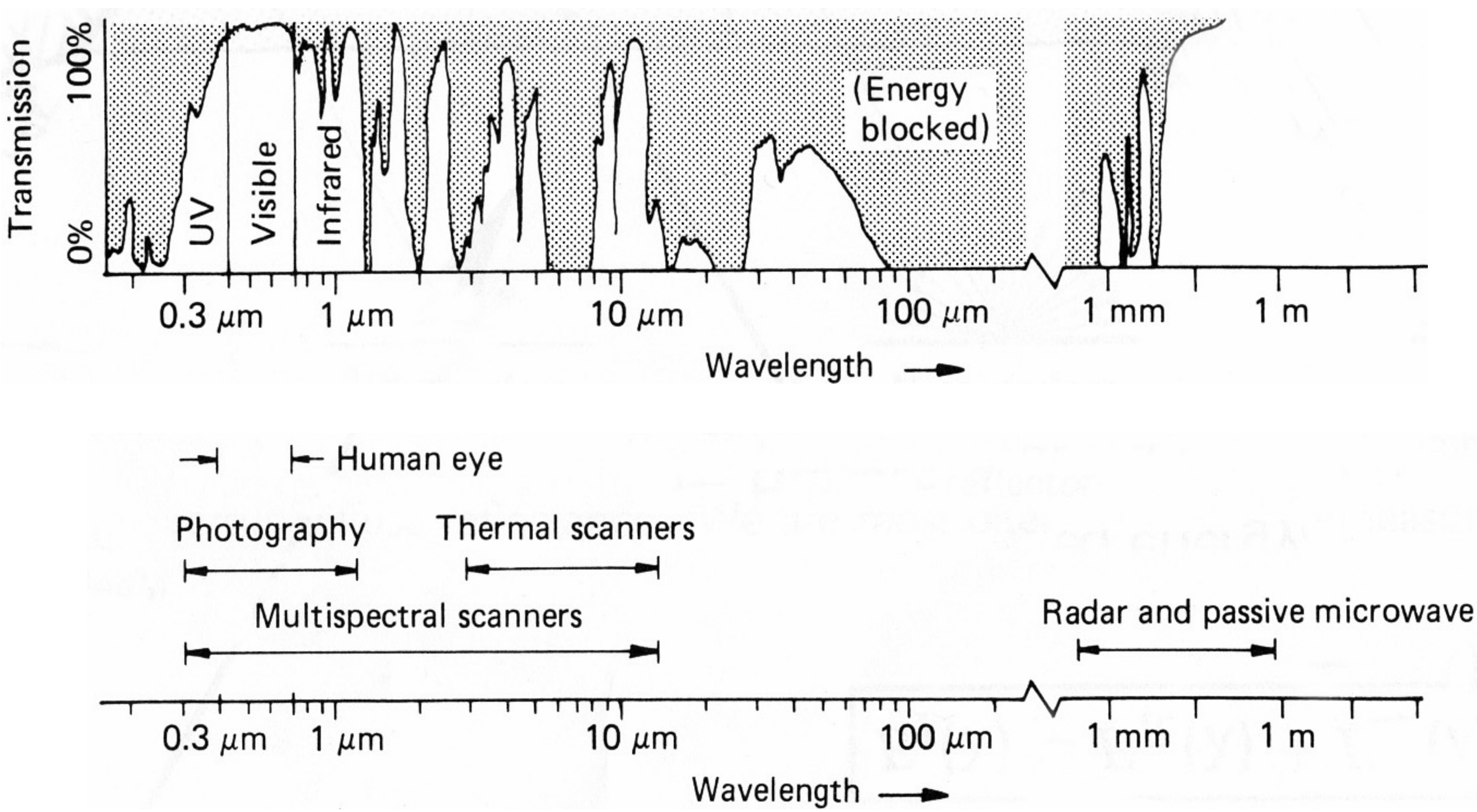
$k_\lambda u$ is a measure of the cumulative depletion that the beam of radiation has experienced as a result of its passage through the layer and is often called the optical depth σ_λ.

Realizing that the hydrostatic equation implies $g \rho \, dz = - q \, dp$

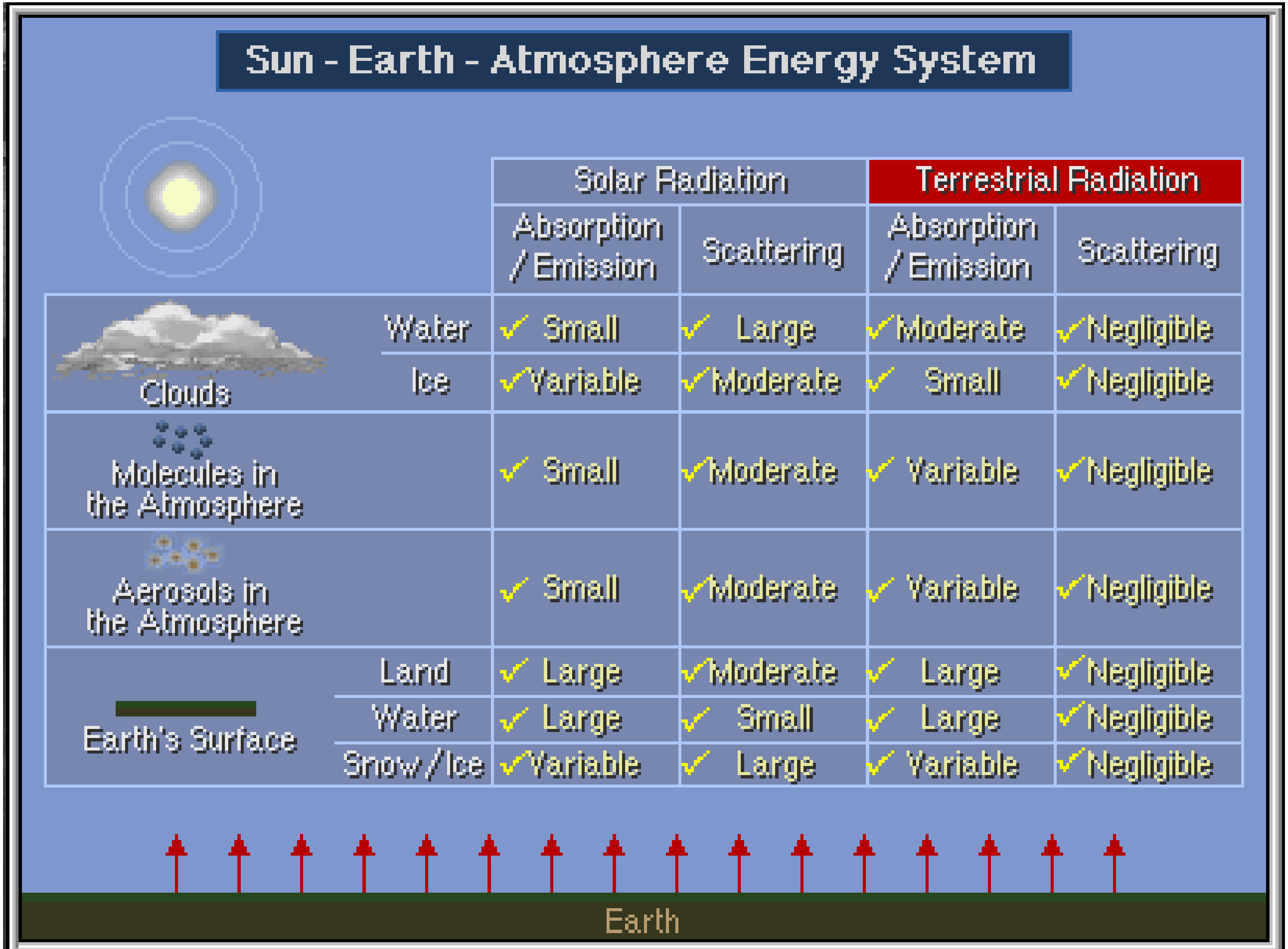
where q is the mixing ratio and ρ is the density of the atmosphere, then

$$u(p) = \int_o^p q \, g^{-1} \, dp \qquad \text{and} \qquad \tau_\lambda (p \to o) = e^{-k_\lambda u(p)} \, .$$

Spectral Characteristics of Atmospheric Transmission and Sensing Systems

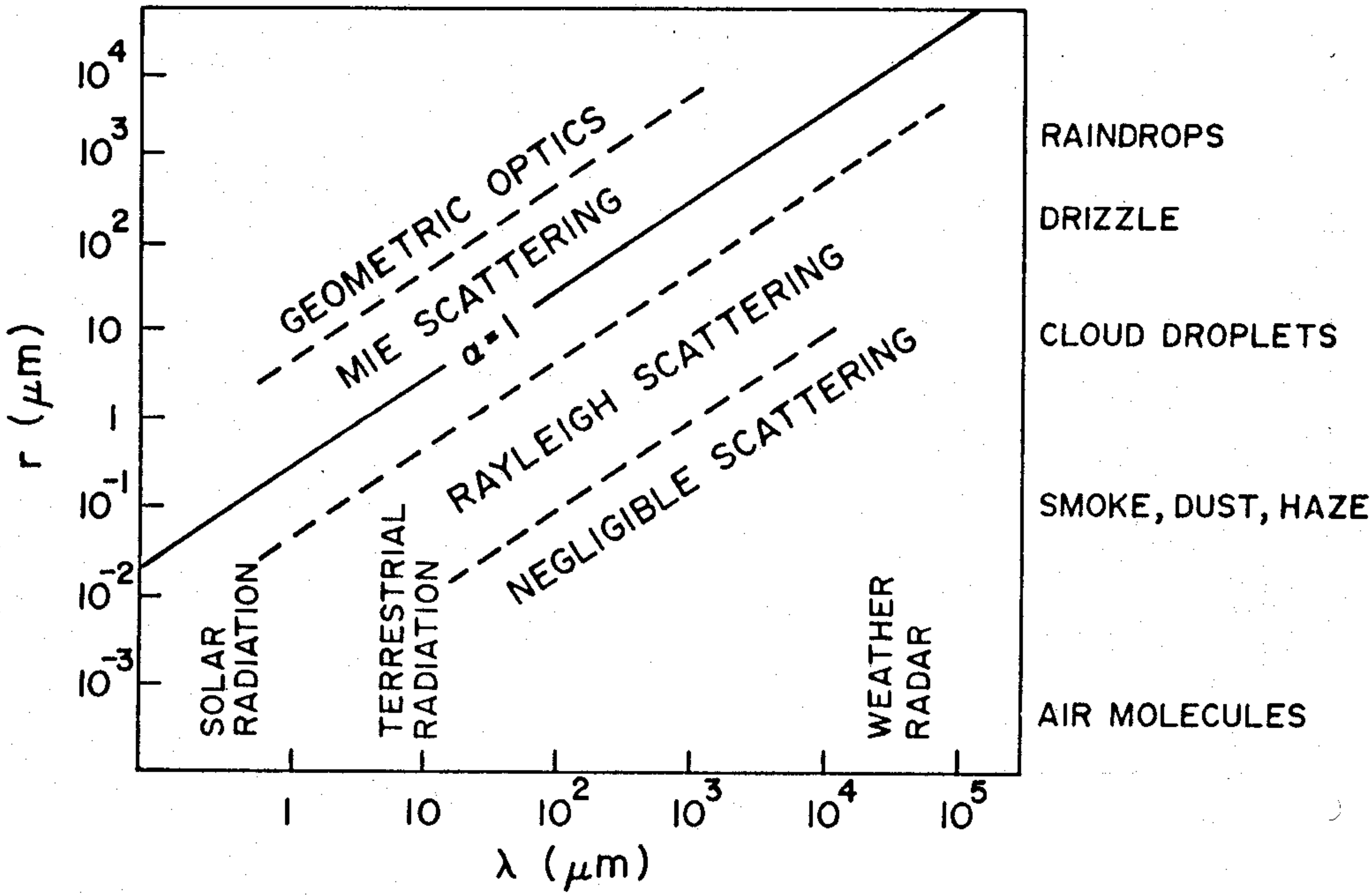


Relative Effects of Radiative Processes



Sun - Earth - Atmosphere Energy System

		Solar Radiation		Terrestrial Radiation	
		Absorption / Emission	Scattering	Absorption / Emission	Scattering
Clouds	Water	✓ Small	✓ Large	✓ Moderate	✓ Negligible
	Ice	✓ Variable	✓ Moderate	✓ Small	✓ Negligible
Molecules in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
Aerosols in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
Earth's Surface	Land	✓ Large	✓ Moderate	✓ Large	✓ Negligible
	Water	✓ Large	✓ Small	✓ Large	✓ Negligible
	Snow / Ice	✓ Variable	✓ Large	✓ Variable	✓ Negligible



GEOMETRIC OPTICS
MIE SCATTERING
α=1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES
r (μm)
λ (μm)
10⁴
10³
10²
10
1
10⁻¹
10⁻²
10⁻³
10⁵

Aerosol Size Distribution

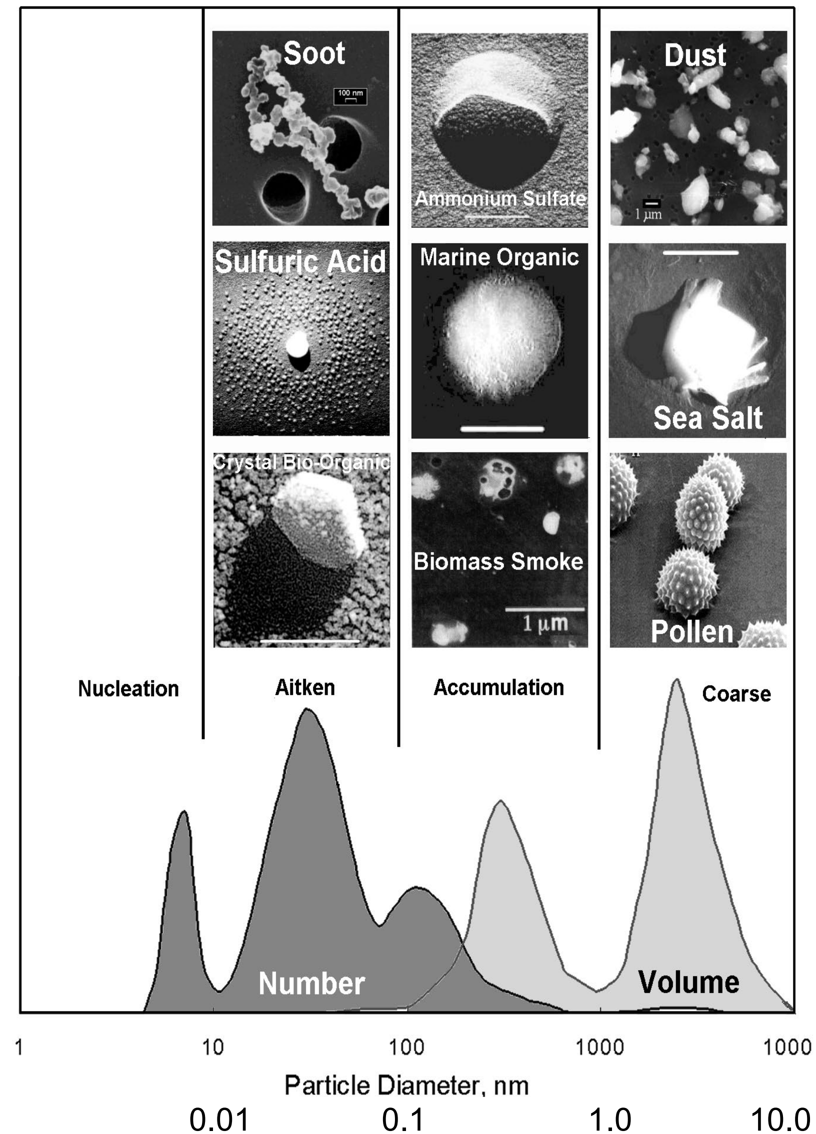
There are 3 modes :

- « **nucleation** »: radius is between 0.002 and 0.05 µm. They result from combustion processes, photo-chemical reactions, etc.

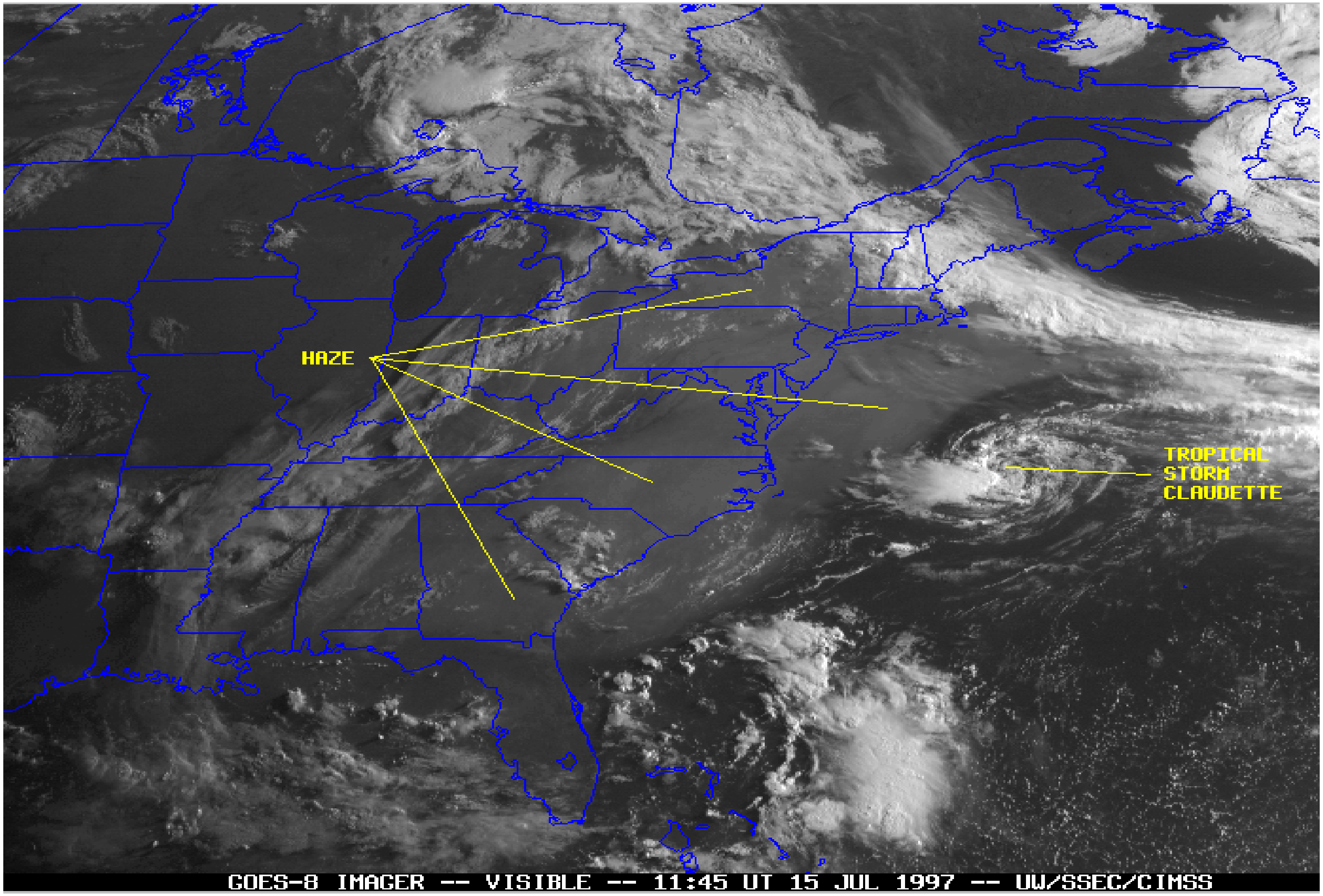
- « **accumulation »:** radius is between 0.05 µm and 0.5 µm. Coagulation processes.

- « **coarse** »: larger than 1 µm. From mechanical processes like aeolian erosion.

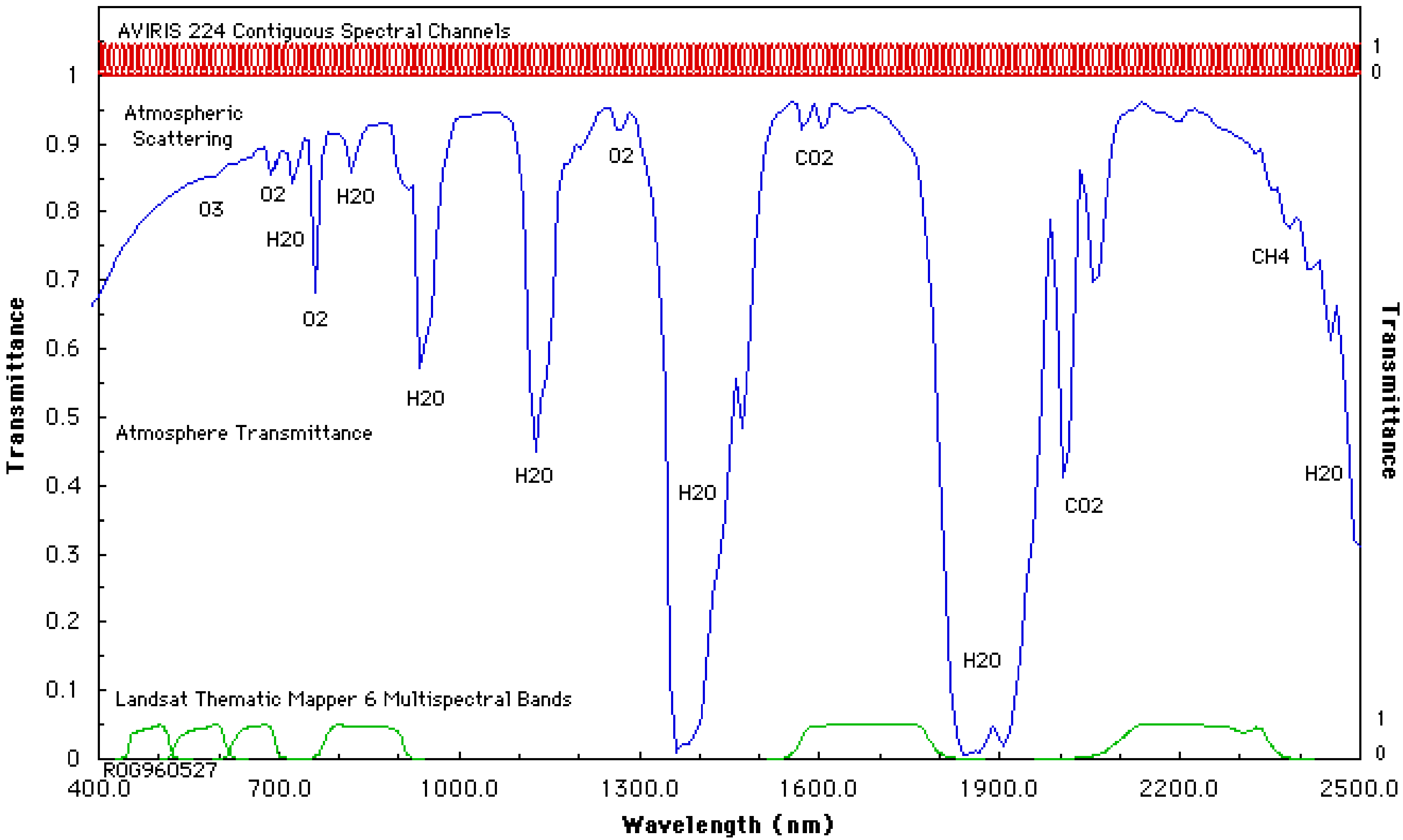
« fine » particles (nucleation and accumulation) result from anthropogenic activities, coarse particles come from natural processes.



Scattering of early morning sun light from haze

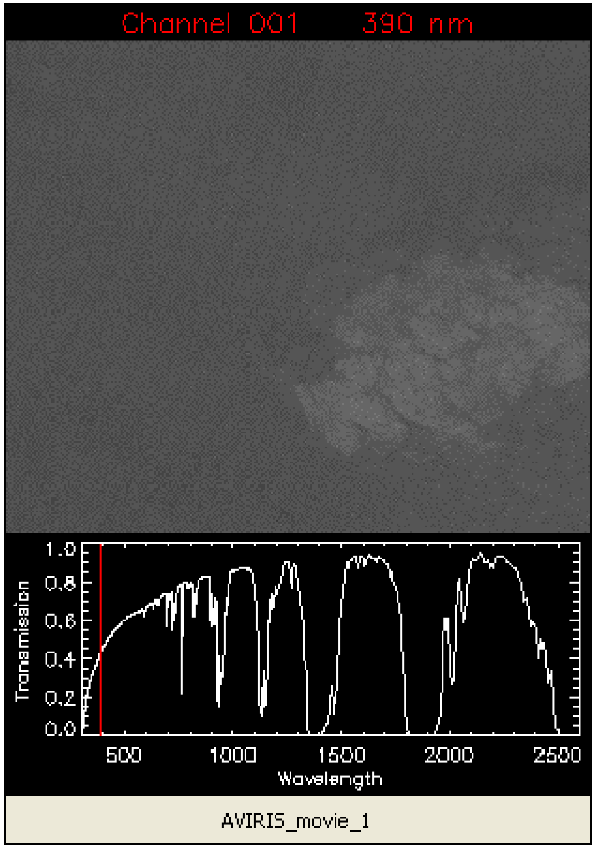


Measurements in the Solar Reflected Spectrum across the region covered by AVIRIS



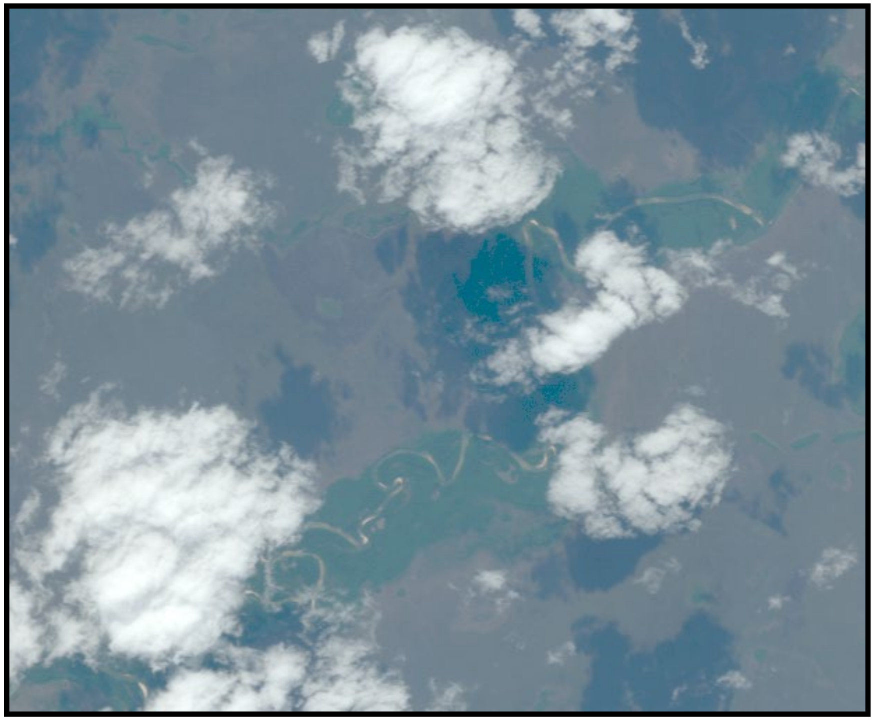
AVIRIS Movie #1

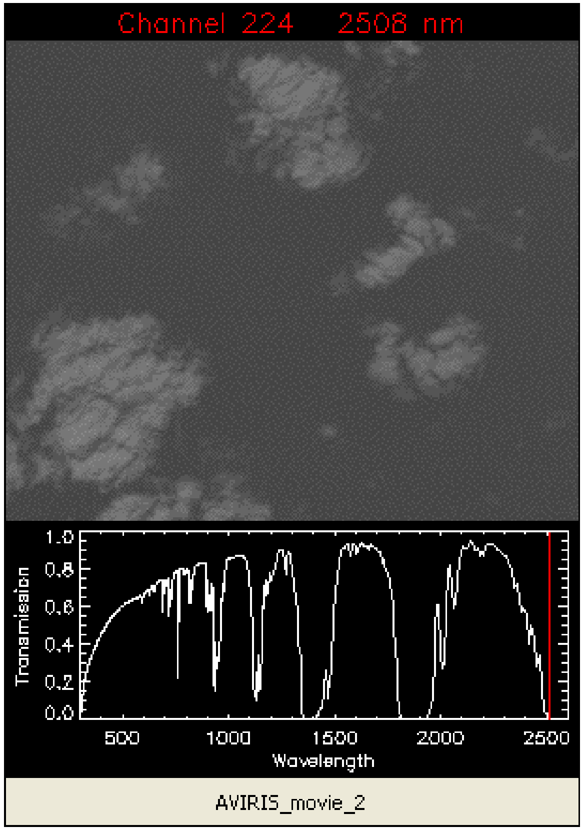
AVIRIS Image - Linden CA 20-Aug-1992
224 Spectral Bands: 0.4 - 2.5 μm
Pixel: 20m x 20m Scene: 10km x 10km



AVIRIS Movie #2

AVIRIS Image - Porto Nacional, Brazil
20-Aug-1995
224 Spectral Bands: 0.4 - 2.5 μm
Pixel: 20m x 20m   Scene: 10km x 10km





Relevant Material in Applications of Meteorological Satellites

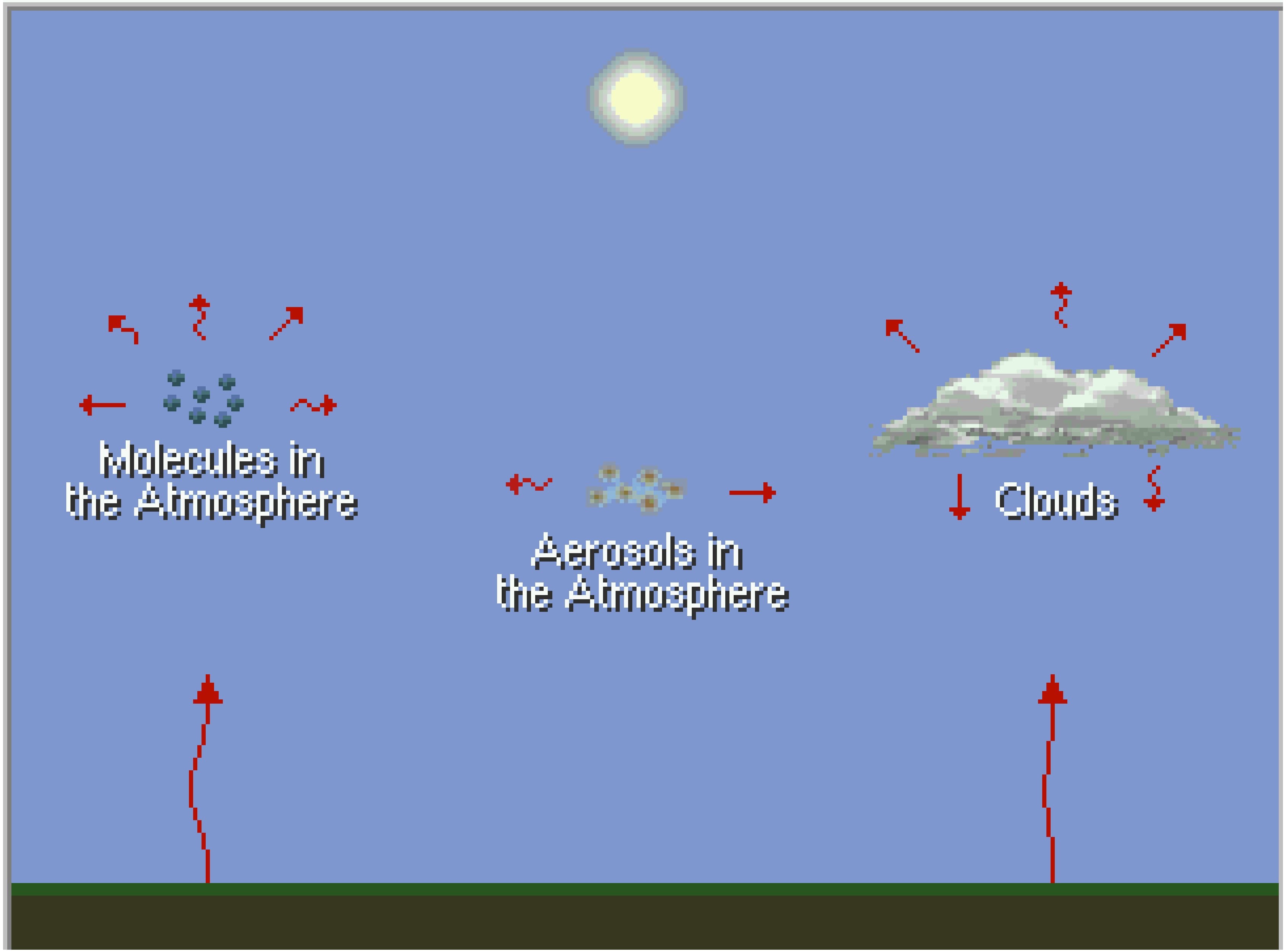
Radiative Transfer Equation

The radiance leaving the earth-atmosphere system sensed by a satellite borne radiometer is the sum of radiation emissions from the earth-surface and each atmospheric level that are transmitted to the top of the atmosphere. Considering the earth's surface to be a blackbody emitter (emissivity equal to unity), the upwelling radiance intensity, I_λ, for a cloudless atmosphere is given by the expression

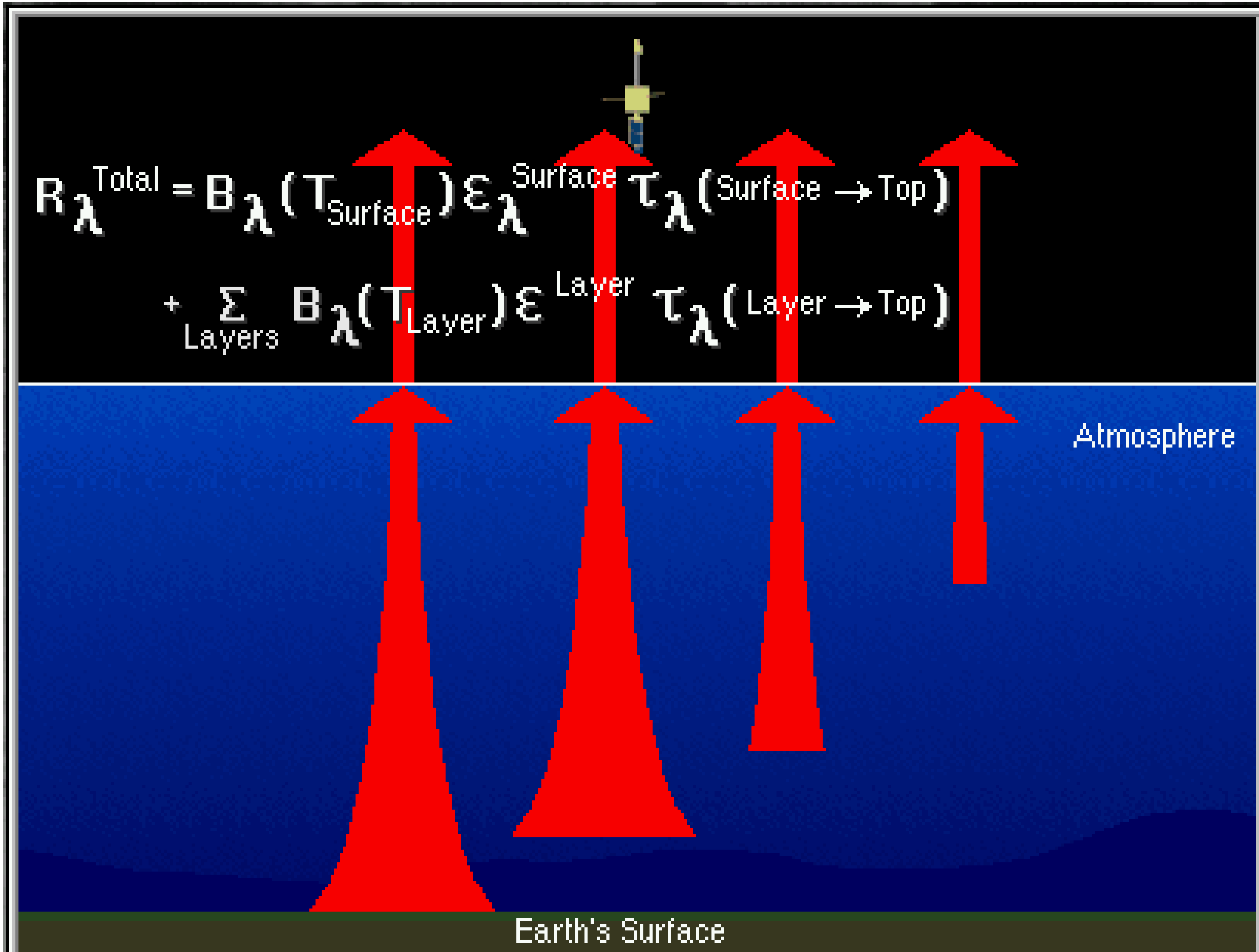
$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T_{sfc})\, \tau_\lambda(sfc - top) + \sum_{layers} \varepsilon_\lambda^{layer} B_\lambda(T_{layer})\, \tau_\lambda(layer - top)$$

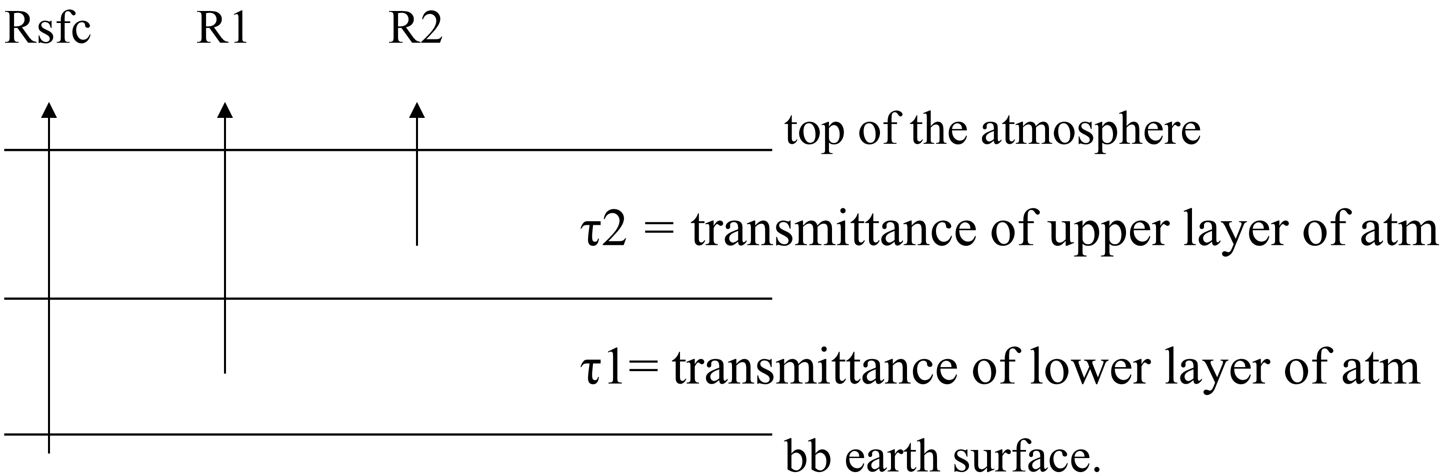
where the first term is the surface contribution and the second term is the atmospheric contribution to the radiance to space.

Re-emission of Infrared Radiation



Radiative Transfer through the Atmosphere





$$R_{obs} = R_{sfc}\, \tau_1\, \tau_2 + R_1\, (1-\tau_1)\, \tau_2 + R_2\, (1-\tau_2)$$

In standard notation,

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s))\, \tau_\lambda(p_s) + \sum_p \varepsilon_\lambda(\Delta p)\, B_\lambda(T(p))\, \tau_\lambda(p)$$

The emissivity of an infinitesimal layer of the atmosphere at pressure p is equal to the absorptance (one minus the transmittance of the layer). Consequently,

$$\varepsilon_\lambda(\Delta p)\, \tau_\lambda(p) = [1 - \tau_\lambda(\Delta p)]\, \tau_\lambda(p)$$

Since transmittance is an exponential function of depth of absorbing constituent,

$$\tau_\lambda(\Delta p)\, \tau_\lambda(p) = \exp\left[- \int_p^{p+\Delta p} k_\lambda\, q\, g^{-1}\, dp\right] * \exp\left[- \int_o^p k_\lambda\, q\, g^{-1}\, dp\right] = \tau_\lambda(p + \Delta p)$$

Therefore

$$\varepsilon_\lambda(\Delta p)\, \tau_\lambda(p) = \tau_\lambda(p) - \tau_\lambda(p + \Delta p) = - \Delta\tau_\lambda(p) .$$

So we can write

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s))\, \tau_\lambda(p_s) - \sum_p B_\lambda(T(p))\, \Delta\tau_\lambda(p) .$$

which when written in integral form reads

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s))\, \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p))\, [\, d\tau_\lambda(p) / dp\,]\, dp .$$

When reflection from the earth surface is also considered, the Radiative Transfer Equation for infrared radiation can be written

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T_s)\, \tau_\lambda(p_s) + \int_{p_s}^{o} B_\lambda(T(p))\, F_\lambda(p)\, [d\tau_\lambda(p)/ dp]\, dp$$

where

$$F_\lambda(p) = \{ 1 + (1 - \varepsilon_\lambda)\, [\tau_\lambda(p_s) / \tau_\lambda(p)]^2 \}$$

The first term is the spectral radiance emitted by the surface and attenuated by the atmosphere, often called the boundary term and the second term is the spectral radiance emitted to space by the atmosphere directly or by reflection from the earth surface.

The atmospheric contribution is the weighted sum of the Planck radiance contribution from each layer, where the weighting function is [$d\tau_\lambda(p) / dp$]. This weighting function is an indication of where in the atmosphere the majority of the radiation for a given spectral band comes from.

Schwarzchild's equation

At wavelengths of terrestrial radiation, absorption and emission are equally important and must be considered simultaneously. Absorption of terrestrial radiation along an upward path through the atmosphere is described by the relation

$$-dL_\lambda^{abs} = L_\lambda \, k_\lambda \, \rho \sec \varphi \, dz \, .$$

Making use of Kirchhoff's law it is possible to write an analogous expression for the emission,

$$dL_\lambda^{em} = B_\lambda \, d\varepsilon_\lambda = B_\lambda \, da_\lambda = B_\lambda \, k_\lambda \, \rho \sec \varphi \, dz \, ,$$

where B_λ is the blackbody monochromatic radiance specified by Planck's law. Together

$$dL_\lambda = -(L_\lambda - B_\lambda) \, k_\lambda \, \rho \sec \varphi \, dz \, .$$

This expression, known as Schwarzchild's equation, is the basis for computations of the transfer of infrared radiation.

Schwarzschild to RTE

$$dL_\lambda = -(L_\lambda - B_\lambda)\, k_\lambda\, \rho\, dz$$

but

$$d\tau_\lambda = \tau_\lambda\, k\, \rho\, dz \quad \text{since} \quad \tau_\lambda = \exp\left[-k_\lambda \int_z^\infty \rho\, dz\right].$$

so

$$\tau_\lambda\, dL_\lambda = -(L_\lambda - B_\lambda)\, d\tau_\lambda$$

$$\tau_\lambda\, dL_\lambda + L_\lambda\, d\tau_\lambda = B_\lambda d\tau_\lambda$$

$$d\,(L_\lambda\, \tau_\lambda) = B_\lambda d\tau_\lambda$$

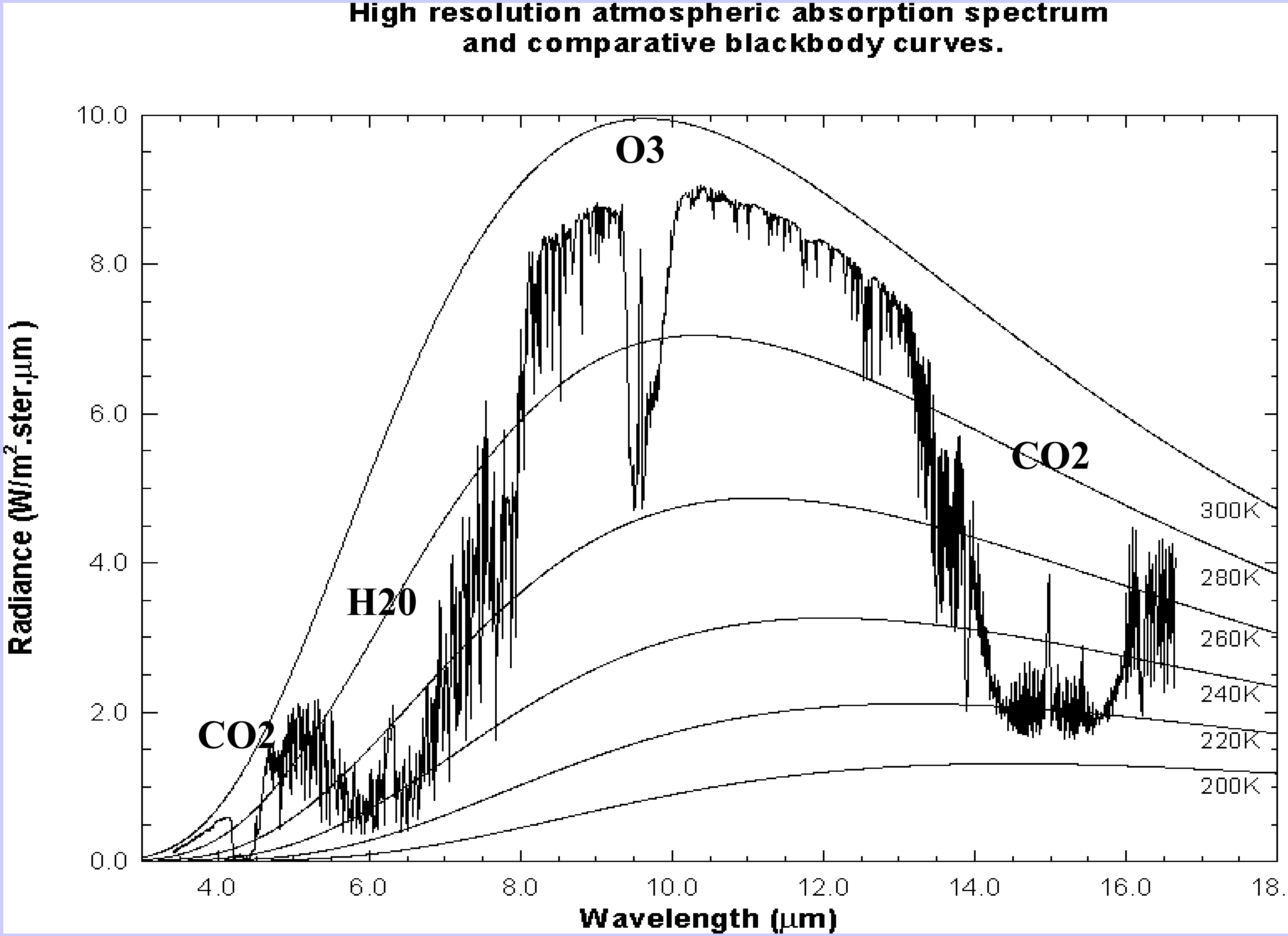
Integrate from 0 to ∞

$$L_\lambda(\infty)\, \tau_\lambda(\infty) - L_\lambda(0)\, \tau_\lambda(0) = \int_0^\infty B_\lambda\, [d\tau_\lambda / dz]\, dz.$$

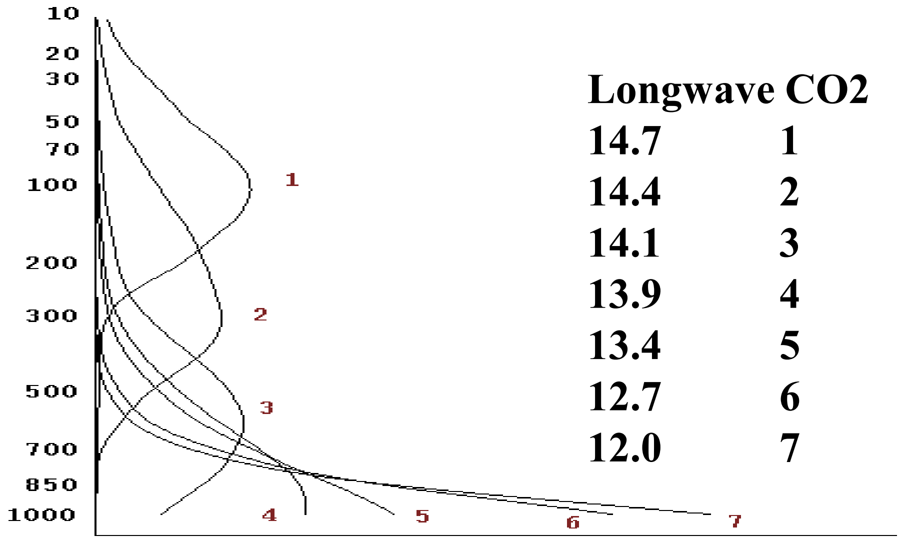
and

$$L_\lambda(\text{sat}) = L_\lambda(\text{sfc})\, \tau_\lambda(\text{sfc}) + \int_0^\infty B_\lambda\, [d\tau_\lambda / dz]\, dz.$$

Earth emitted spectra overlaid on Planck function envelopes

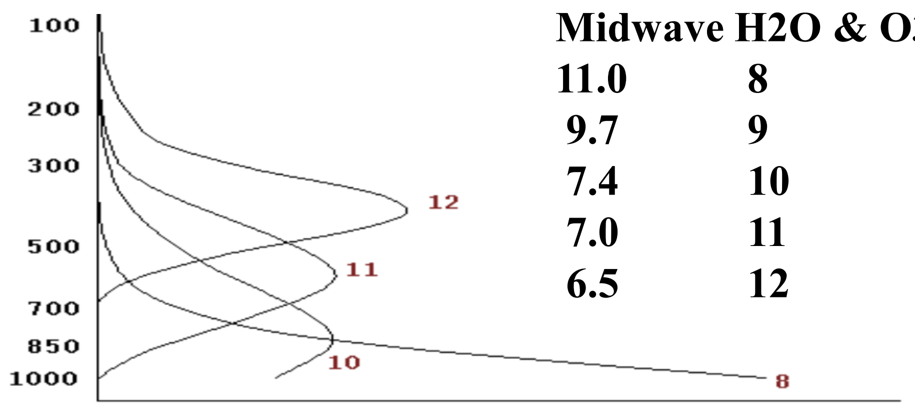


Weighting Functions



Longwave CO2

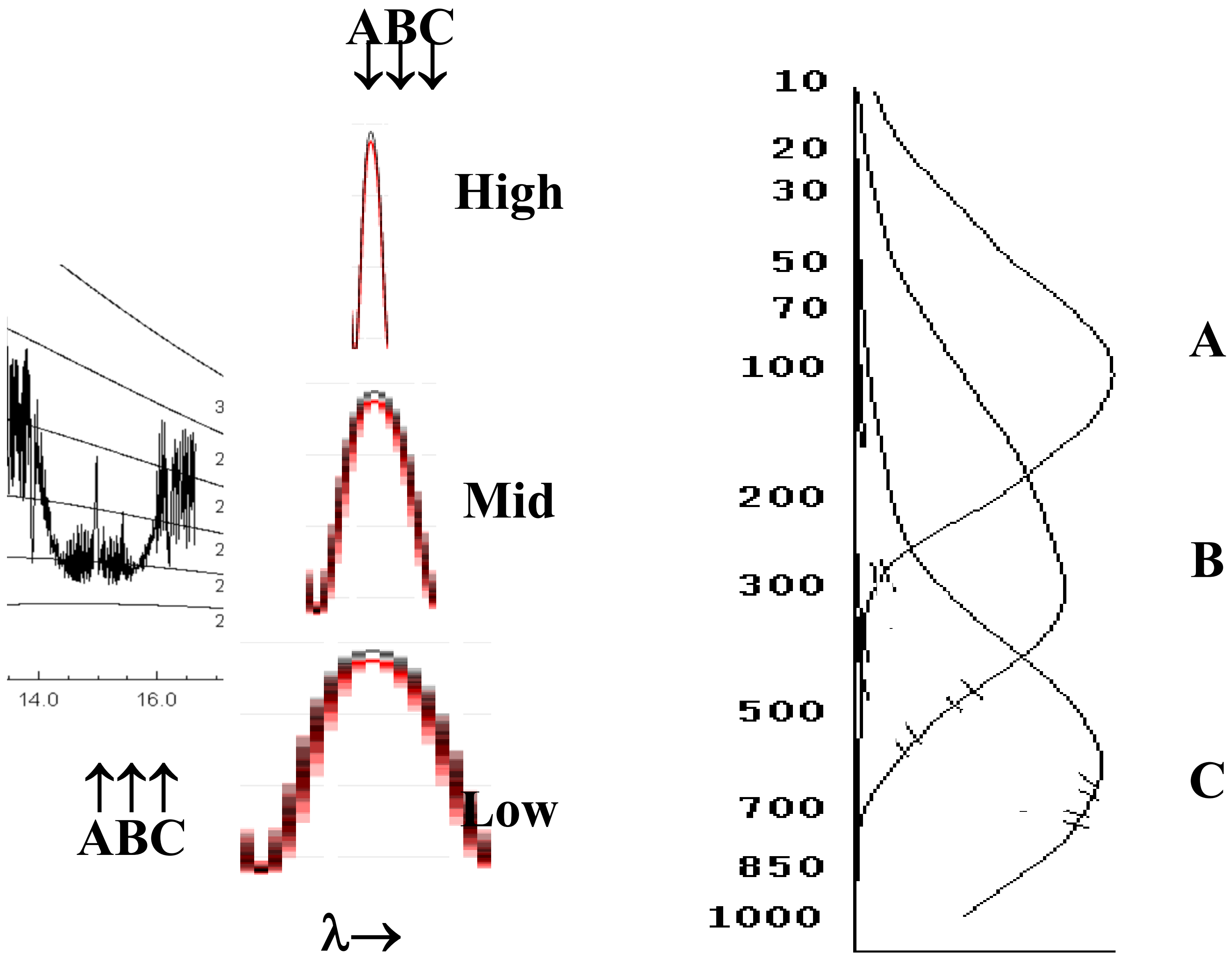
14.7	1	680	CO2, strat temp
14.4	2	696	CO2, strat temp
14.1	3	711	CO2, upper trop temp
13.9	4	733	CO2, mid trop temp
13.4	5	748	CO2, lower trop temp
12.7	6	790	H2O, lower trop moisture
12.0	7	832	H2O, dirty window



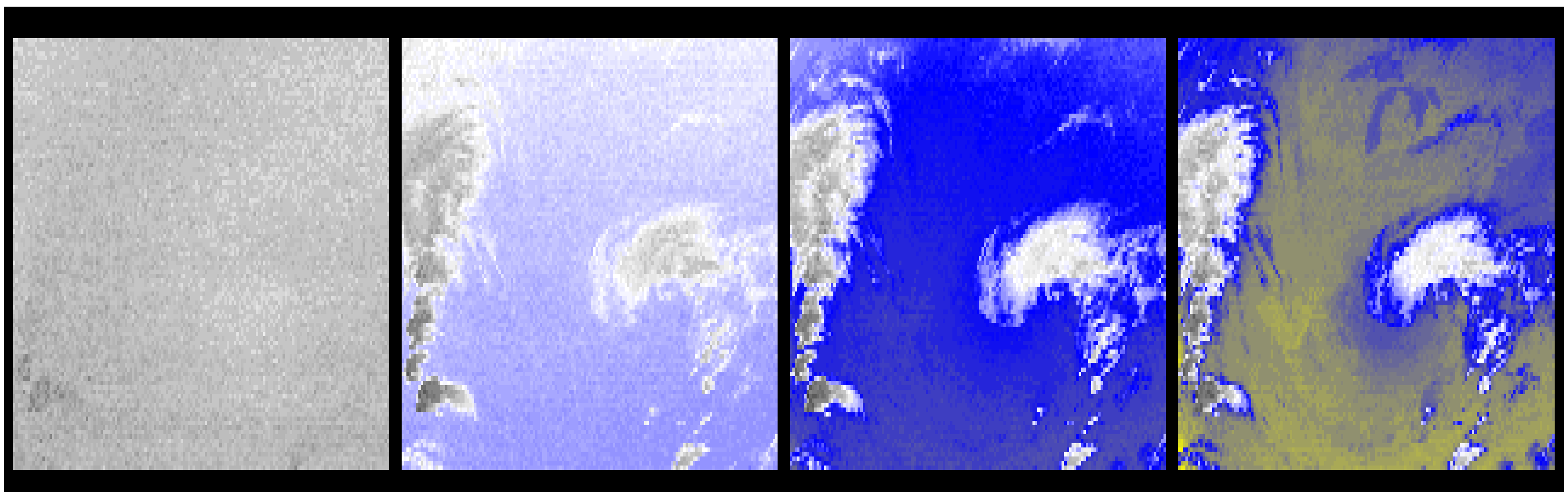
Midwave H2O & O3

11.0	8	907	window
9.7	9	1030	O3, strat ozone
7.4	10	1345	H2O, lower mid trop moisture
7.0	11	1425	H2O, mid trop moisture
6.5	12	1535	H2O, upper trop moisture

line broadening with pressure helps to explain weighting functions

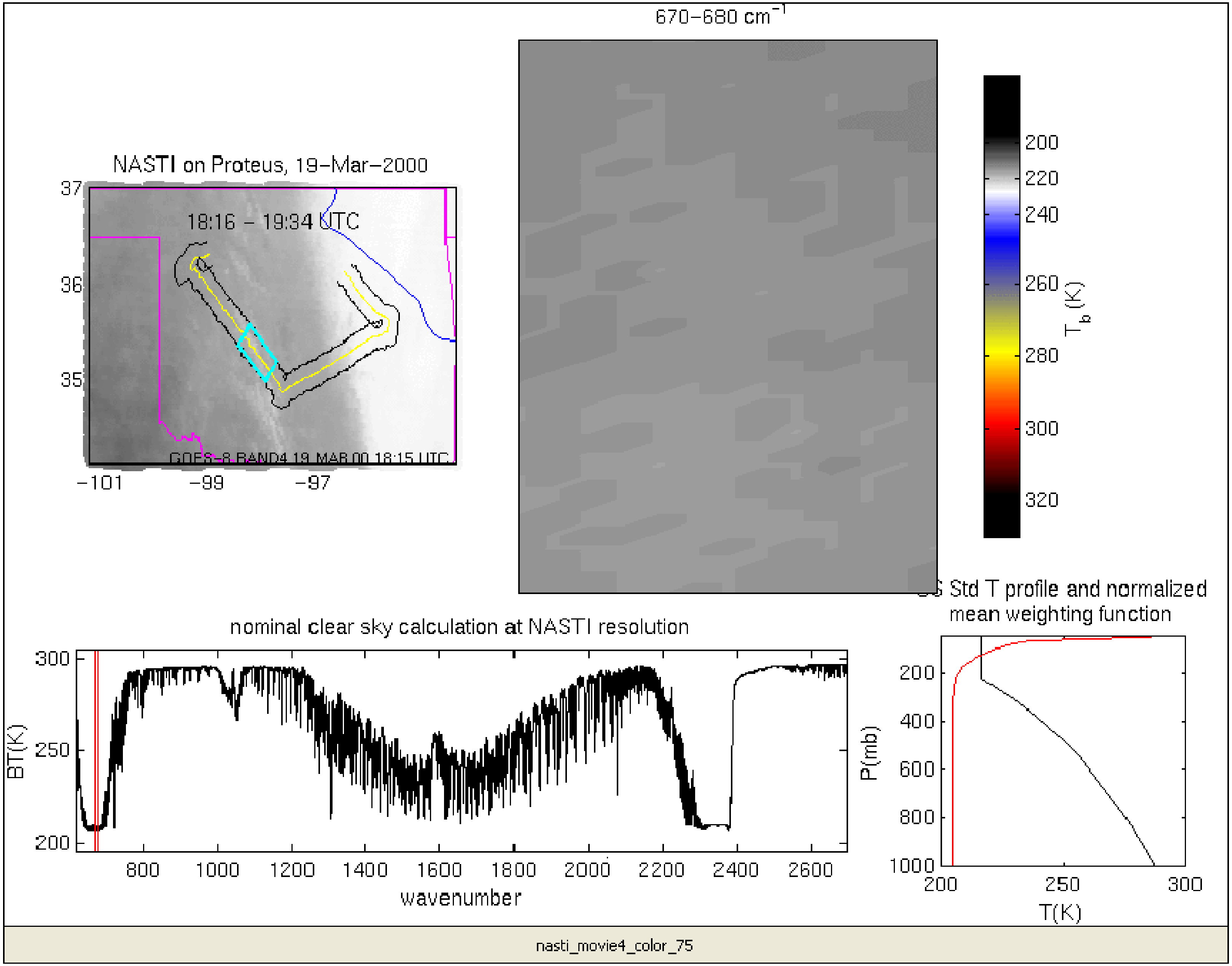


CO2 channels see to different levels in the atmosphere



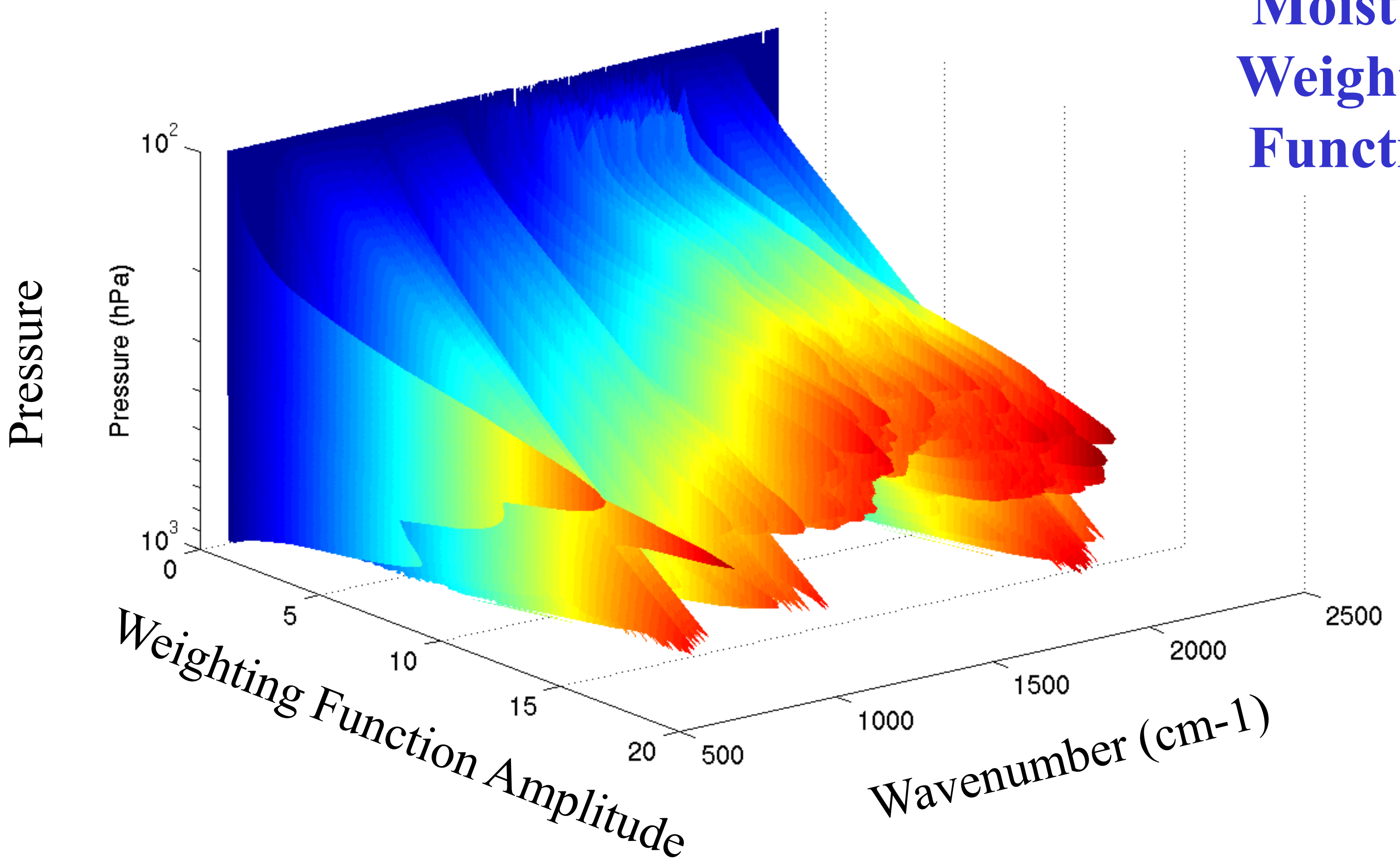
14.2 um 13.9 um 13.6 um 13.3 um

Improvements with Hyperspectral IR Data



These water vapor weighting functions reflect the radiance sensitivity of the specific channels to a water vapor % change at a specific level (equivalent to dR/dlnq scaled by dlnp).

Moisture Weighting Functions



UW/CIMSS

The advanced sounder has more and sharper weighting functions

Characteristics of RTE

* Radiance arises from deep and overlapping layers

* The radiance observations are not independent

* There is no unique relation between the spectrum of the outgoing radiance and T(p) or Q(p)

* T(p) is buried in an exponent in the denominator in the integral

* Q(p) is implicit in the transmittance

* Boundary conditions are necessary for a solution; the better the first guess the better the final solution

To investigate the RTE further consider the atmospheric contribution to the radiance to space of an infinitesimal layer of the atmosphere at height z, $dI_\lambda(z) = B_\lambda(T(z))\, d\tau_\lambda(z)$.

Assume a well-mixed isothermal atmosphere where the density drops off exponentially with height $\rho = \rho_o \exp(-\gamma z)$, and assume k_λ is independent of height, so that the optical depth can be written for normal incidence

$$\sigma_\lambda = \int_z^\infty k_\lambda \rho \, dz = \gamma^{-1} k_\lambda \rho_o \exp(-\gamma z)$$

and the derivative with respect to height

$$\frac{d\sigma_\lambda}{dz} = - k_\lambda \rho_o \exp(-\gamma z) = -\gamma \sigma_\lambda \, .$$

Therefore, we may obtain an expression for the detected radiance per unit thickness of the layer as a function of optical depth,

$$\frac{dI_\lambda(z)}{dz} = B_\lambda(T_{const}) \frac{d\tau_\lambda(z)}{dz} = B_\lambda(T_{const})\, \gamma\, \sigma_\lambda \exp(-\sigma_\lambda) \, .$$

The level which is emitting the most detected radiance is given by

$$\frac{d}{dz}\left\{\frac{dI_\lambda(z)}{dz}\right\} = 0 \text{ , or where } \sigma_\lambda = 1.$$

Most of monochromatic radiance detected is emitted by layers near level of unit optical depth.

<u>Profile Retrieval from Sounder Radiances</u>

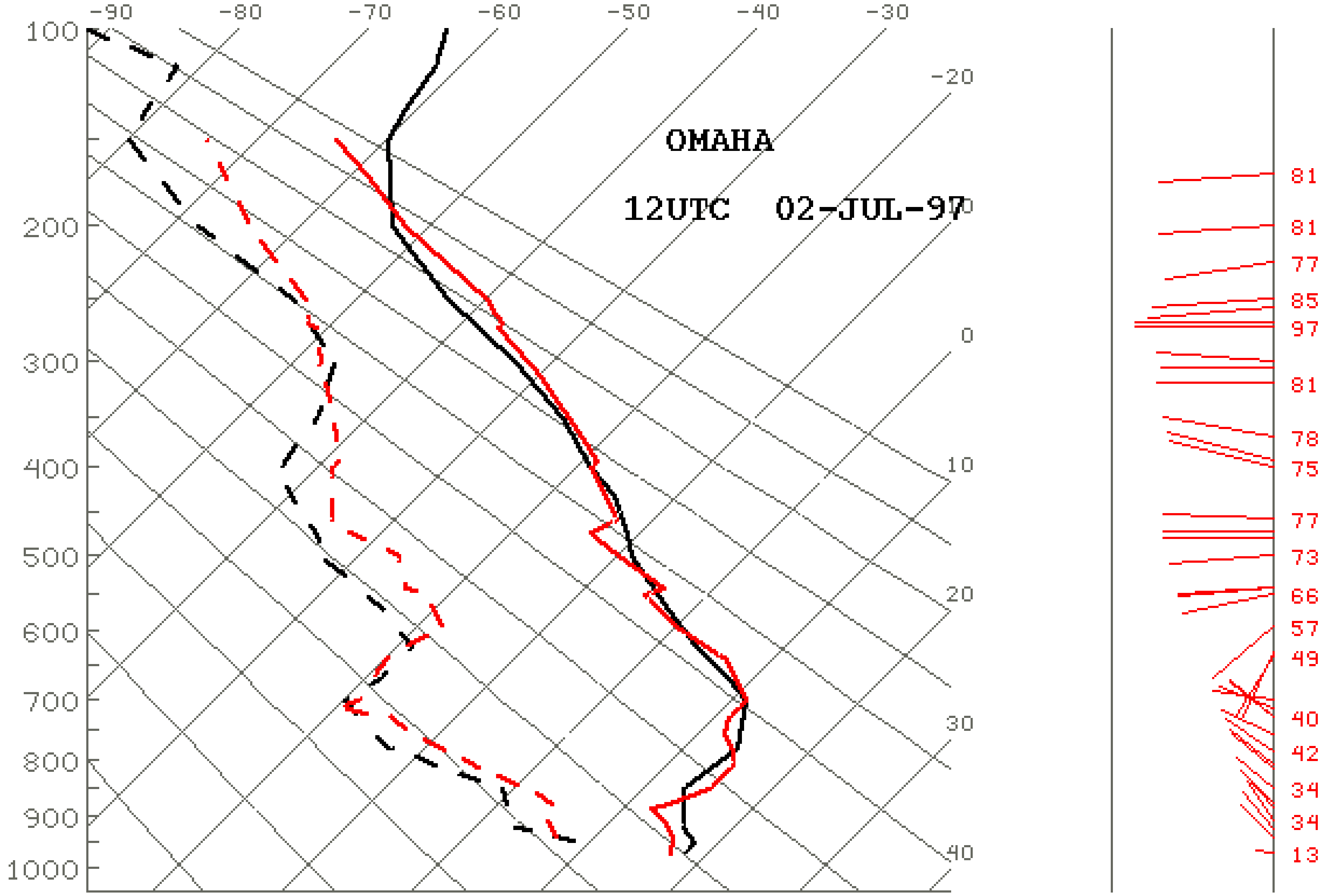
$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s))\, \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p))\, F_\lambda(p)\, [\, d\tau_\lambda(p) / dp \,]\, dp \,.$$

I1, I2, I3, ... , In are measured with the sounder
P(sfc) and T(sfc) come from ground based conventional observations
$\tau_\lambda(p)$ are calculated with physics models (using for CO2 and O3)
$\varepsilon_\lambda^{sfc}$ is estimated from a priori information (or regression guess)

First guess solution is inferred from (1) in situ radiosonde reports,
(2) model prediction, or (3) blending of (1) and (2)

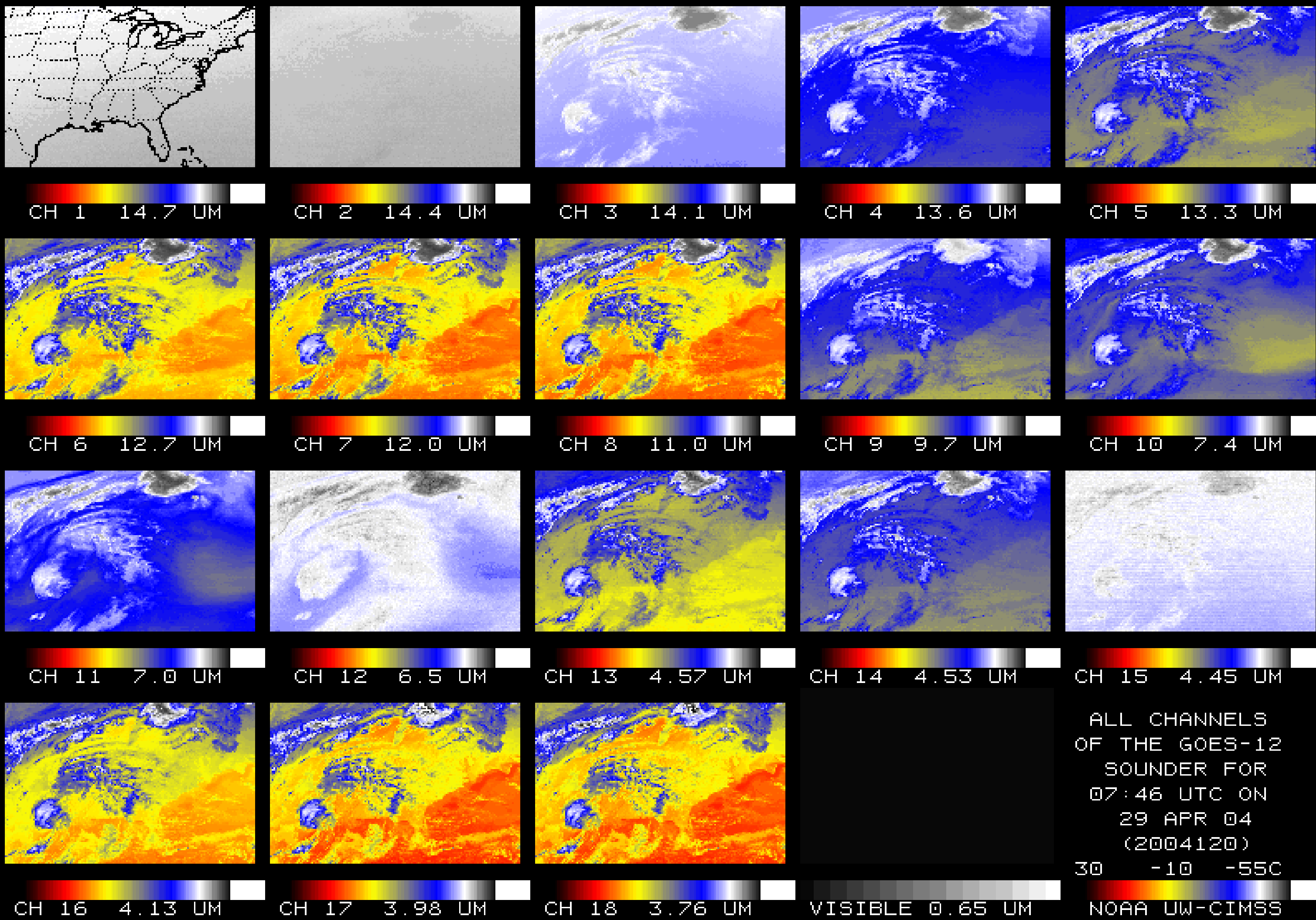
Profile retrieval from perturbing guess to match measured sounder radiances

Example GOES Sounding

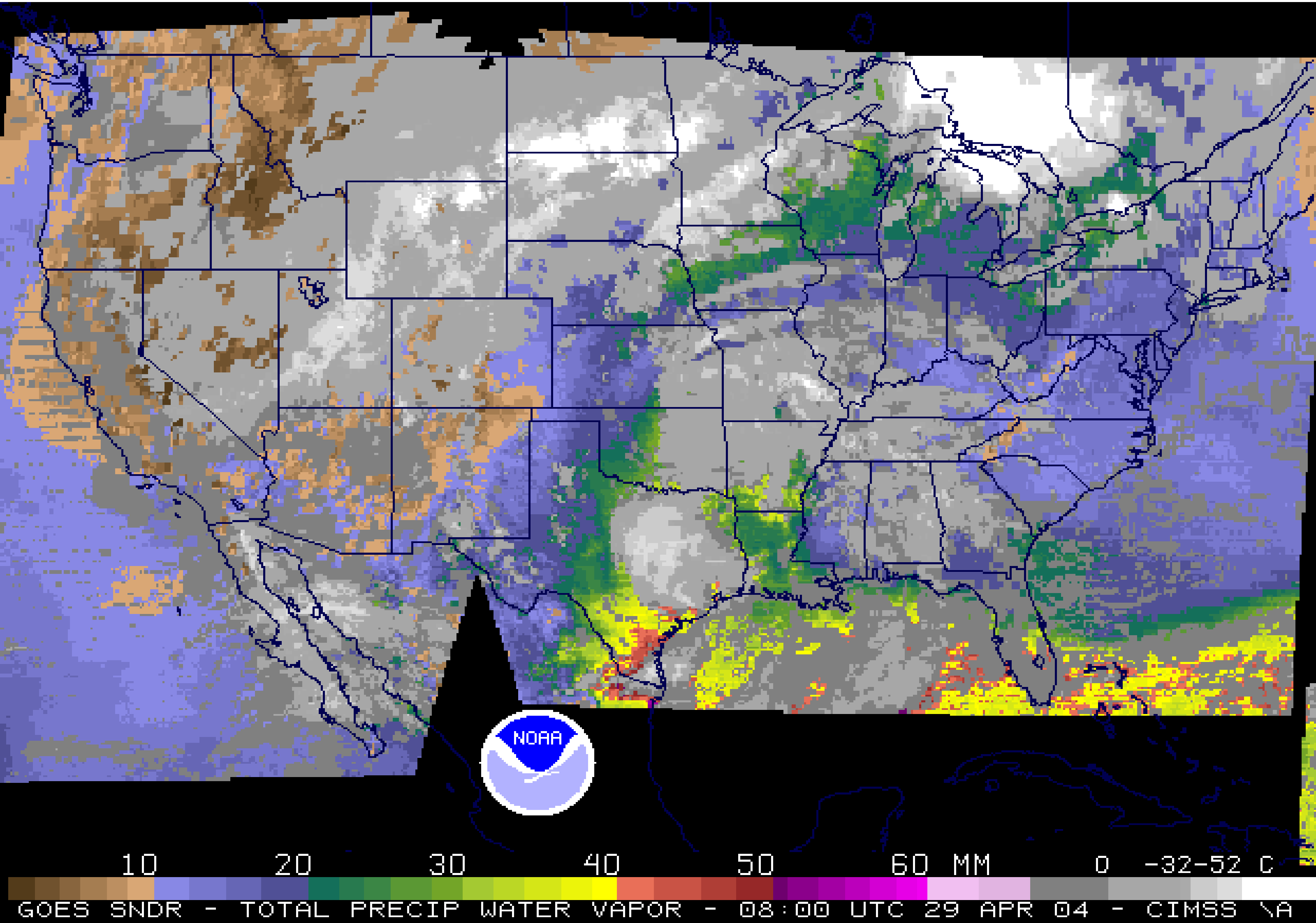


GMT	ID	TOTAL	EQUIP	FMAX	CVT	L.I.	KINX	PW	
021153	267	30				11	-10	12	GOES-8 RTVL
021200	72558	36				10	-4	14	RAOB

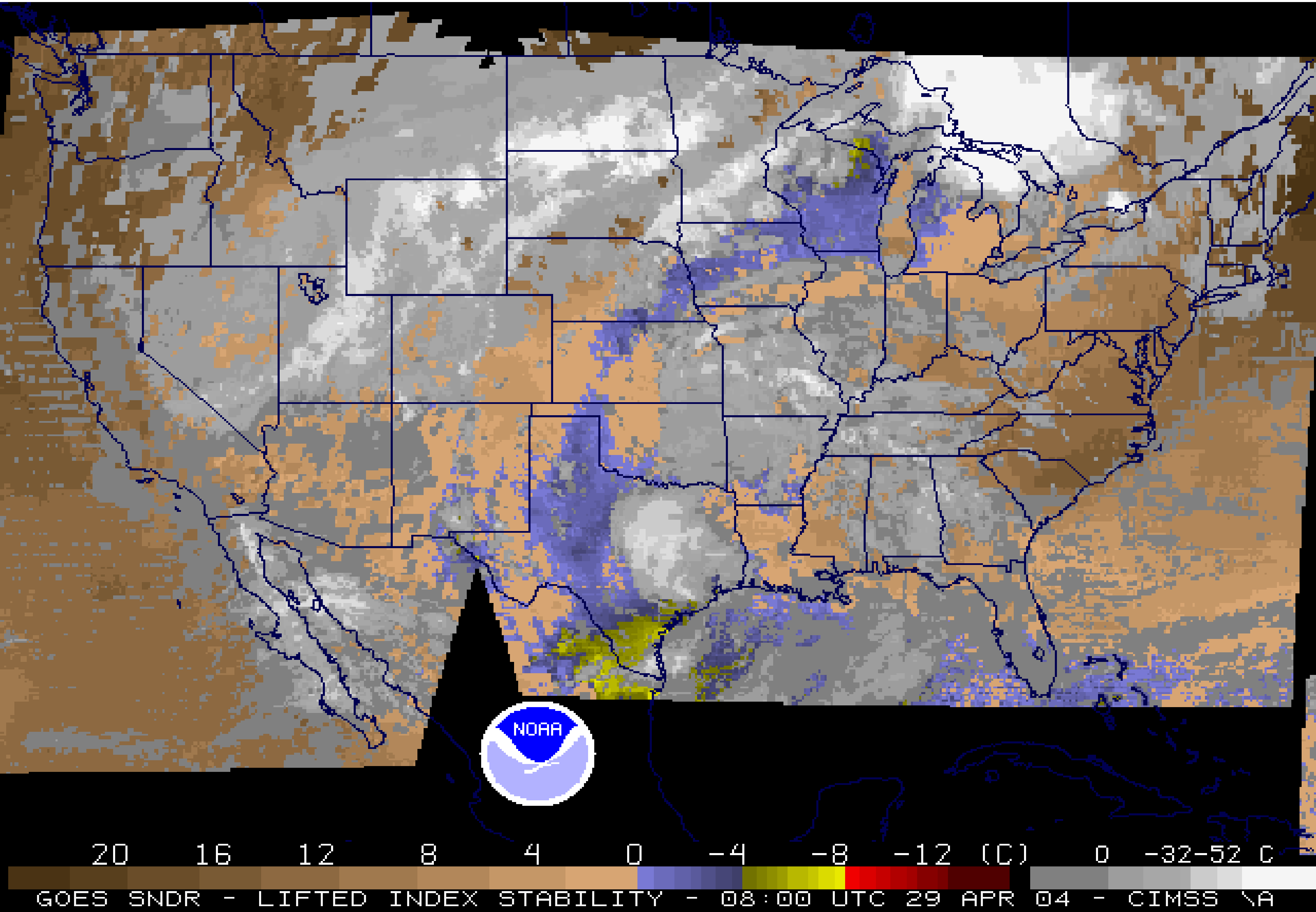
GOES-12 Sounder – Brightness Temperature (Radiances) – 12 bands

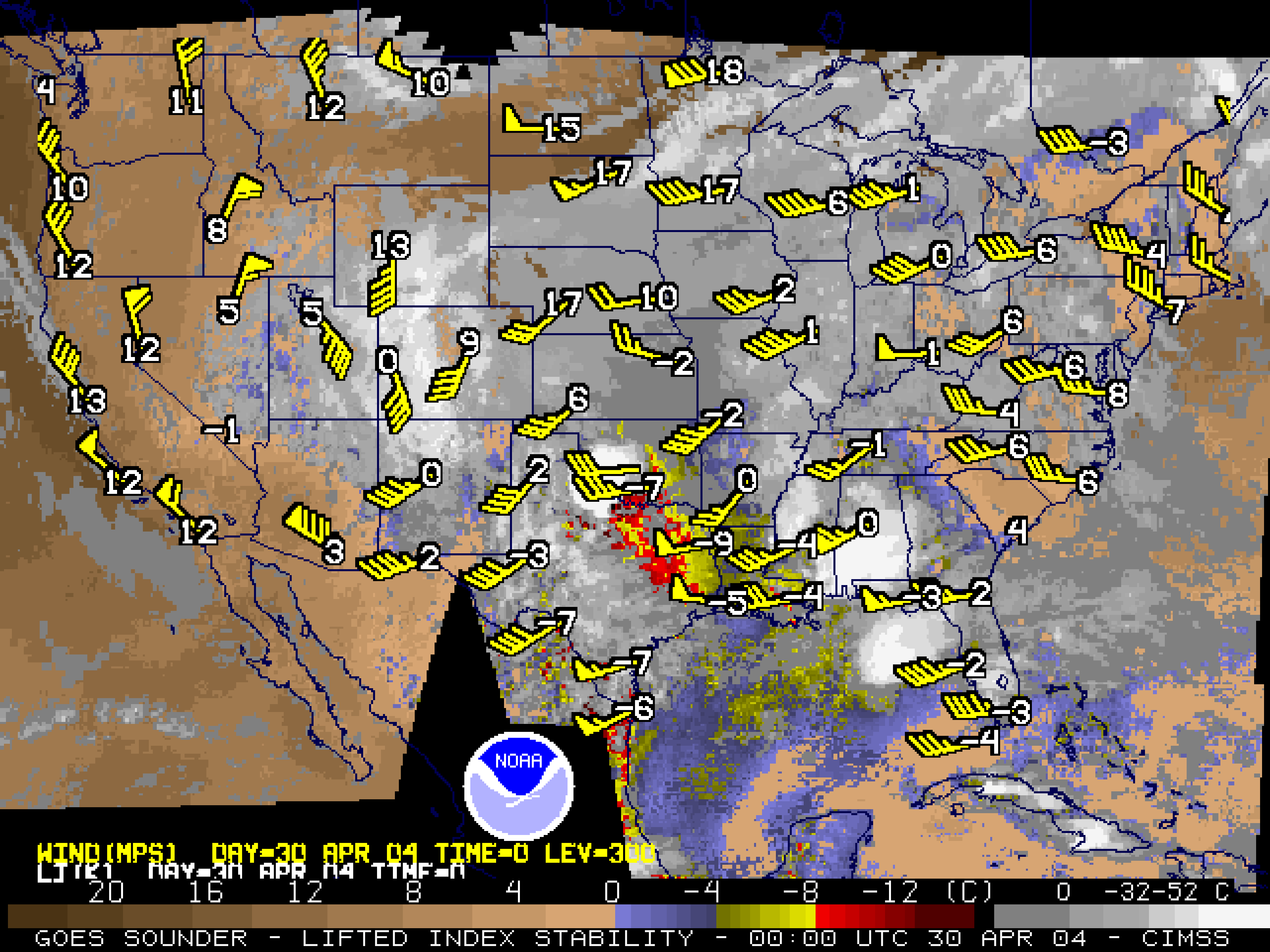


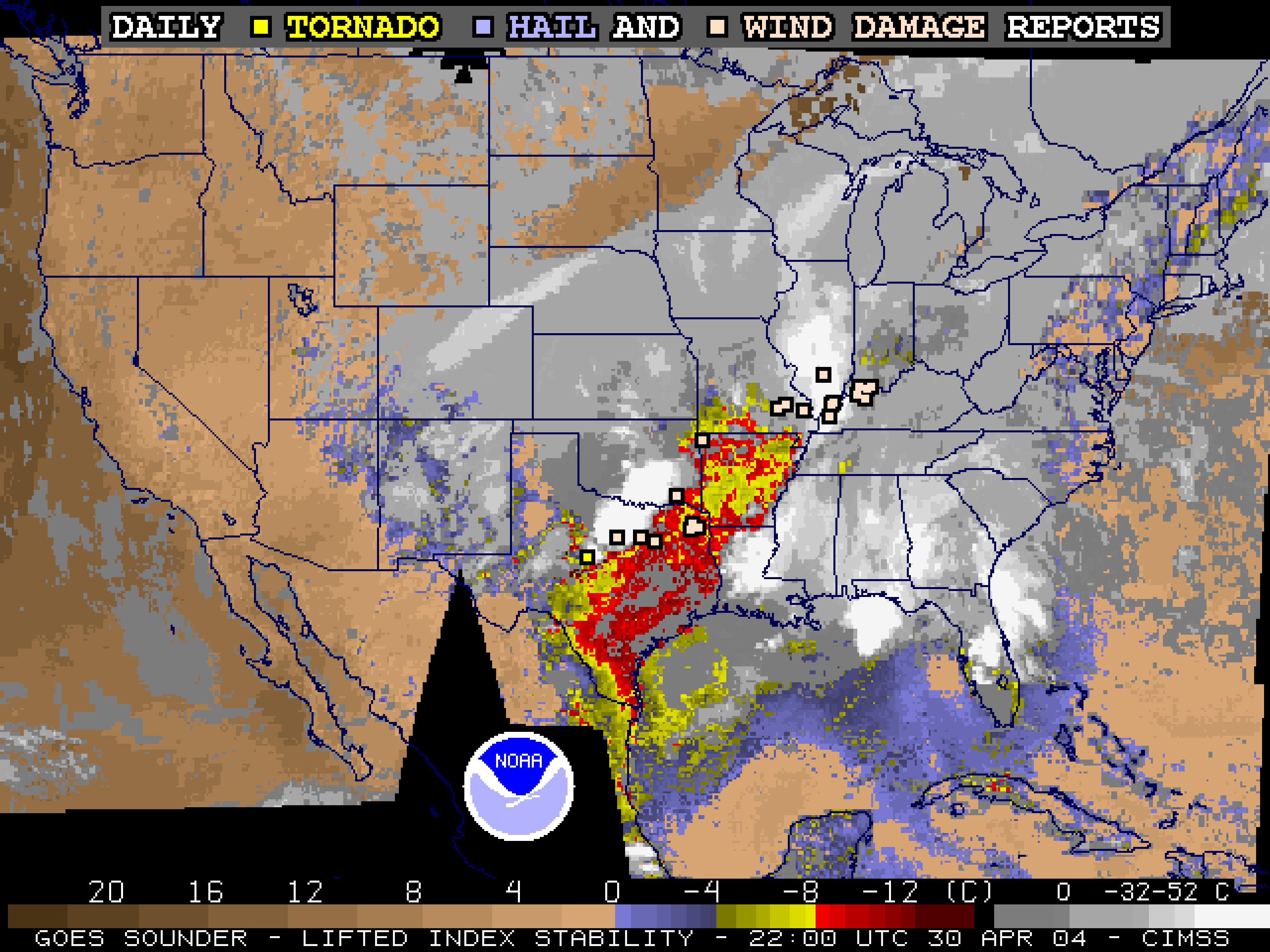
GOES Sounders –Total Precipitable Water



GOES Sounders –Lifted Index Stability




NOAA
WIND(MPS) DAY=30 APR 04 TIME=0 LEV=300
LI(C) DAY=30 APR 04 TIME=0
20 16 12 8 4 0 -4 -8 -12 (C) 0 -32 -52 C
GOES SOUNDER - LIFTED INDEX STABILITY - 00:00 UTC 30 APR 04 - CIMSS


DAILY TORNADO HAIL AND WIND DAMAGE REPORTS
NOAA
20 16 12 8 4 0 -4 -8 -12 (C) 0 -32-52 C
GOES SOUNDER - LIFTED INDEX STABILITY - 22:00 UTC 30 APR 04 - CIMSS

Sounder Retrieval Products

$$I_\lambda = \varepsilon_\lambda(sfc)\, B_\lambda(T(ps))\, \tau_\lambda(ps) - \int_o^{ps} B_\lambda(T(p))\, F_\lambda(p)\, [\, d\tau_\lambda(p) / dp \,]\, dp \,.$$

Direct

brightness temperatures

Derived in Clear Sky

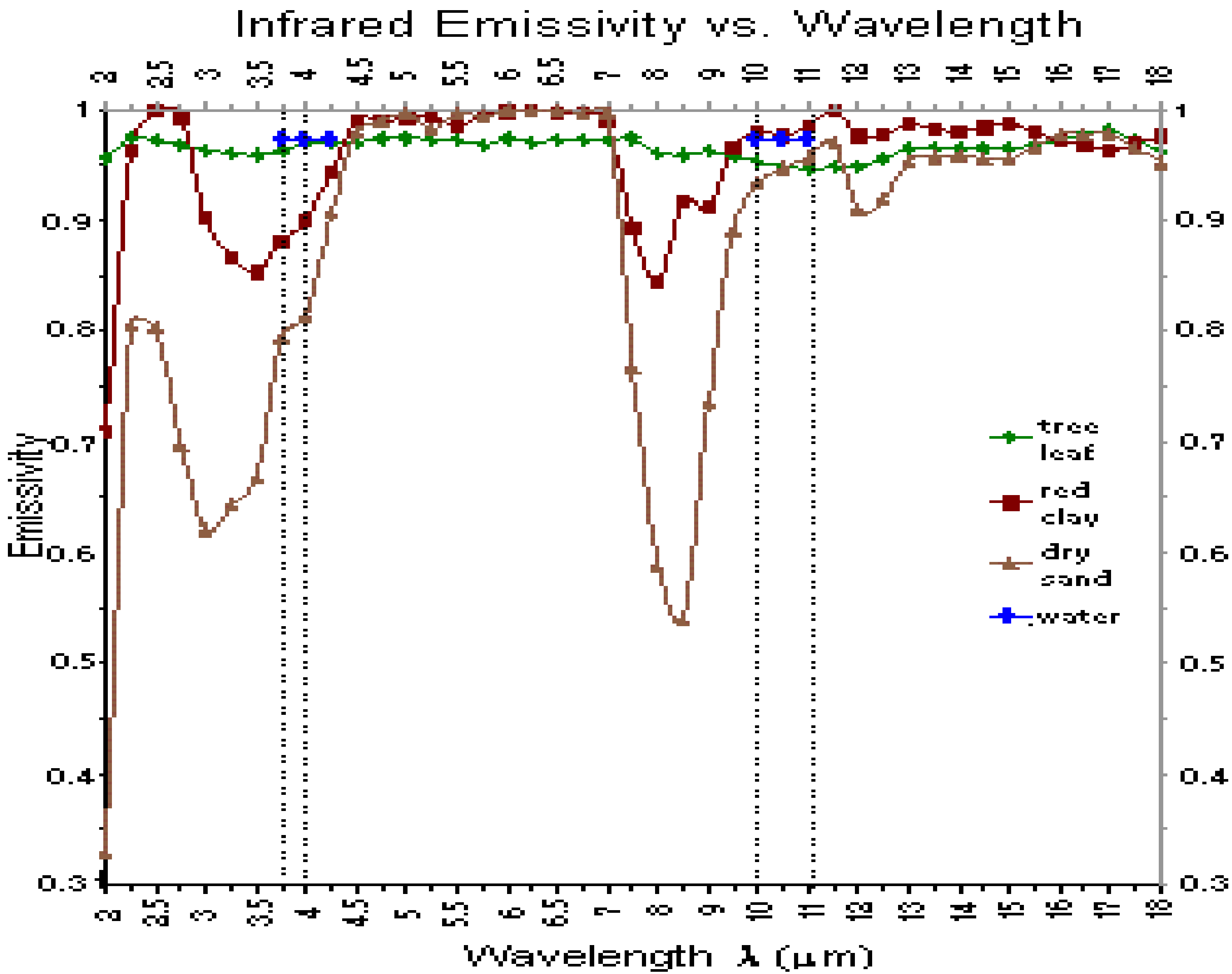
20 retrieved temperatures (at mandatory levels)
20 geo-potential heights (at mandatory levels)
11 dewpoint temperatures (at 300 hPa and below)
3 thermal gradient winds (at 700, 500, 400 hPa)
1 total precipitable water vapor
1 surface skin temperature
2 stability index (lifted index, CAPE)

Derived in Cloudy conditions

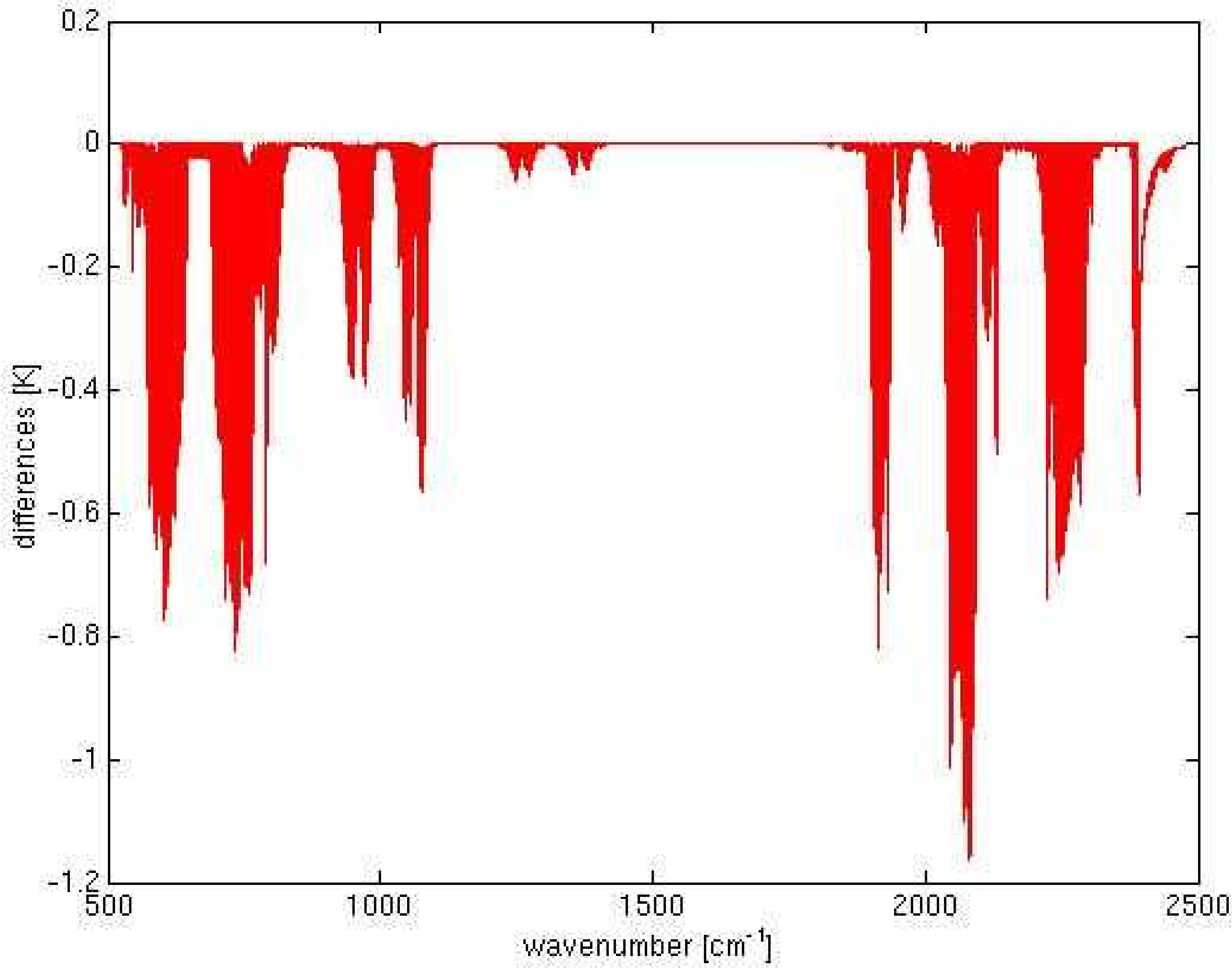
3 cloud parameters (amount, cloud top pressure, and cloud top temperature)

Mandatory Levels (in hPa)

sfc	780	300	70
1000	700	250	50
950	670	200	30
920	500	150	20
850	400	100	10


Infrared Emissivity vs. Wavelength
Emissivity
Wavelength λ (µm)
tree leaf
red clay
dry sand
water
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
2.5
3
3.5
4
4.5
5
5.5
6
6.5
7
8
9
10
11
12
13
14
15
16
17
18
PND/COMET

BT differences resulting from 10 ppmv change in CO2 concentration



Microwave RTE

Lectures in Benevento
June 2007

Paul Menzel
UW/CIMSS/AOS

WAVELENGTH			FREQUENCY		WAVENUMBER
cm	μm	Å	Hz	GHz	cm^{-1}
10^{-5} Near Ultraviolet (UV)	0.1	1,000	$3x10^{15}$		
$4x10^{-5}$ Visible	0.4	4,000	$7.5x10^{14}$		
$7.5x10^{-5}$ Near Infrared (IR)	0.75	7,500	$4x10^{14}$		13,333
$2x10^{-3}$ Far Infrared (IR)	20	$2x10^{5}$	$1.5x10^{13}$		500
0.1 Microwave (MW)	10^{3}		$3x10^{11}$	300	10

Radiation is governed by Planck's Law

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

In microwave region $c_2/\lambda T << 1$ so that

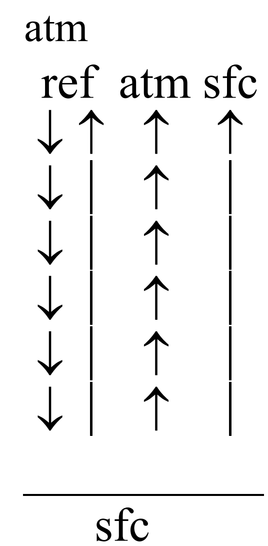
$$e^{c_2/\lambda T} = 1 + c_2/\lambda T + \text{second order}$$

And classical Rayleigh Jeans radiation equation emerges

$$B_\lambda(T) \approx [c_1 / c_2] [T / \lambda^4]$$

Radiance is linear function of brightness temperature.

Microwave Form of RTE



$$I^{sfc}_{\lambda} = \varepsilon_{\lambda} B_{\lambda}(T_s)\, \tau_{\lambda}(p_s) + (1-\varepsilon_{\lambda})\, \tau_{\lambda}(p_s) \int_{o}^{p_s} B_{\lambda}(T(p)) \frac{\partial \tau'_{\lambda}(p)}{\partial \ln p}\, d \ln p$$

$$I_{\lambda} = \varepsilon_{\lambda} B_{\lambda}(T_s)\, \tau_{\lambda}(p_s) + (1-\varepsilon_{\lambda})\, \tau_{\lambda}(p_s) \int_{o}^{p_s} B_{\lambda}(T(p)) \frac{\partial \tau'_{\lambda}(p)}{\partial \ln p}\, d \ln p$$
$$+ \int_{p_s}^{o} B_{\lambda}(T(p)) \frac{\partial \tau_{\lambda}(p)}{\partial \ln p}\, d \ln p$$

In the microwave region $c_2 /\lambda T << 1$, so the Planck radiance is linearly proportional to the temperature

$$B_{\lambda}(T) \approx [c_1 / c_2]\, [T / \lambda^4]$$

So

$$T_{b\lambda} = \varepsilon_{\lambda} T_s(p_s)\, \tau_{\lambda}(p_s) + \int_{p_s}^{o} T(p)\, F_{\lambda}(p) \frac{\partial \tau_{\lambda}(p)}{\partial \ln p}\, d \ln p$$

where

$$F_{\lambda}(p) = \{ 1 + (1 - \varepsilon_{\lambda}) \left[\frac{\tau_{\lambda}(p_s)}{\tau_{\lambda}(p)}\right]^2 \} .$$

The transmittance to the surface can be expressed in terms of transmittance to the top of the atmosphere by remembering

$$\tau'_\lambda(p) = \exp\left[- \frac{1}{g} \int_p^{p_s} k_\lambda(p)\, g(p)\, dp \right]$$

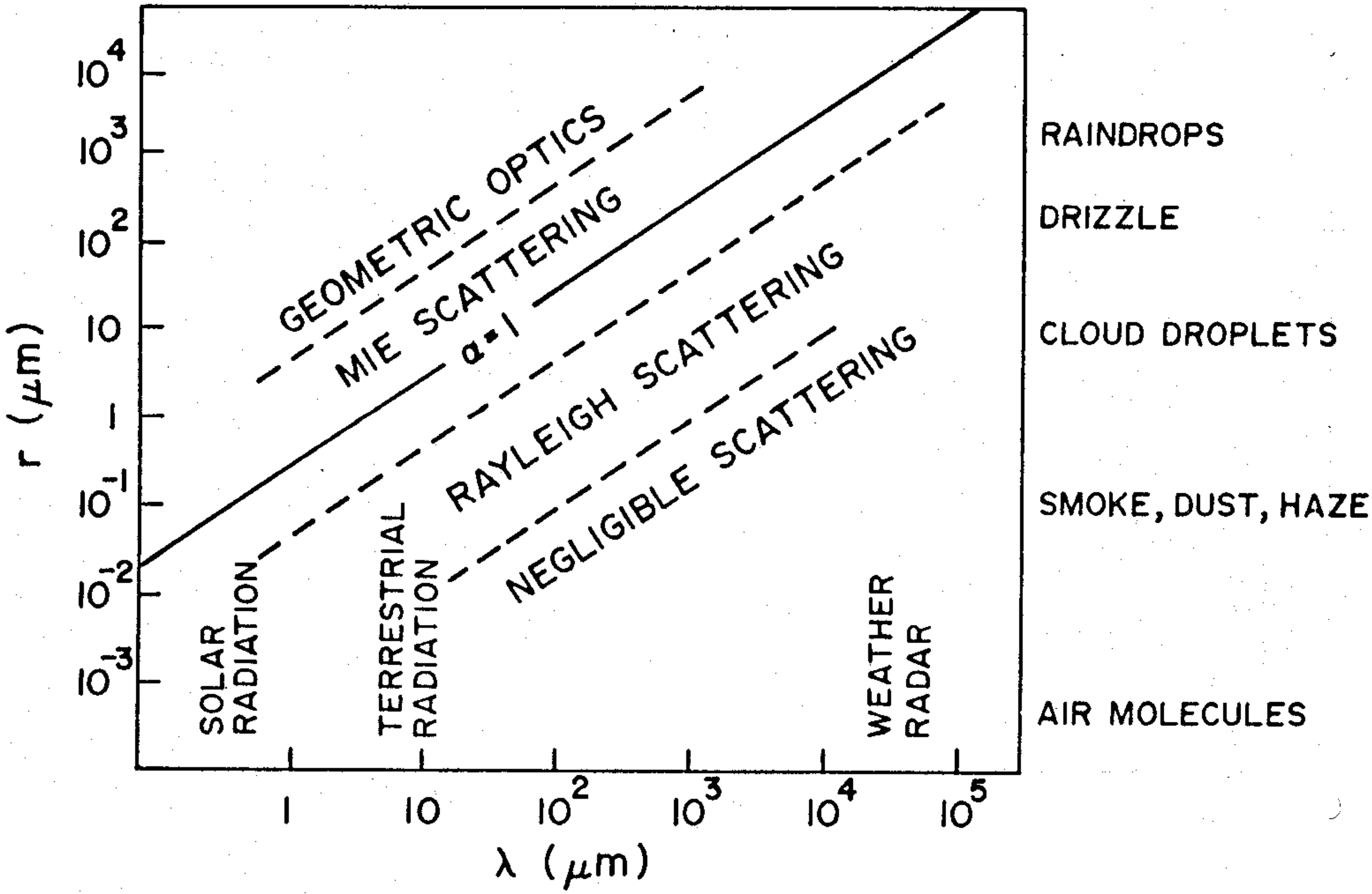
$$= \exp\left[- \int_o^{p_s} + \int_o^{p} \right]$$

$$= \tau_\lambda(p_s) \,/\, \tau_\lambda(p) \ .$$

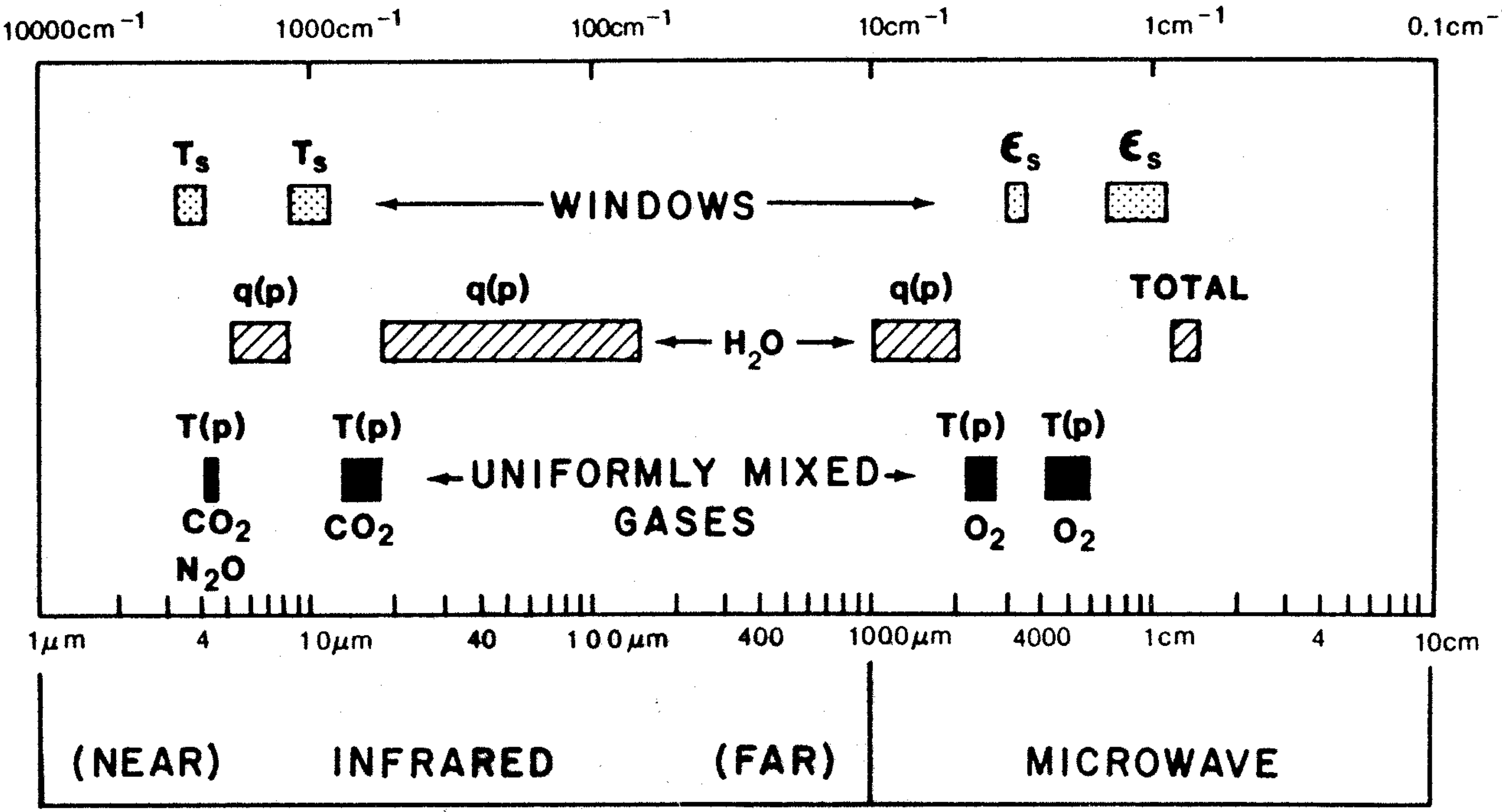
So

$$\frac{\partial \tau'_\lambda(p)}{\partial \ln p} = - \frac{\tau_\lambda(p_s)}{(\tau_\lambda(p))^2} \frac{\partial \tau_\lambda(p)}{\partial \ln p} \ .$$

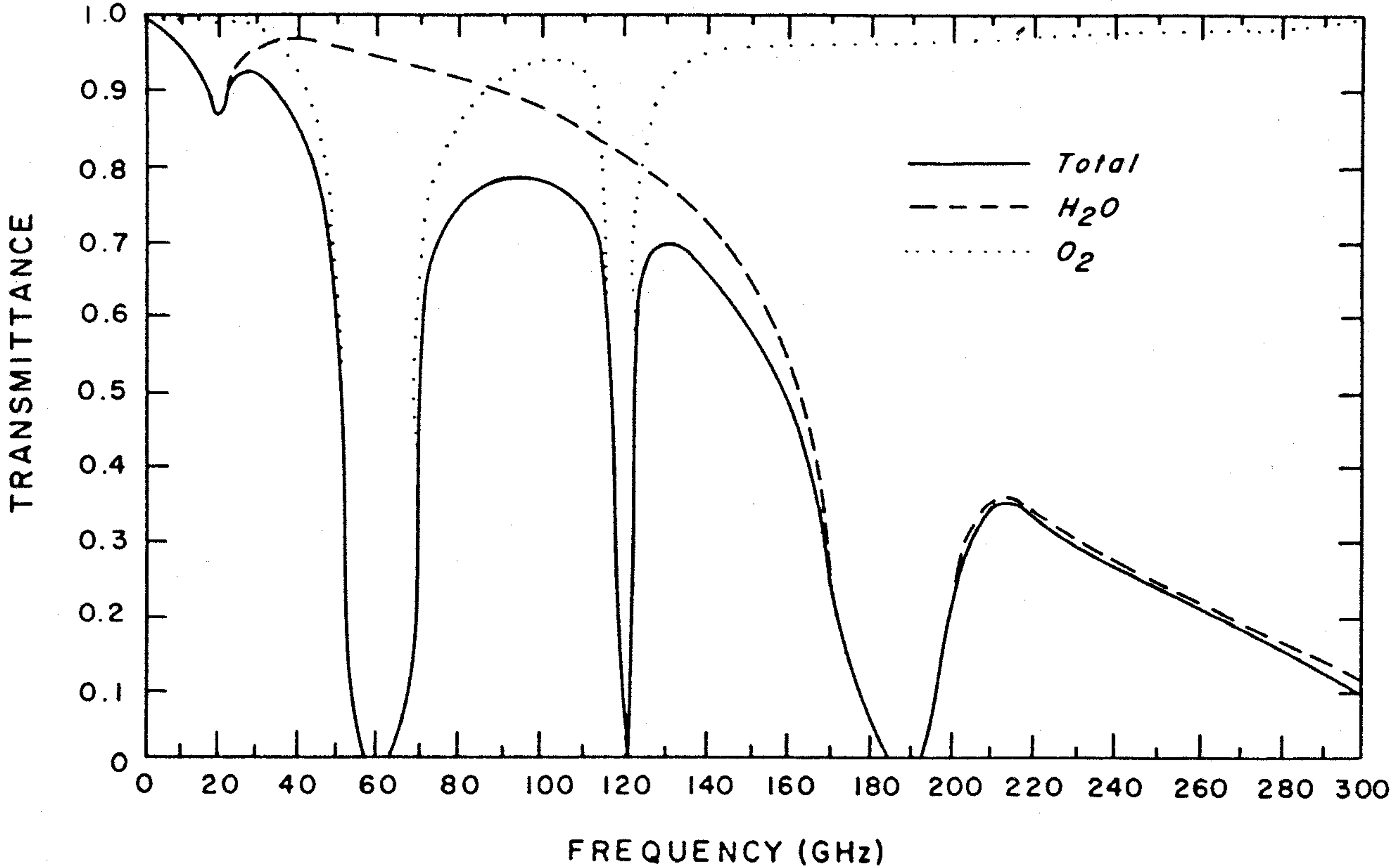
[remember that $\tau_\lambda(p_s, p)\, \tau_\lambda(p, 0) = \tau_\lambda(p_s, 0)$ and $\tau_\lambda(p_s, p) = \tau_\lambda(p, p_s)$]



GEOMETRIC OPTICS
MIE SCATTERING
α = 1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
r (μm)
λ (μm)
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES



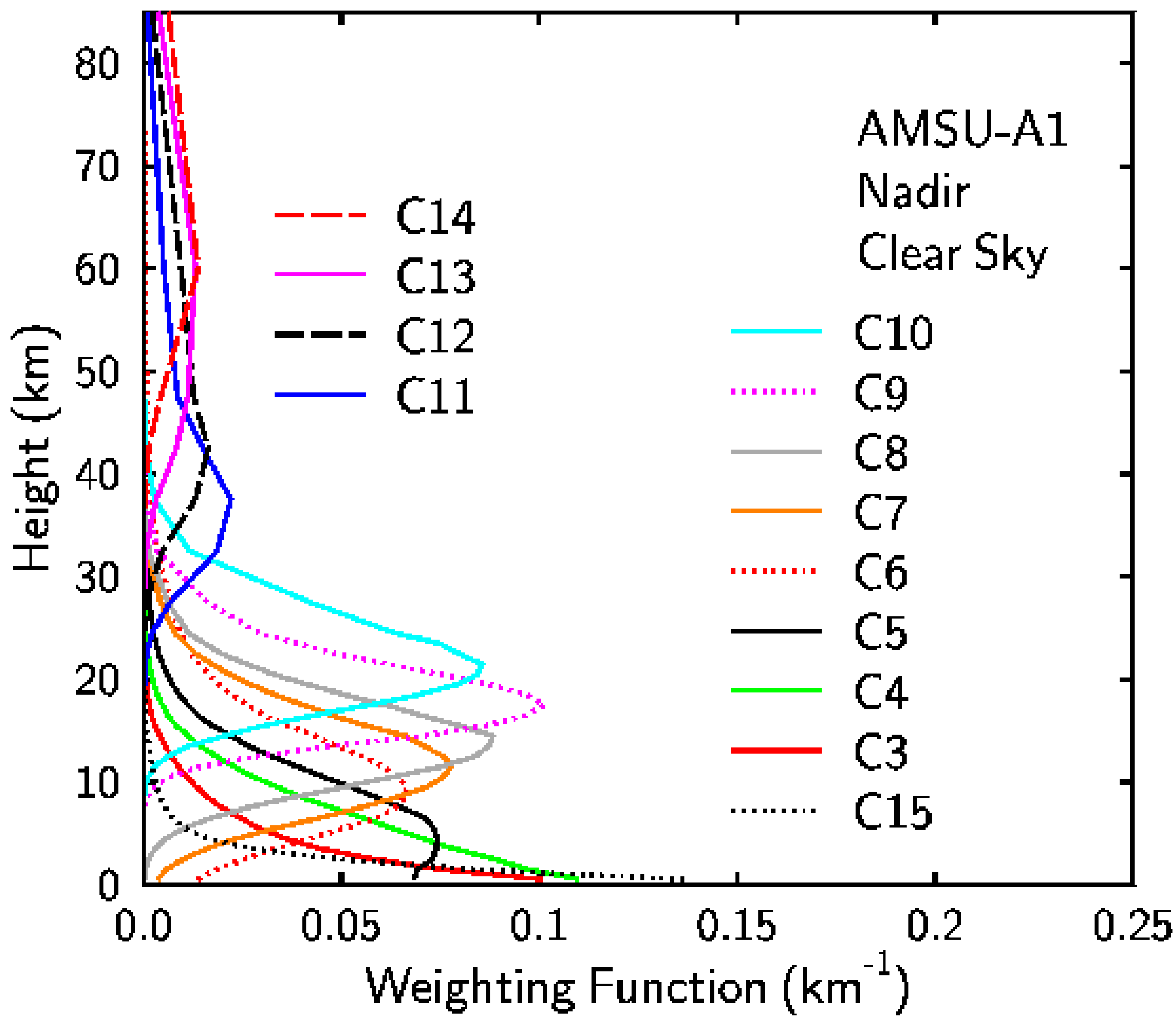
Spectral regions used for remote sensing of the earth atmosphere and surface from satellites. ε indicates emissivity, q denotes water vapour, and T represents temperature.

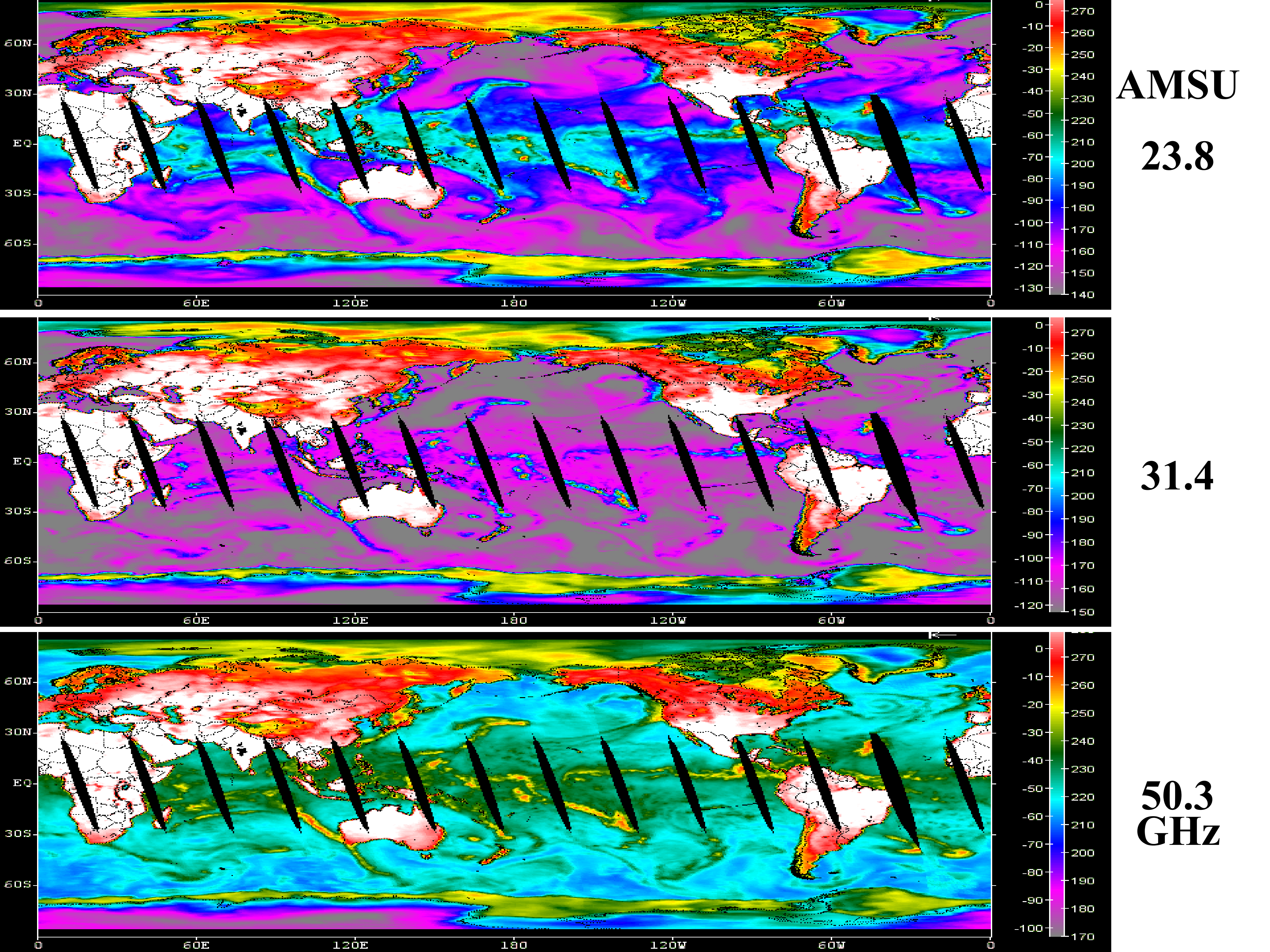


TRANSMITTANCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
FREQUENCY (GHz)
Total
H_2O
O_2

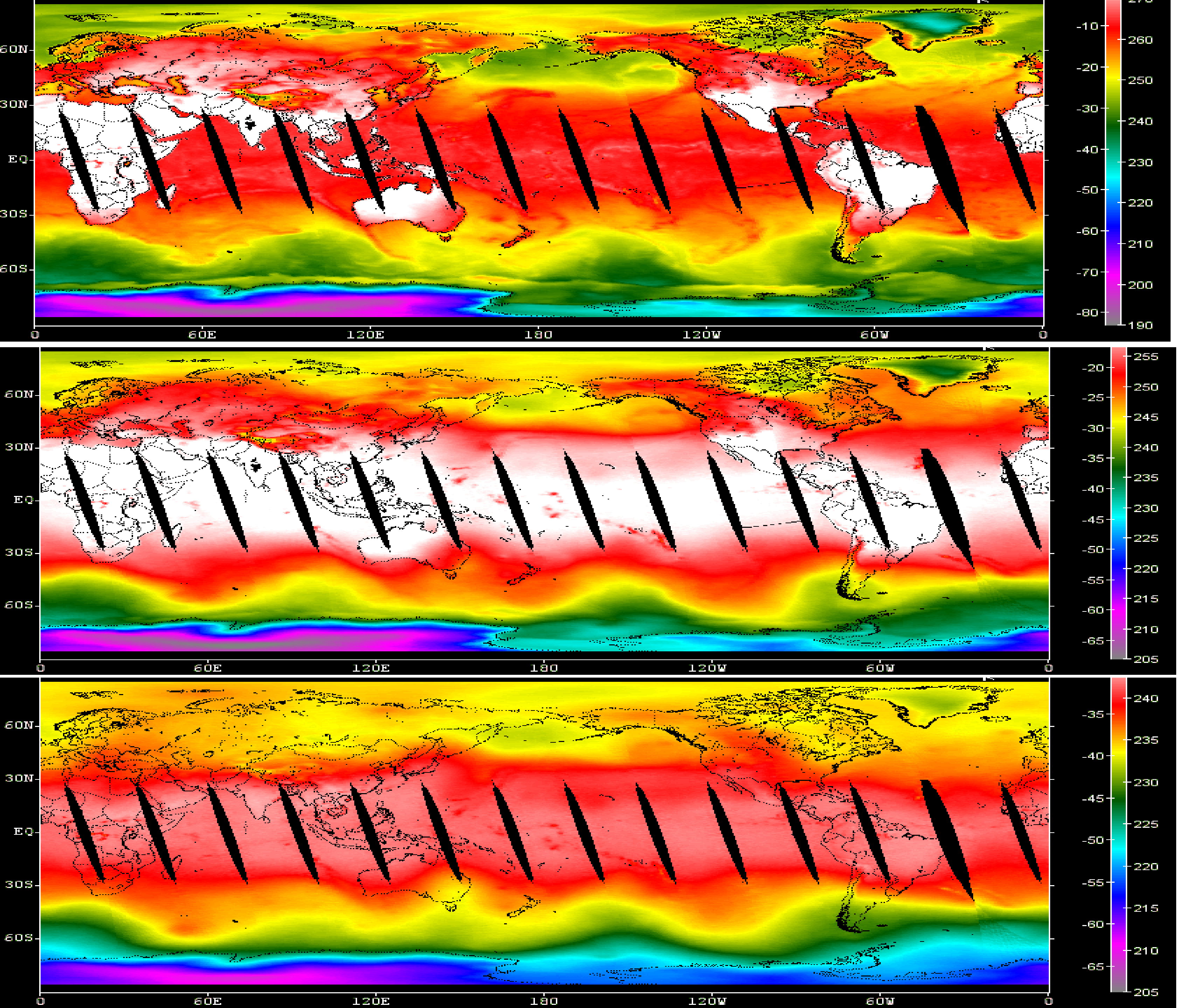
Microwave spectral bands

23.8 GHz	dirty window H_2O absorption
31.4 GHz	window
60 GHz	O_2 sounding
120 GHz	O_2 sounding
183 GHz	H_2O sounding


AMSU-A1
Nadir
Clear Sky
C14
C13
C12
C11
C10
C9
C8
C7
C6
C5
C4
C3
C15
Height (km)
Weighting Function (km^{-1})
0
10
20
30
40
50
60
70
80
0.0
0.05
0.1
0.15
0.2
0.25



AMSU
23.8
31.4
50.3
GHz
60N
30N
EQ
30S
60S
0
60E
120E
180
120W
60W

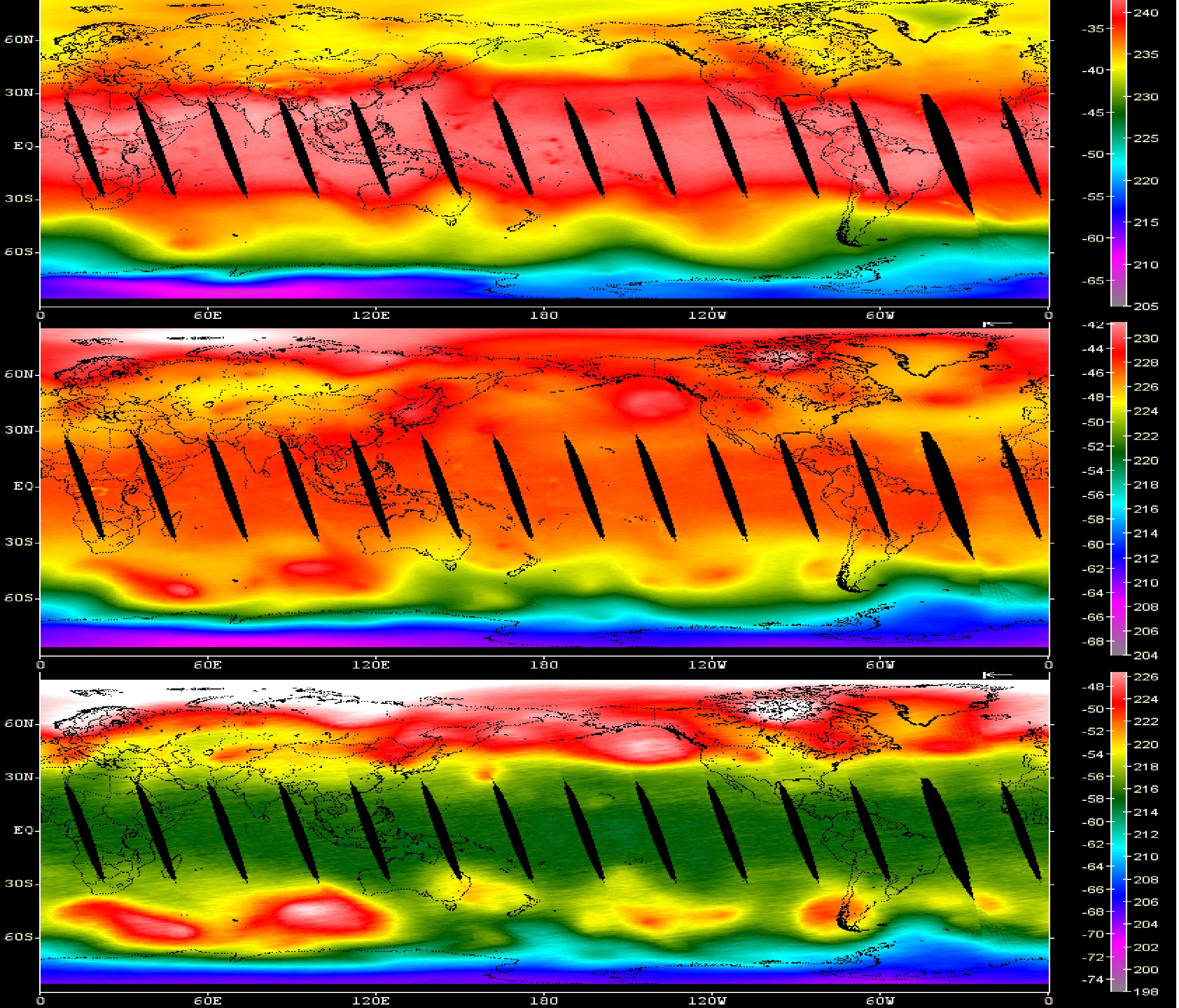


AMSU

52.8

53.6

54.4
GHz

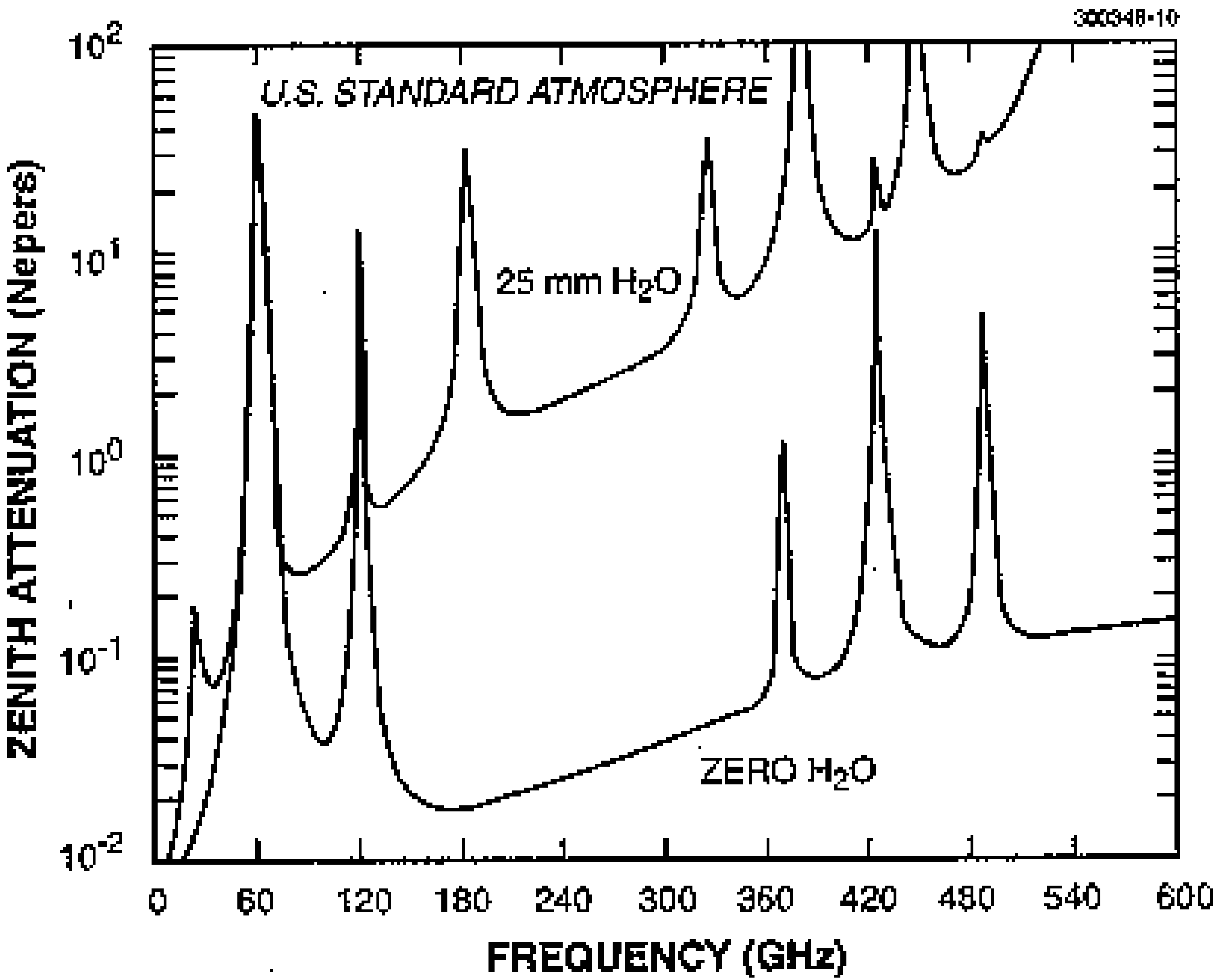


AMSU

54.4

54.9

55.5
GHz



300348-10
U.S. STANDARD ATMOSPHERE
25 mm H_2O
ZERO H_2O
ZENITH ATTENUATION (Nepers)
10^2
10^1
10^0
10^{-1}
10^{-2}
0 60 120 180 240 300 360 420 480 540 600
FREQUENCY (GHz)