

Radiation and the Radiative Transfer Equation

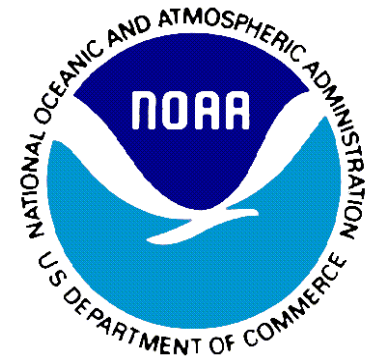
Lectures in Ostuni
June 2006

Paul Menzel
NOAA/NESDIS/ORA

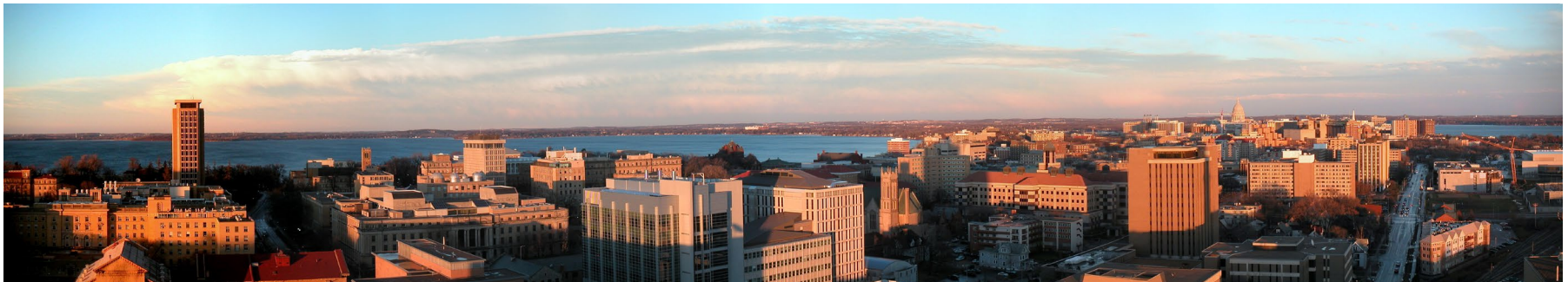
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REMOTE SENSING APPLICATIONS WITH METEOROLOGICAL SATELLITES

by

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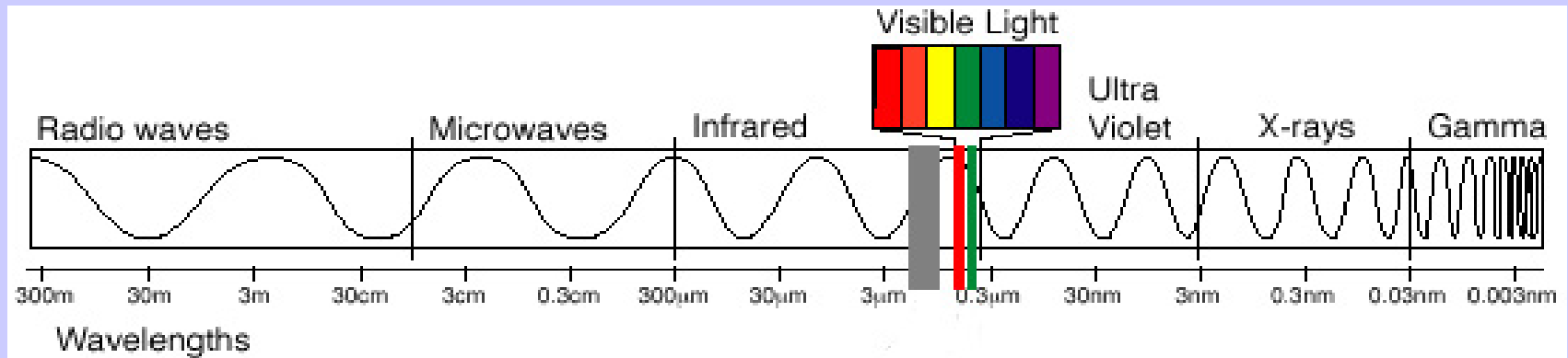
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All satellite remote sensing systems involve the measurement of electromagnetic radiation.

Electromagnetic radiation has the properties of both waves and discrete particles, although the two are never manifest simultaneously.

Electromagnetic radiation is usually quantified according to its wave-like properties; for many applications it considered to be a continuous train of sinusoidal shapes.

The Electromagnetic Spectrum



Remote sensing uses radiant energy that is reflected and emitted from Earth at various “wavelengths” of the electromagnetic spectrum

Our eyes are sensitive to the visible portion of the EM spectrum

Terminology of radiant energy

**Energy from
the Earth Atmosphere**

over time is

Flux

which strikes the detector area

Irradiance

at a given wavelength interval

**Monochromatic
Irradiance**

over a solid angle on the Earth

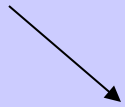
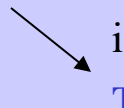
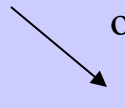
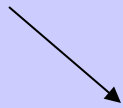
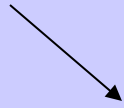
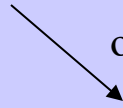
**Radiance observed by
satellite radiometer**

is described by

The Planck function

can be inverted to

Brightness temperature



Definitions of Radiation

QUANTITY	SYMBOL	UNITS
Energy	dQ	Joules
Flux	dQ/dt	Joules/sec = Watts
Irradiance	dQ/dt/dA	Watts/meter²
Monochromatic Irradiance	dQ/dt/dA/dλ or dQ/dt/dA/dν	W/m²/micron W/m²/cm⁻¹
Radiance	dQ/dt/dA/dλ/dΩ or dQ/dt/dA/dν/dΩ	W/m²/micron/ster W/m²/cm⁻¹/ster

Radiation from the Sun

The rate of energy transfer by electromagnetic radiation is called the radiant flux, which has units of energy per unit time. It is denoted by

$$F = dQ / dt$$

and is measured in joules per second or watts. For example, the radiant flux from the sun is about 3.90×10^{26} W.

The radiant flux per unit area is called the irradiance (or radiant flux density in some texts). It is denoted by

$$E = dQ / dt / dA$$

and is measured in watts per square metre. The irradiance of electromagnetic radiation passing through the outermost limits of the visible disk of the sun (which has an approximate radius of 7×10^8 m) is given by

$$E (\text{sun sfc}) = \frac{3.90 \times 10^{26}}{4\pi (7 \times 10^8)^2} = 6.34 \times 10^7 \text{ W m}^{-2} .$$

The solar irradiance arriving at the earth can be calculated by realizing that the flux is a constant, therefore

$$E (\text{earth sfc}) \times 4\pi R_{\text{es}}^2 = E (\text{sun sfc}) \times 4\pi R_{\text{s}}^2,$$

where R_{es} is the mean earth to sun distance (roughly 1.5×10^{11} m) and R_{s} is the solar radius. This yields

$$E (\text{earth sfc}) = 6.34 \times 10^7 (7 \times 10^8 / 1.5 \times 10^{11})^2 = 1380 \text{ W m}^{-2}.$$

The irradiance per unit wavelength interval at wavelength λ is called the monochromatic irradiance,

$$E_{\lambda} = dQ / dt / dA / d\lambda ,$$

and has the units of watts per square metre per micrometer. With this definition, the irradiance is readily seen to be

$$E = \int_0^{\infty} E_{\lambda} d\lambda .$$

In general, the irradiance upon an element of surface area may consist of contributions which come from an infinity of different directions. It is sometimes necessary to identify the part of the irradiance that is coming from directions within some specified infinitesimal arc of solid angle $d\Omega$. The irradiance per unit solid angle is called the radiance,

$$I = dQ / dt / dA / d\lambda / d\Omega,$$

and is expressed in watts per square metre per micrometer per steradian. This quantity is often also referred to as intensity and denoted by the letter B (when referring to the Planck function).

If the zenith angle, θ , is the angle between the direction of the radiation and the normal to the surface, then the component of the radiance normal to the surface is then given by $I \cos \theta$. The irradiance represents the combined effects of the normal component of the radiation coming from the whole hemisphere; that is,

$$E = \int_{\Omega} I \cos \theta d\Omega \quad \text{where in spherical coordinates } d\Omega = \sin \theta d\theta d\phi .$$

Radiation whose radiance is independent of direction is called isotropic radiation. In this case, the integration over $d\Omega$ can be readily shown to be equal to π so that

$$E = \pi I .$$

Radiation is governed by Planck's Law

$$B(\lambda, T) = \frac{c_2}{\lambda T} \left\{ \frac{c_1}{\lambda^5} [e^{-1}] \right\}$$

Summing the Planck function at one temperature over all wavelengths yields the energy of the radiating source

$$E = \int_{\lambda} B(\lambda, T) = \sigma T^4$$

Brightness temperature is uniquely related to radiance for a given wavelength by the Planck function.

Using wavelengths

Planck's Law
$$B(\nu, T) = \frac{c_1}{\lambda^5} \left[e^{-\frac{c_2}{\lambda T}} - 1 \right]^{-1} \quad (\text{mW/m}^2/\text{ster/cm})$$

where

λ = wavelengths in cm

T = temperature of emitting surface (deg K)

$c_1 = 1.191044 \times 10^{-5}$ (mW/m²/ster/cm⁻⁴)

$c_2 = 1.438769$ (cm deg K)

Wien's Law

$$dB(\lambda_{\max}, T) / d\lambda = 0 \text{ where } \lambda(\max) = .2897/T$$

indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\lambda_{\max}, T) \sim T^{**5}$.

Stefan-Boltzmann Law
$$E = \pi \int_0^{\infty} B(\lambda, T) d\lambda = \sigma T^4, \text{ where } \sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{deg}^4.$$

states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

Brightness Temperature

$$T = \frac{c_1}{\lambda^5 B_\lambda} \left[\ln \left(\frac{c_1}{\lambda^5 B_\lambda} + 1 \right) \right]^{-1}$$

is determined by inverting Planck function

Using wavenumbers

$$B(\nu, T) = \frac{c_2 \nu / T}{c_1 \nu^3} [e^{-c_2 \nu / T} - 1]^{-1}$$

(mW/m²/ster/cm⁻¹)

$$\nu(\text{max in cm}^{-1}) = 1.95T$$

$$B(\nu_{\text{max}}, T) \sim T^{**3}.$$

$$E = \pi \int_0^{\infty} B(\nu, T) d\nu = \sigma T^4,$$

$$T = \frac{c_2 \nu}{c_1 \nu^3} [\ln\left(\frac{c_1 \nu^3}{B_\nu} + 1\right)]$$

Using wavelengths

$$B(\lambda, T) = \frac{c_2 / \lambda T}{c_1 \lambda^5} [e^{-c_2 / \lambda T} - 1]^{-1}$$

(mW/m²/ster/μm)

$$\lambda(\text{max in cm})T = 0.2897$$

$$B(\lambda_{\text{max}}, T) \sim T^{**5}.$$

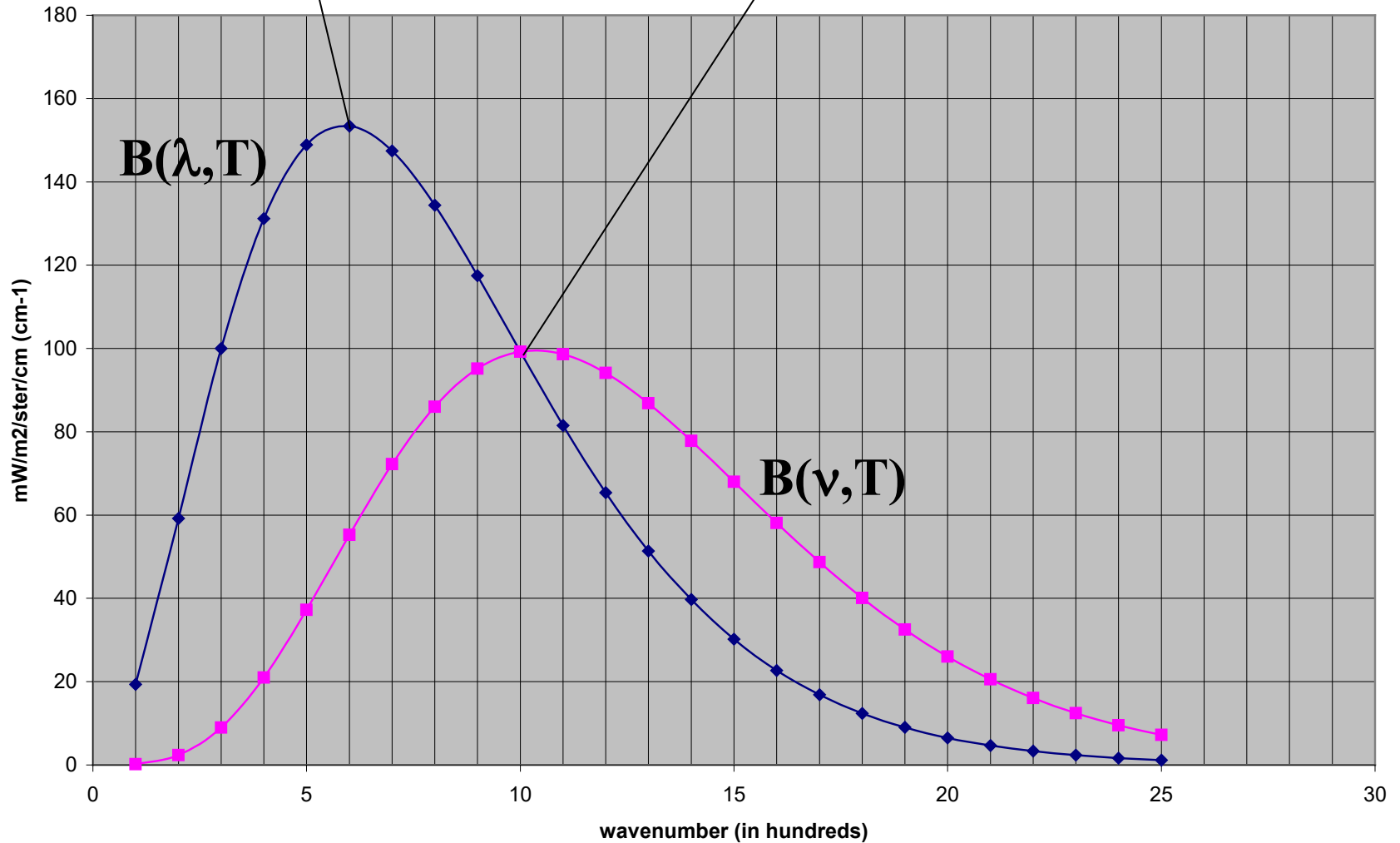
$$E = \pi \int_0^{\infty} B(\lambda, T) d\lambda = \sigma T^4,$$

$$T = \frac{c_2}{\lambda \ln\left(\frac{c_1}{\lambda^5 B_\lambda} + 1\right)}$$

$$B(\lambda_{\max}, T) \sim T^5$$

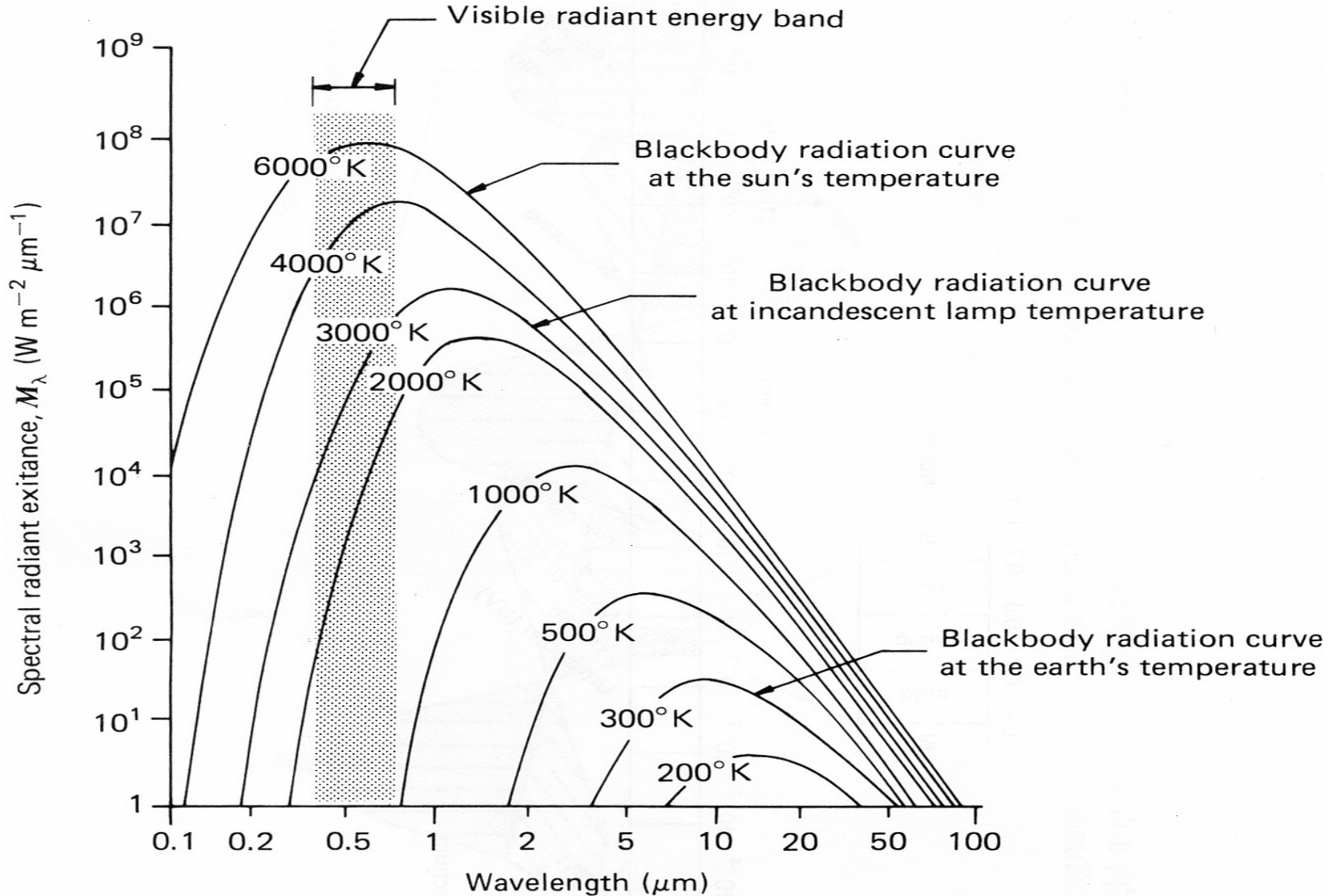
$$B(\nu_{\max}, T) \sim T^3$$

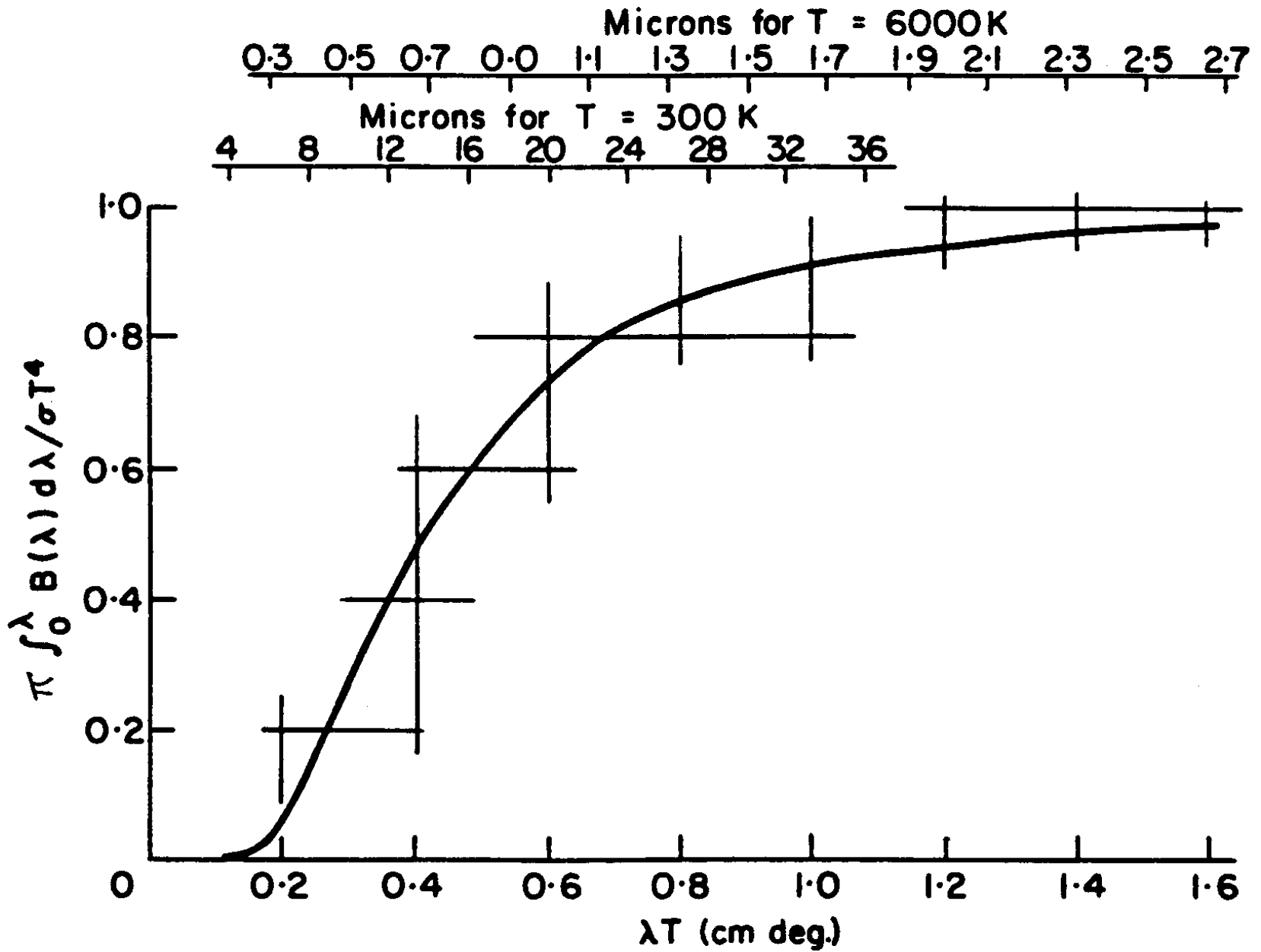
Planck Radiances

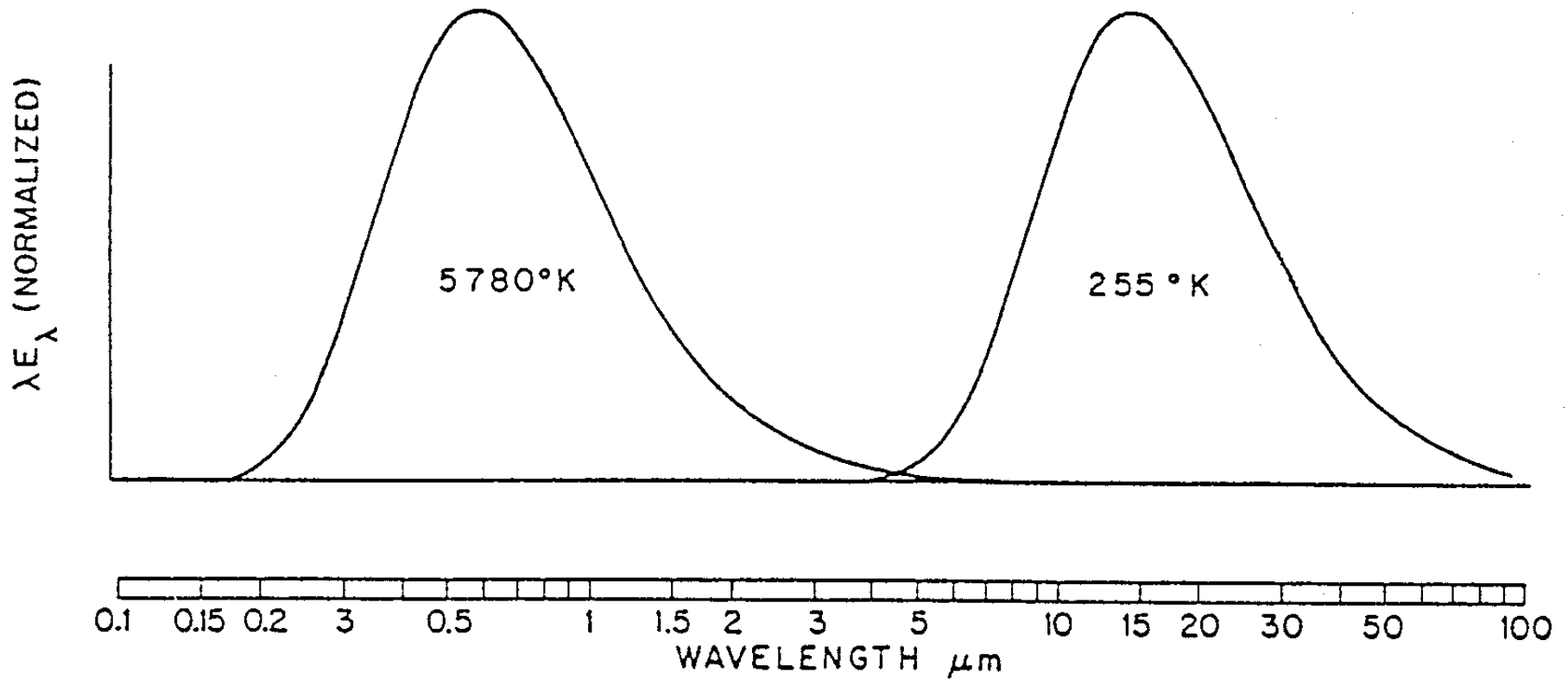


$B(\lambda, T)$ versus $B(\nu, T)$

Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures



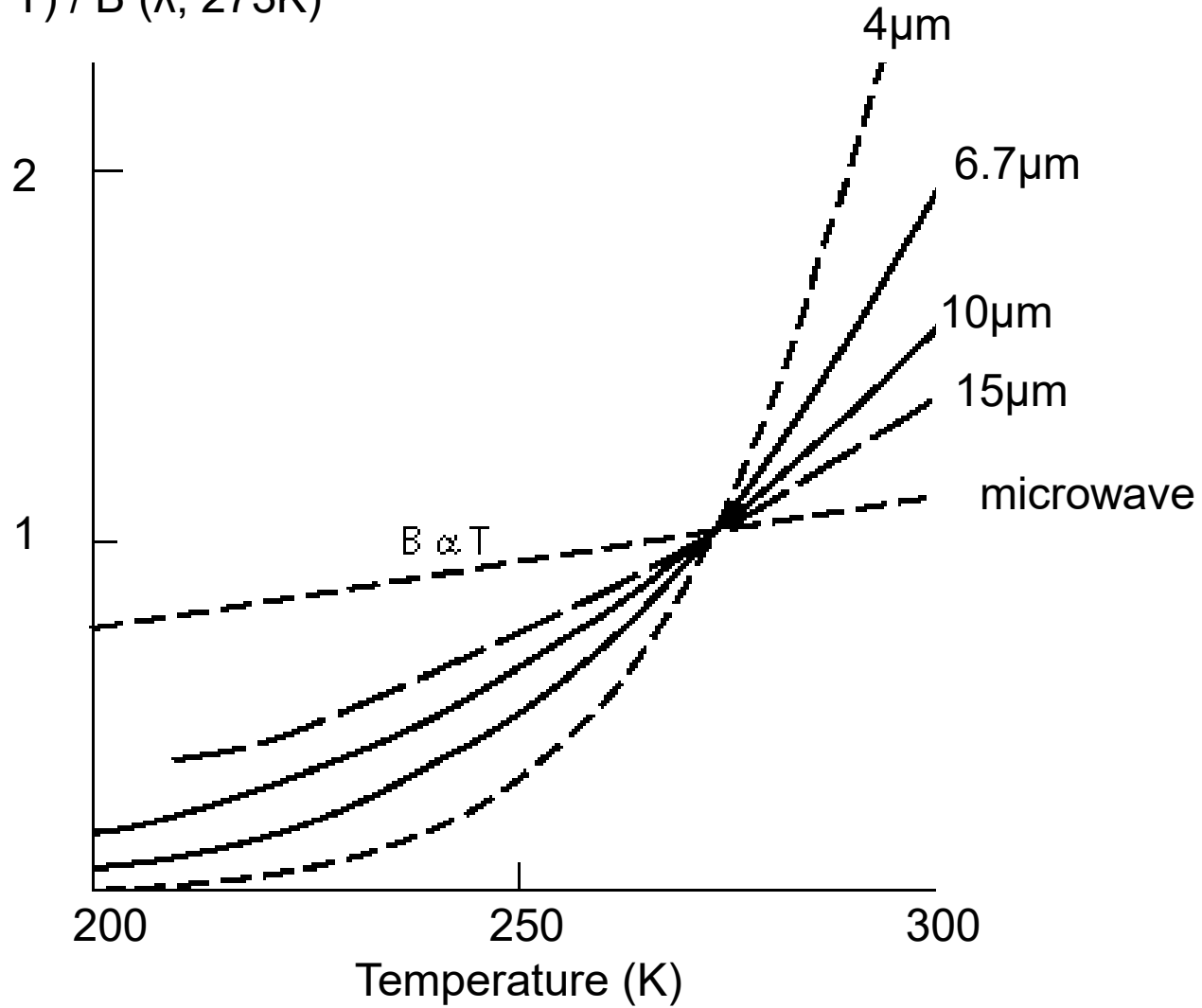




Normalized black body spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength so that the area under the curves is proportional to irradiance.

Temperature Sensitivity of $B(\lambda, T)$ for typical earth scene temperatures

$B(\lambda, T) / B(\lambda, 273K)$



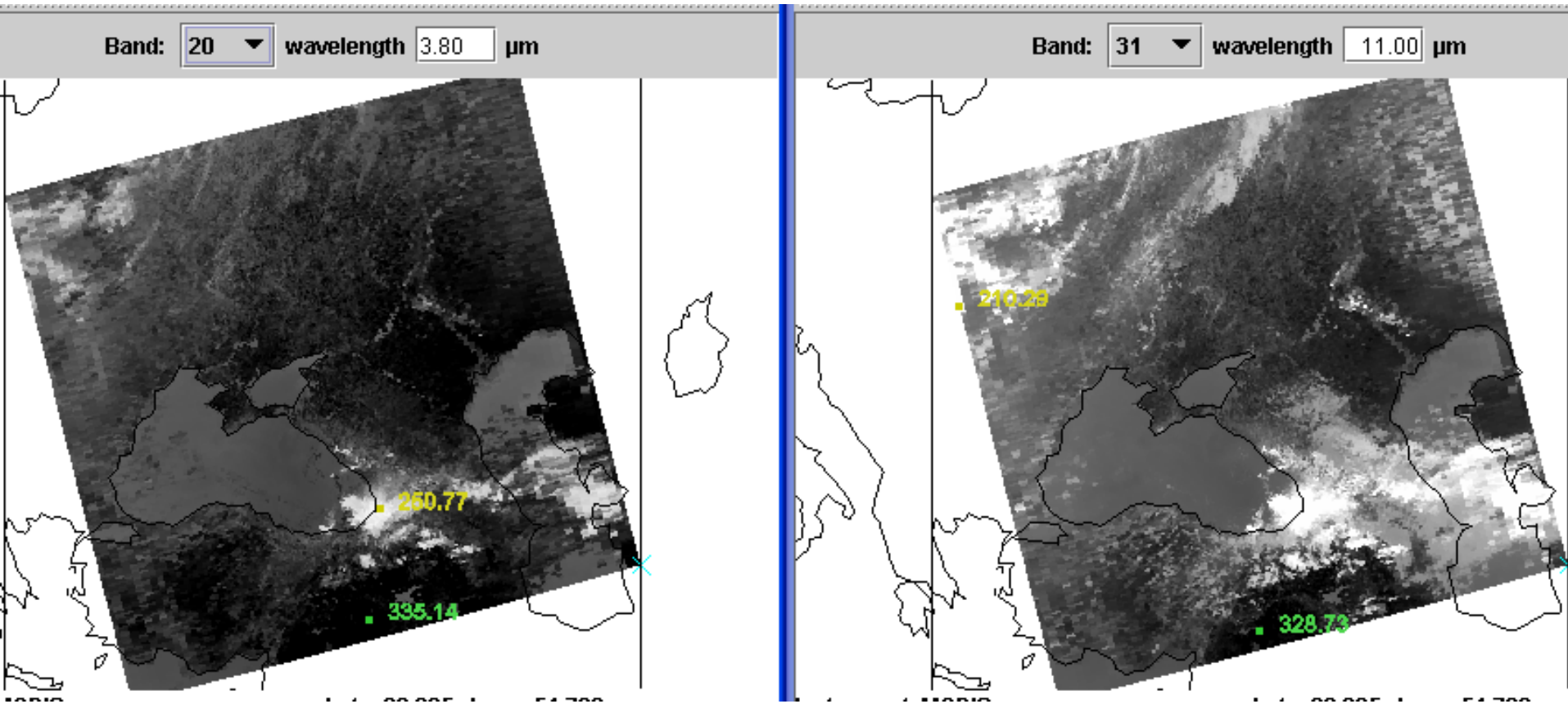
Temperature sensitivity, or the percentage change in radiance corresponding to a percentage change in temperature, α , is defined as

$$dB/B = \alpha dT/T.$$

The temperature sensitivity indicates the power to which the Planck radiance depends on temperature, since B proportional to T^α satisfies the equation. For infrared wavelengths,

$$\alpha = c_2\nu/T = c_2/\lambda T.$$

Wavenumber	Typical Scene Temperature	Temperature Sensitivity
700	220	4.58
900	300	4.32
1200	300	5.76
1600	240	9.59
2300	220	15.04
2500	300	11.99



Cloud edges and broken clouds appear different in 11 and 4 um images.

$$T(11)^{**4}=(1-N)*Tclr^{**4}+N*Tcld^{**4}\sim(1-N)*300^{**4}+N*200^{**4}$$

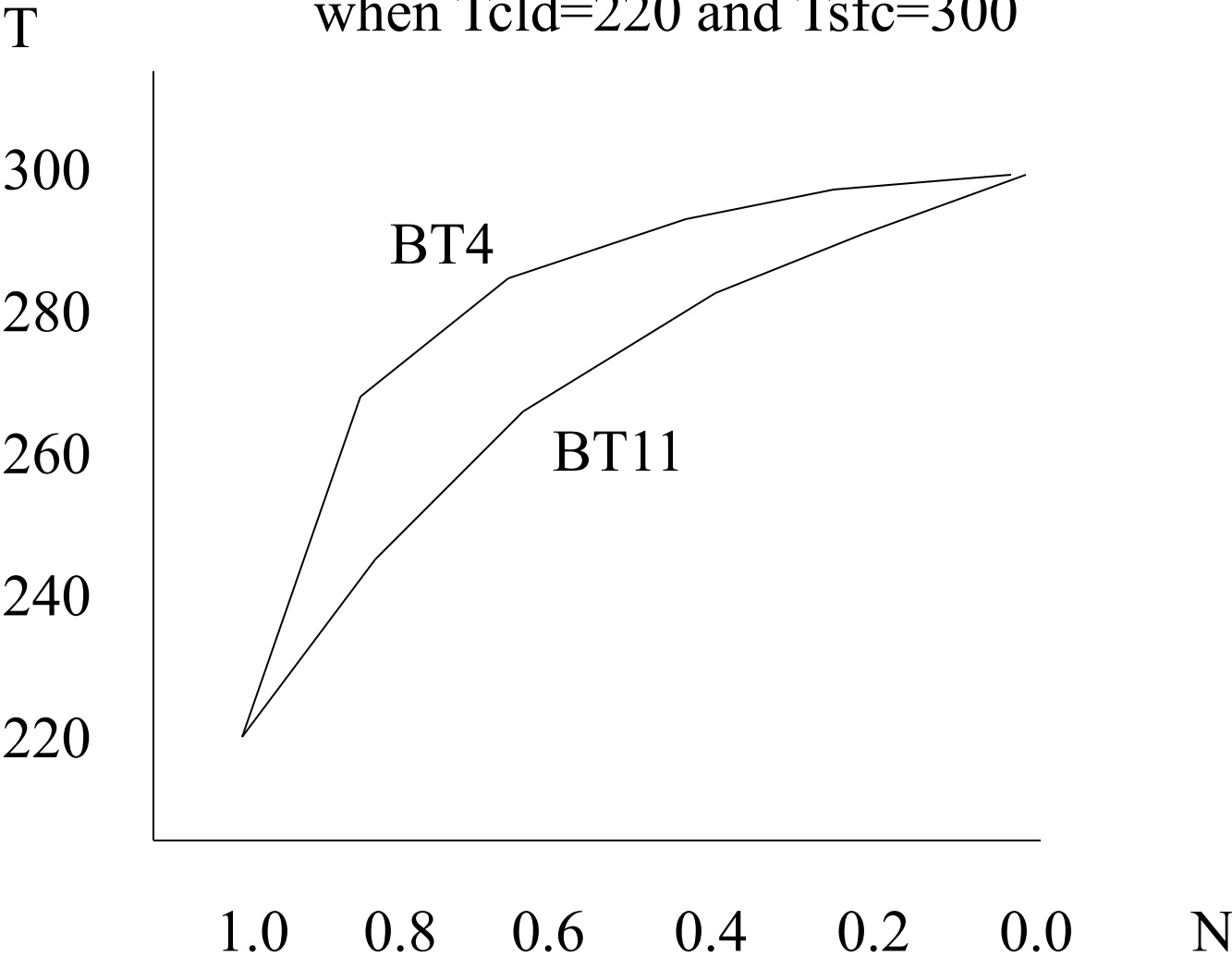
$$T(4)^{**12}=(1-N)*Tclr^{**12}+N*Tcld^{**12}\sim(1-N)*300^{**12}+N*200^{**12}$$

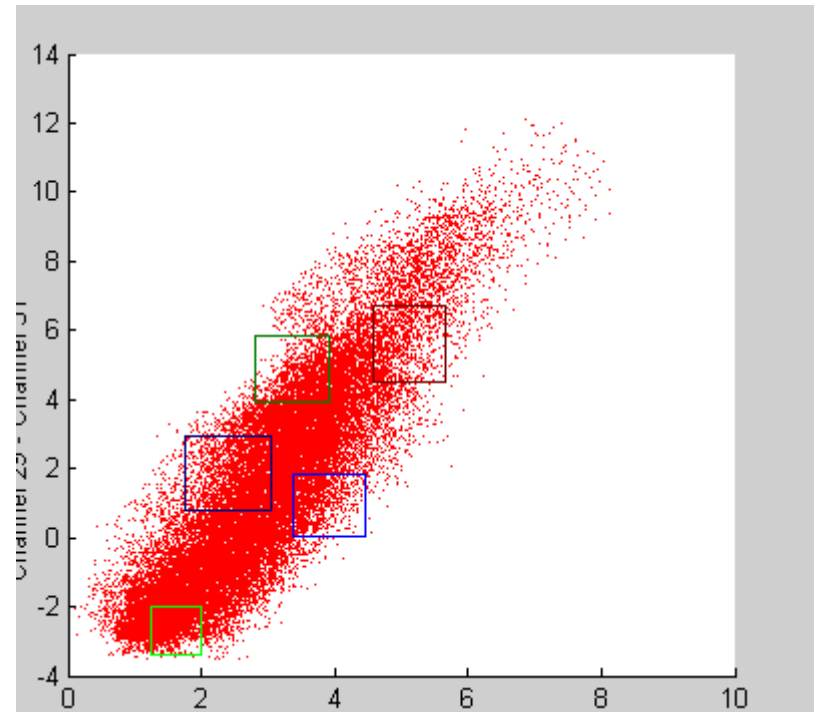
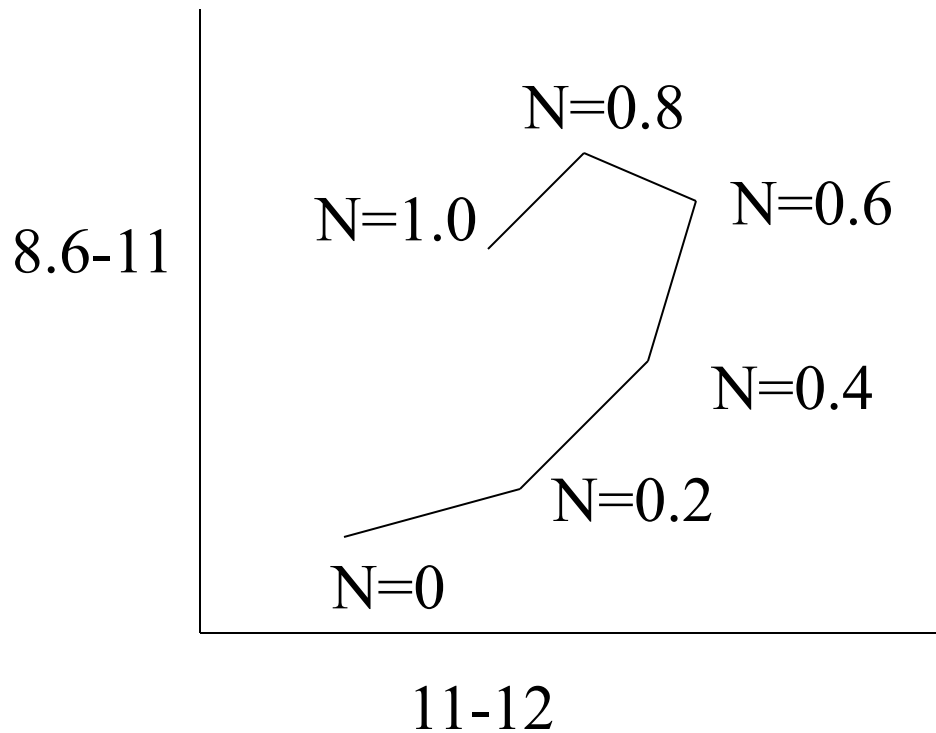
Cold part of pixel has more influence for B(11) than B(4)

Table 6.1 Longwave and Shortwave Window Planck Radiances ($\text{mW/m}^2/\text{ster/cm}^{-1}$) and Brightness Temperatures (degrees K) as a function of Fractional Cloud Amount (for cloud of 220 K and surface of 300 K) using $B(T) = (1-N)*B(T_{\text{sfc}}) + N*B(T_{\text{cld}})$.

Cloud Fraction N	Longwave Window		Shortwave Window		$T_s - T_1$
	Rad	Temp	Rad	Temp	
1.0	23.5	220	.005	220	0
.8	42.0	244	.114	267	23
.6	60.5	261	.223	280	19
.4	79.0	276	.332	289	13
.2	97.5	289	.441	295	6
.0	116.0	300	.550	300	0

BT SW and LW for different cloud amounts
when $T_{cld}=220$ and $T_{sfc}=300$





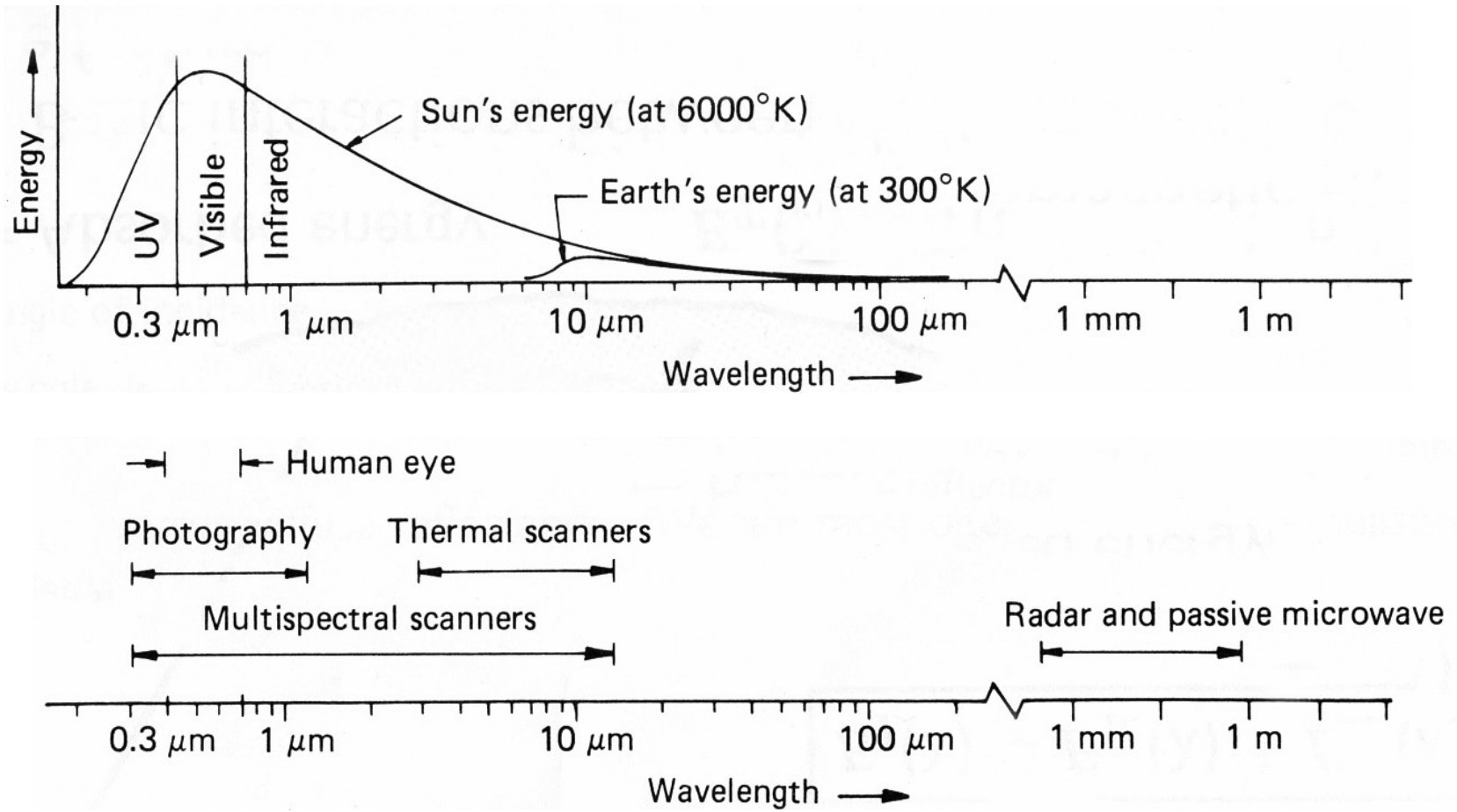
Broken clouds appear different in 8.6, 11 and 12 um images;
 assume $T_{clr}=300$ and $T_{cld}=230$

$$T(11)-T(12)=[(1-N)*B_{11}(T_{clr})+N*B_{11}(T_{cld})]^{-1} \\ - [(1-N)*B_{12}(T_{clr})+N*B_{12}(T_{cld})]^{-1}$$

$$T(8.6)-T(11)=[(1-N)*B_{8.6}(T_{clr})+N*B_{8.6}(T_{cld})]^{-1} \\ - [(1-N)*B_{11}(T_{clr})+N*B_{11}(T_{cld})]^{-1}$$

Cold part of pixel has more influence at longer wavelengths

Spectral Characteristics of Energy Sources and Sensing Systems



Relevant Material in Applications of Meteorological Satellites

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CHAPTER 5 - THE RADIATIVE TRANSFER EQUATION (RTE)

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Emission, Absorption, Reflection, and Scattering

Blackbody radiation B_λ represents the upper limit to the amount of radiation that a real substance may emit at a given temperature for a given wavelength.

Emissivity ε_λ is defined as the fraction of emitted radiation R_λ to Blackbody radiation,

$$\varepsilon_\lambda = R_\lambda / B_\lambda .$$

In a medium at thermal equilibrium, what is absorbed is emitted (what goes in comes out) so

$$a_\lambda = \varepsilon_\lambda .$$

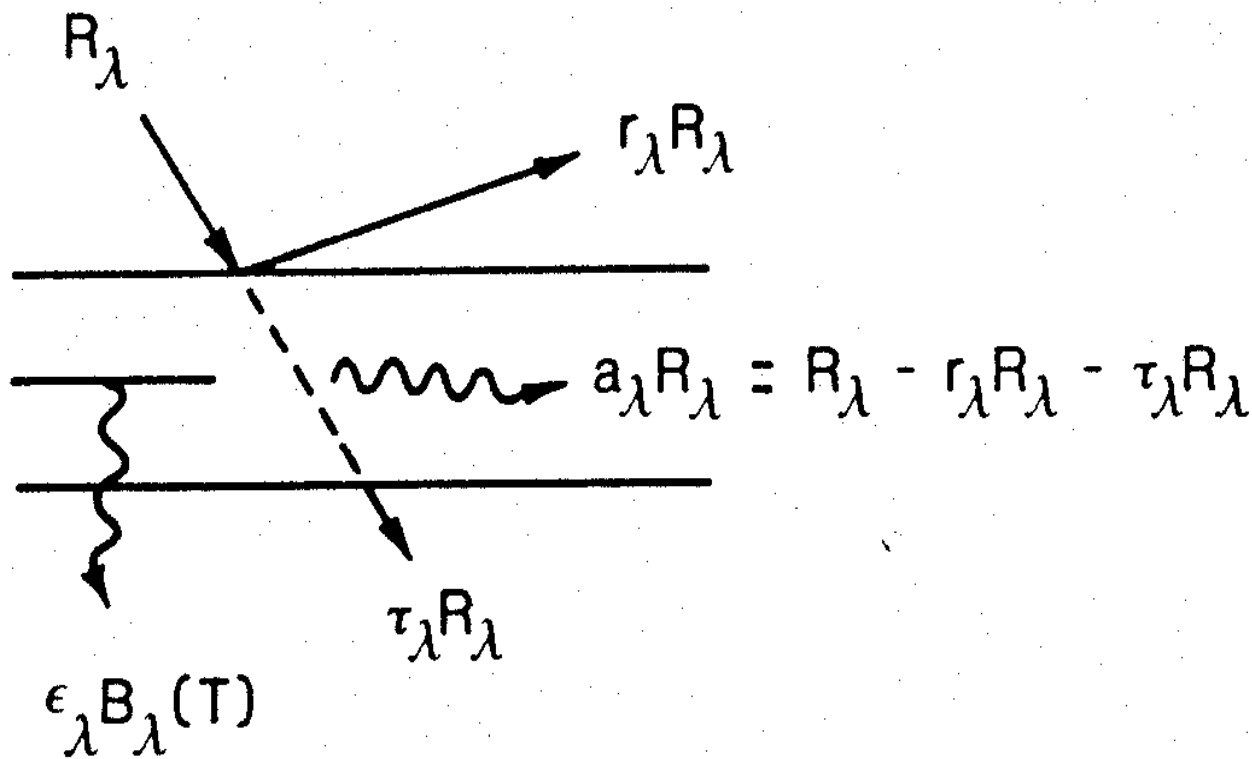
Thus, materials which are strong absorbers at a given wavelength are also strong emitters at that wavelength; similarly weak absorbers are weak emitters.

If a_λ , r_λ , and τ_λ represent the fractional absorption, reflectance, and transmittance, respectively, then conservation of energy says

$$a_\lambda + r_\lambda + \tau_\lambda = 1 .$$

For a blackbody $a_\lambda = 1$, it follows that $r_\lambda = 0$ and $\tau_\lambda = 0$ for blackbody radiation. Also, for a perfect window $\tau_\lambda = 1$, $a_\lambda = 0$ and $r_\lambda = 0$. For any opaque surface $\tau_\lambda = 0$, so radiation is either absorbed or reflected $a_\lambda + r_\lambda = 1$.

At any wavelength, strong reflectors are weak absorbers (i.e., snow at visible wavelengths), and weak reflectors are strong absorbers (i.e., asphalt at visible wavelengths).



‘ENERGY
CONSERVATION’

Planetary Albedo

Planetary albedo is defined as the fraction of the total incident solar irradiance, S , that is reflected back into space. Radiation balance then requires that the absorbed solar irradiance is given by

$$E = (1 - A) S/4.$$

The factor of one-fourth arises because the cross sectional area of the earth disc to solar radiation, πr^2 , is one-fourth the earth radiating surface, $4\pi r^2$.

Thus recalling that $S = 1380 \text{ Wm}^{-2}$, if the earth albedo is 30 percent,

$$\text{then } E = 241 \text{ Wm}^{-2}.$$

Incoming
solar

Outgoing IR

$\downarrow E$

$\uparrow (1-a_1) Y_s$ $\uparrow Y_a$

top of the atmosphere

$\downarrow (1-a_s) E$

$\uparrow Y_s$

$\downarrow Y_a$

earth surface.

$$Y_s = \frac{(2-a_s)}{(2-a_L)} E = \sigma T_s^4$$

Selective Absorption and Transmission

Assume that the earth behaves like a blackbody and that the atmosphere has an absorptivity a_S for incoming solar radiation and a_L for outgoing longwave radiation. Let Y_a be the irradiance emitted by the atmosphere (both upward and downward); Y_s the irradiance emitted from the earth's surface; and E the solar irradiance absorbed by the earth-atmosphere system. Then, radiative equilibrium requires

$$\begin{aligned} E - (1-a_L) Y_s - Y_a &= 0, \text{ at the top of the atmosphere,} \\ (1-a_S) E - Y_s + Y_a &= 0, \text{ at the surface.} \end{aligned}$$

Solving yields

$$\begin{aligned} Y_s &= \frac{(2-a_S)}{(2-a_L)} E, \text{ and} \\ Y_a &= \frac{(2-a_L) - (1-a_L)(2-a_S)}{(2-a_L)} E. \end{aligned}$$

Since $a_L > a_S$, the irradiance and hence the radiative equilibrium temperature at the earth surface is increased by the presence of the atmosphere. With $a_L = .8$ and $a_S = .1$ and $E = 241 \text{ Wm}^{-2}$, Stefans Law yields a blackbody temperature at the surface of 286 K, in contrast to the 255 K it would be if the atmospheric absorptance was independent of wavelength ($a_S = a_L$). The atmospheric gray body temperature in this example turns out to be 245 K.

Transmittance

Transmission through an absorbing medium for a given wavelength is governed by the number of intervening absorbing molecules (path length u) and their absorbing power (k_λ) at that wavelength. Beer's law indicates that transmittance decays exponentially with increasing path length

$$\tau_\lambda (z \rightarrow \infty) = e^{-k_\lambda u (z)}$$

where the path length is given by $u (z) = \int_z^\infty \rho dz$.

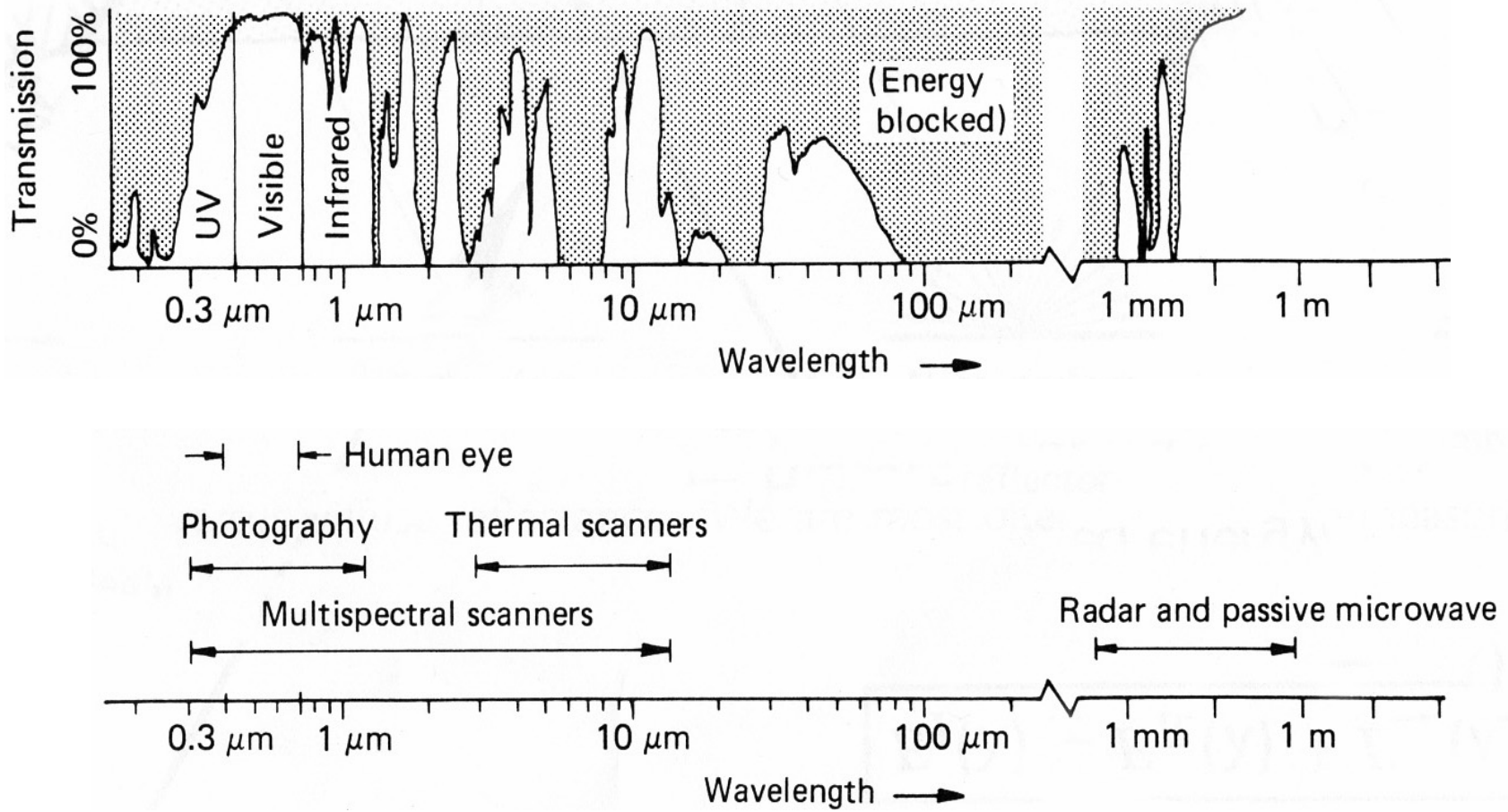
$k_\lambda u$ is a measure of the cumulative depletion that the beam of radiation has experienced as a result of its passage through the layer and is often called the optical depth σ_λ .

Realizing that the hydrostatic equation implies $g \rho dz = -q dp$

where q is the mixing ratio and ρ is the density of the atmosphere, then

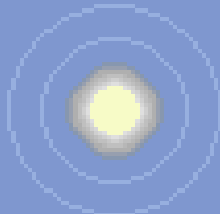
$$u (p) = \int_0^p q g^{-1} dp \quad \text{and} \quad \tau_\lambda (p \rightarrow 0) = e^{-k_\lambda u (p)} .$$

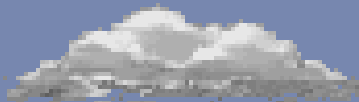

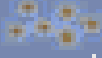

Spectral Characteristics of Atmospheric Transmission and Sensing Systems

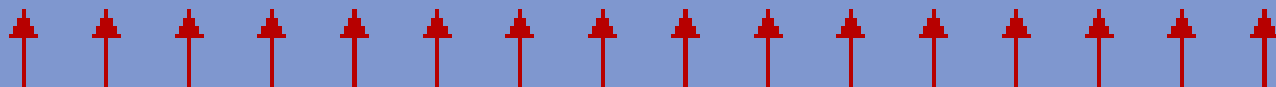


Relative Effects of Radiative Processes

Sun - Earth - Atmosphere Energy System

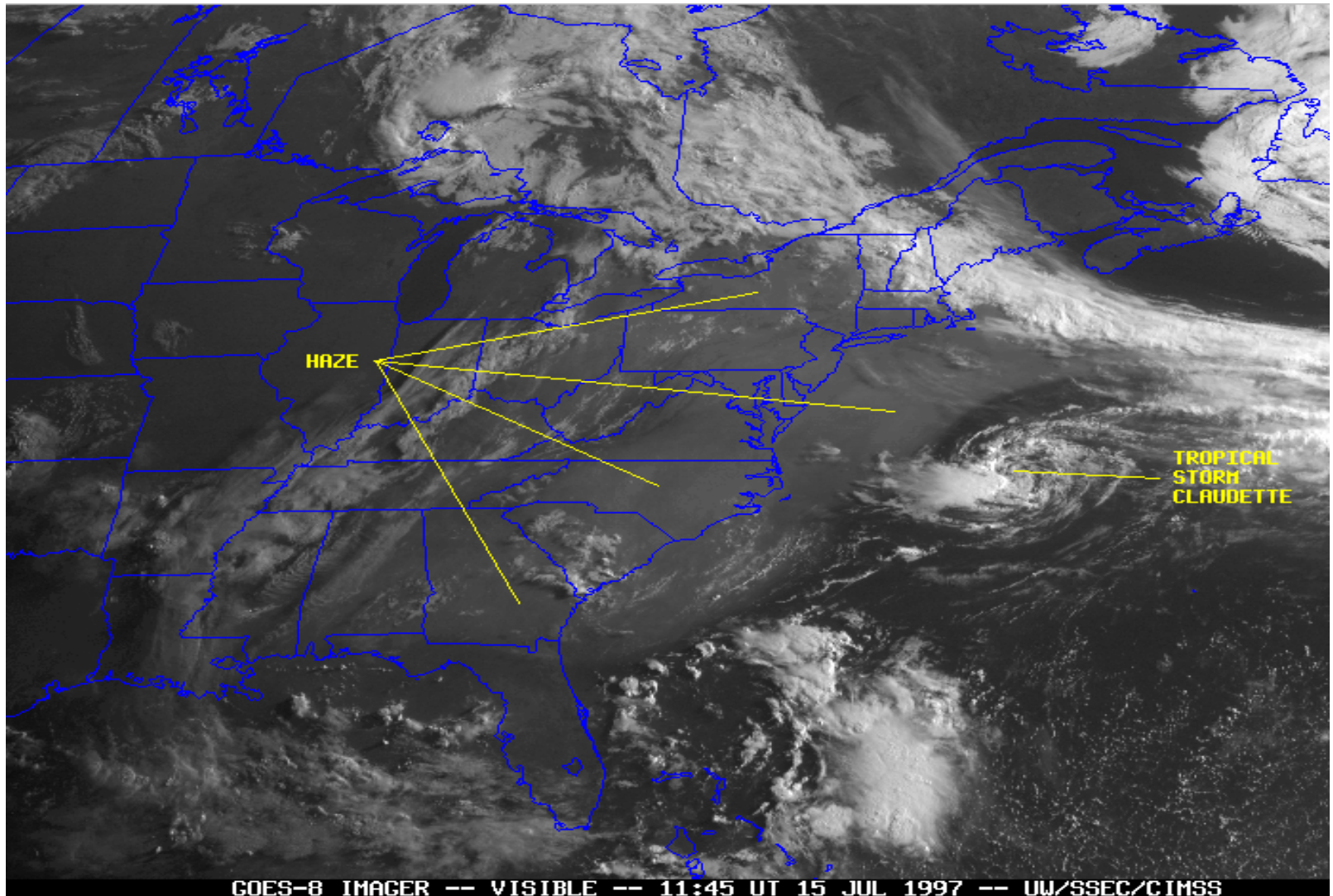


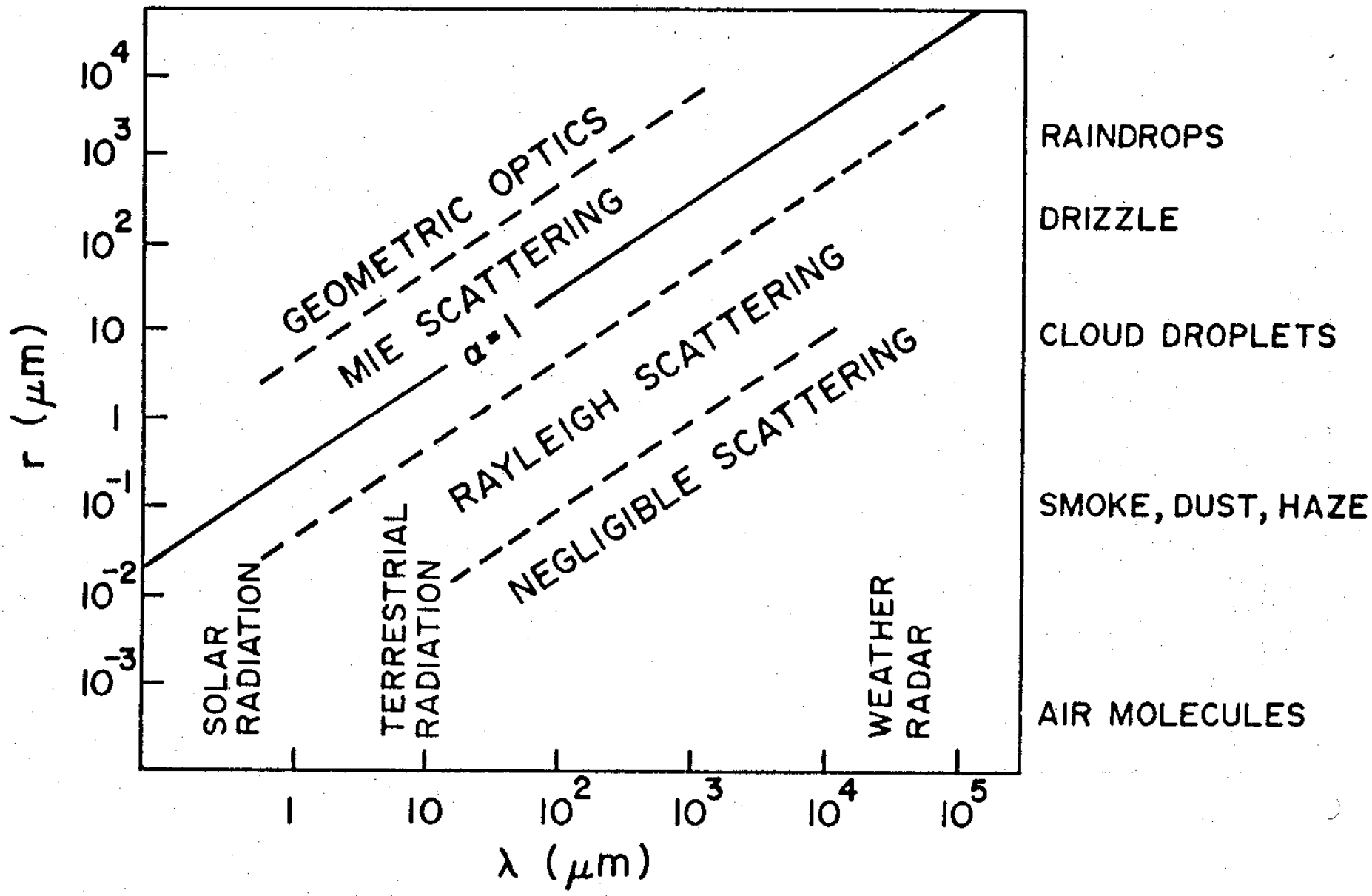
		Solar Radiation		Terrestrial Radiation	
		Absorption / Emission	Scattering	Absorption / Emission	Scattering
 Clouds	Water	✓ Small	✓ Large	✓ Moderate	✓ Negligible
	Ice	✓ Variable	✓ Moderate	✓ Small	✓ Negligible
 Molecules in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
 Aerosols in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
 Earth's Surface	Land	✓ Large	✓ Moderate	✓ Large	✓ Negligible
	Water	✓ Large	✓ Small	✓ Large	✓ Negligible
	Snow / Ice	✓ Variable	✓ Large	✓ Variable	✓ Negligible



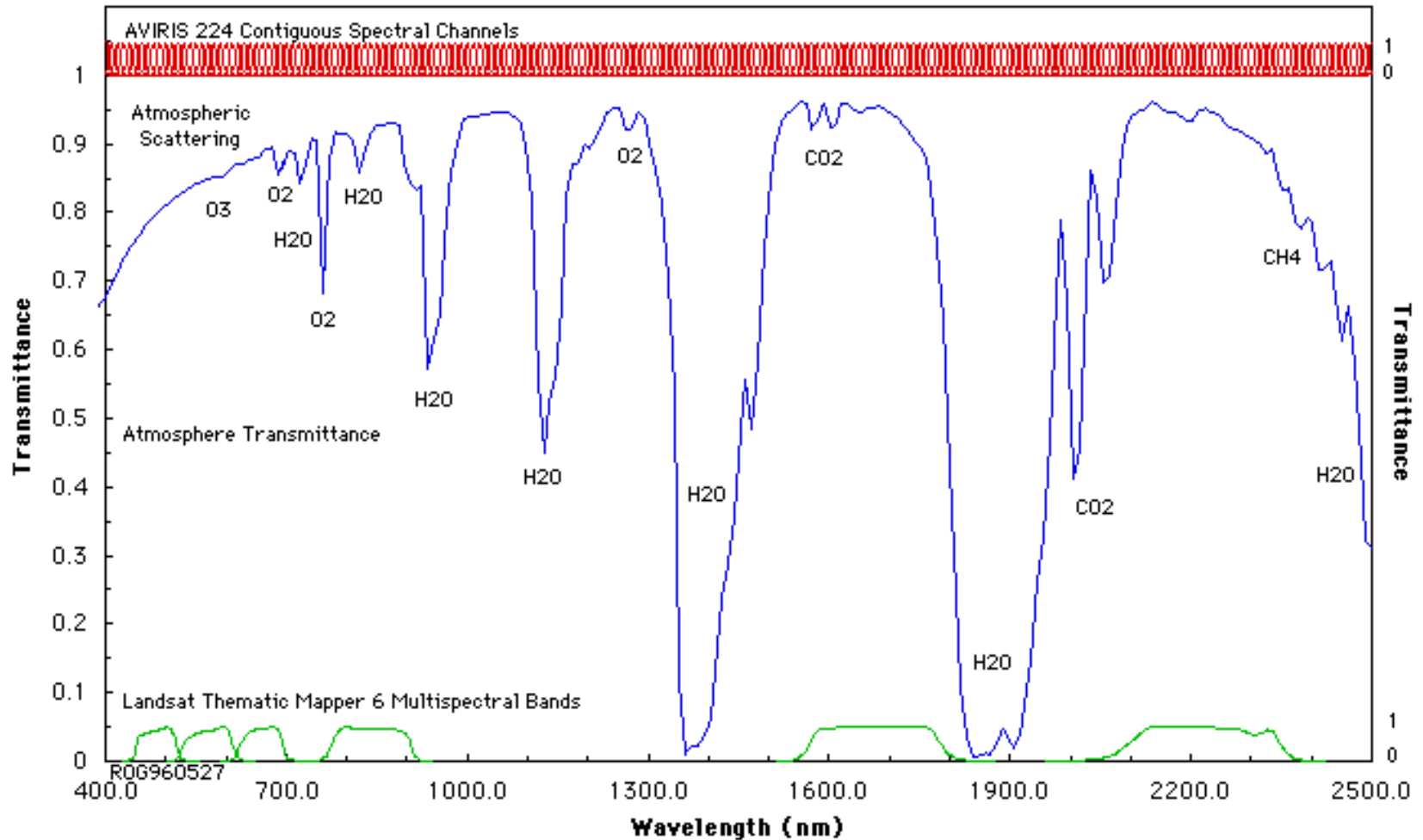
Earth

Scattering of early morning sun light from haze



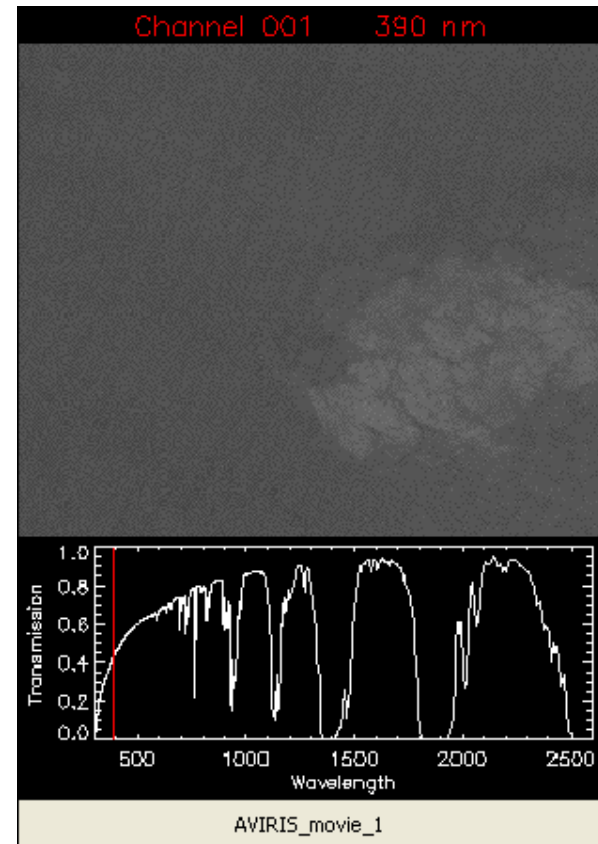


Measurements in the Solar Reflected Spectrum across the region covered by AVIRIS

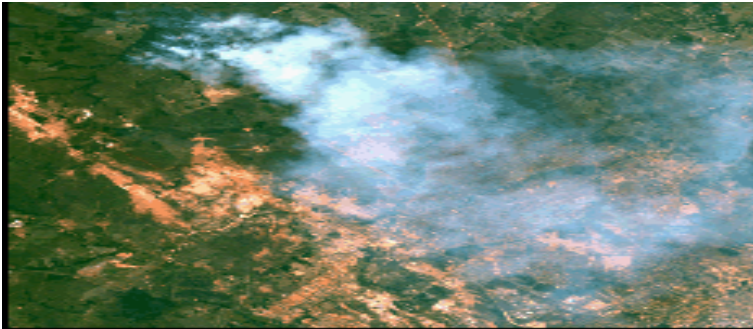


AVIRIS Movie #1

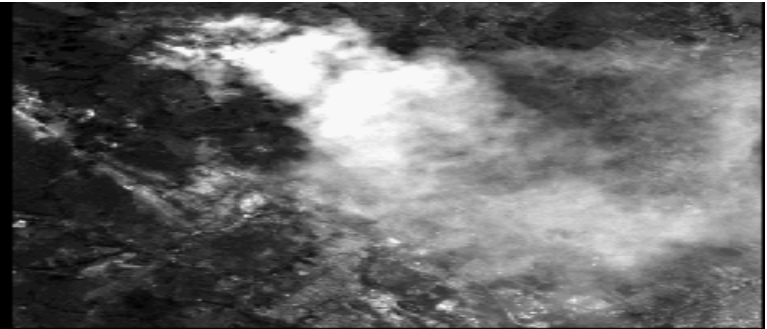
AVIRIS Image - Linden CA 20-Aug-1992
224 Spectral Bands: 0.4 - 2.5 μm
Pixel: 20m x 20m Scene: 10km x 10km



Cuiaba Brazil mosaic on 25 August 1995 shows a forest clearing fire.



True color



500.5 nm



1000.2 nm



1501.4 nm



2000.5 nm



2508.5 nm

Relevant Material in Applications of Meteorological Satellites

CHAPTER 2 - NATURE OF RADIATION

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Radiative Transfer Equation

The radiance leaving the earth-atmosphere system sensed by a satellite borne radiometer is the sum of radiation emissions from the earth-surface and each atmospheric level that are transmitted to the top of the atmosphere. Considering the earth's surface to be a blackbody emitter (emissivity equal to unity), the upwelling radiance intensity, I_λ , for a cloudless atmosphere is given by the expression

$$I_\lambda = \varepsilon_\lambda^{\text{sfc}} B_\lambda(T_{\text{sfc}}) \tau_\lambda(\text{sfc} - \text{top}) + \sum_{\text{layers}} \varepsilon_\lambda^{\text{layer}} B_\lambda(T_{\text{layer}}) \tau_\lambda(\text{layer} - \text{top})$$

where the first term is the surface contribution and the second term is the atmospheric contribution to the radiance to space.

In standard notation,

$$I_{\lambda} = \varepsilon_{\lambda}^{\text{sfc}} B_{\lambda}(T(p_s)) \tau_{\lambda}(p_s) + \sum_p \varepsilon_{\lambda}(\Delta p) B_{\lambda}(T(p)) \tau_{\lambda}(p)$$

The emissivity of an infinitesimal layer of the atmosphere at pressure p is equal to the absorptance (one minus the transmittance of the layer). Consequently,

$$\varepsilon_{\lambda}(\Delta p) \tau_{\lambda}(p) = [1 - \tau_{\lambda}(\Delta p)] \tau_{\lambda}(p)$$

Since transmittance is an exponential function of depth of absorbing constituent,

$$\tau_{\lambda}(\Delta p) \tau_{\lambda}(p) = \exp \left[- \int_p^{p+\Delta p} k_{\lambda} q g^{-1} dp \right] * \exp \left[- \int_0^p k_{\lambda} q g^{-1} dp \right] = \tau_{\lambda}(p + \Delta p)$$

Therefore

$$\varepsilon_{\lambda}(\Delta p) \tau_{\lambda}(p) = \tau_{\lambda}(p) - \tau_{\lambda}(p + \Delta p) = - \Delta \tau_{\lambda}(p) .$$

So we can write

$$I_{\lambda} = \varepsilon_{\lambda}^{\text{sfc}} B_{\lambda}(T(p_s)) \tau_{\lambda}(p_s) - \sum_p B_{\lambda}(T(p)) \Delta \tau_{\lambda}(p) .$$

which when written in integral form reads

$$I_{\lambda} = \varepsilon_{\lambda}^{\text{sfc}} B_{\lambda}(T(p_s)) \tau_{\lambda}(p_s) - \int_0^{p_s} B_{\lambda}(T(p)) [d\tau_{\lambda}(p) / dp] dp .$$

When reflection from the earth surface is also considered, the Radiative Transfer Equation for infrared radiation can be written

$$I_{\lambda} = \varepsilon_{\lambda}^{\text{sfc}} B_{\lambda}(T_s) \tau_{\lambda}(p_s) + \int_{p_s}^0 B_{\lambda}(T(p)) F_{\lambda}(p) [d\tau_{\lambda}(p) / dp] dp$$

where

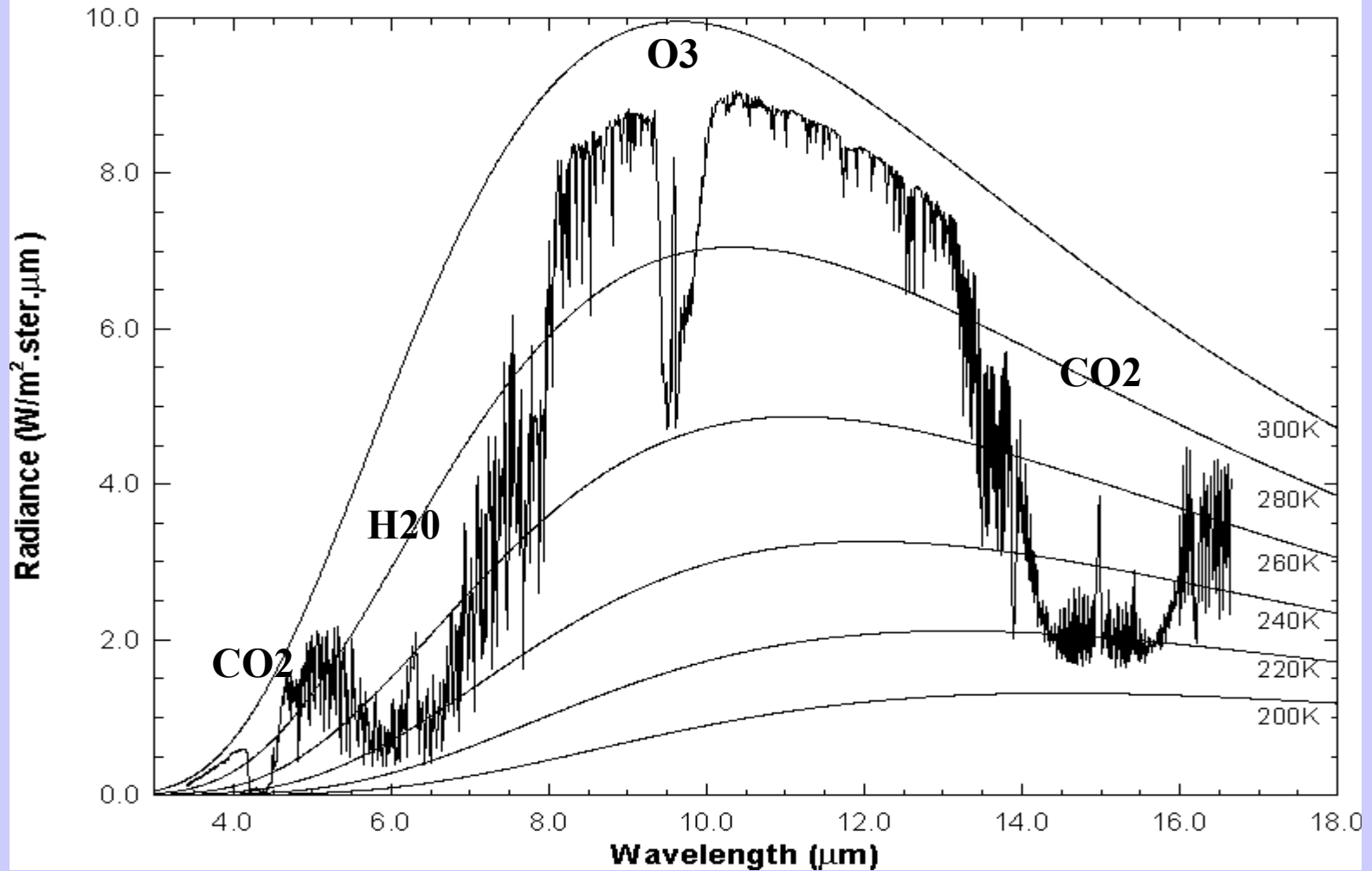
$$F_{\lambda}(p) = \{ 1 + (1 - \varepsilon_{\lambda}) [\tau_{\lambda}(p_s) / \tau_{\lambda}(p)]^2 \}$$

The first term is the spectral radiance emitted by the surface and attenuated by the atmosphere, often called the boundary term and the second term is the spectral radiance emitted to space by the atmosphere directly or by reflection from the earth surface.

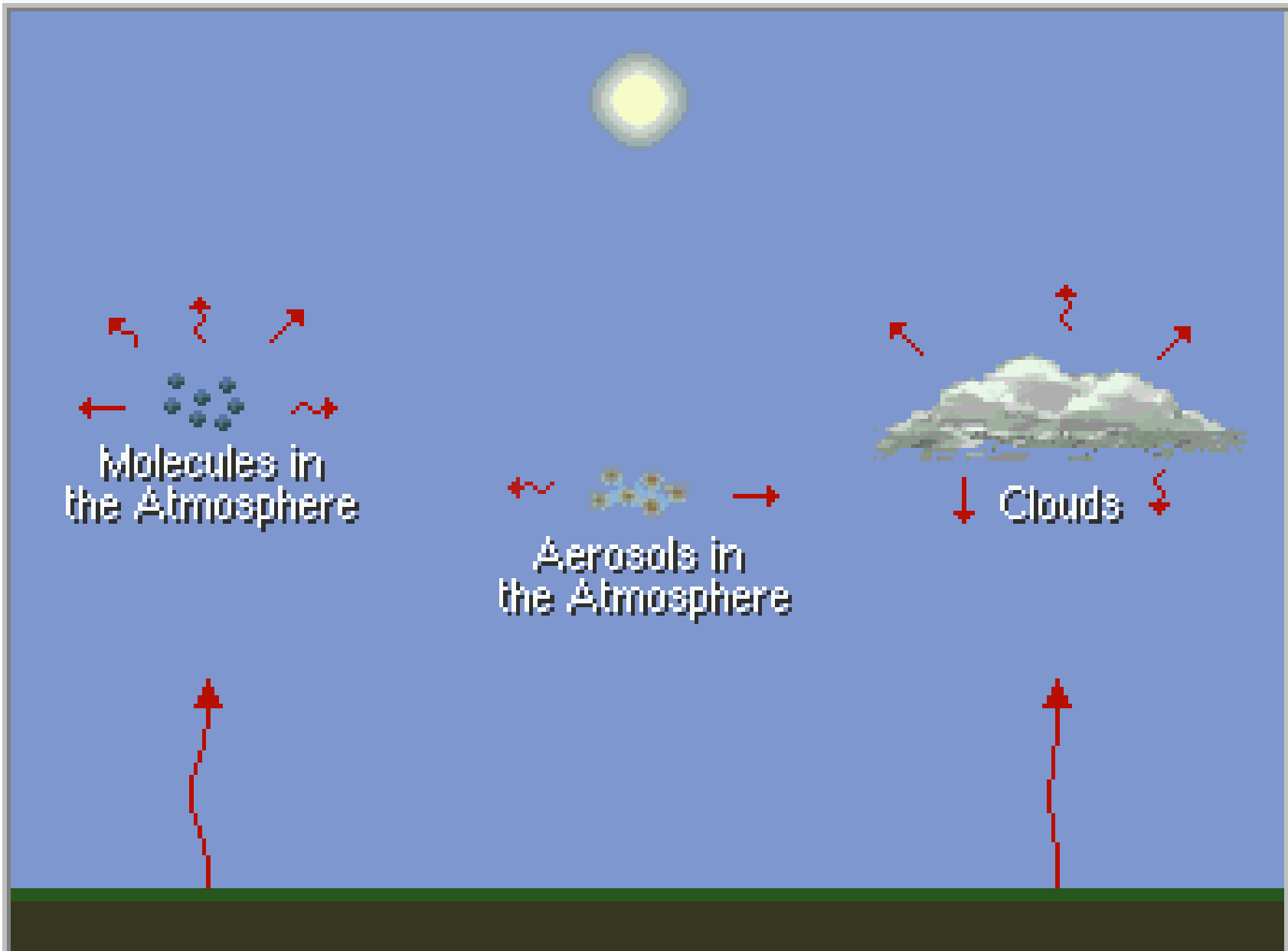
The atmospheric contribution is the weighted sum of the Planck radiance contribution from each layer, where the weighting function is $[d\tau_{\lambda}(p) / dp]$. This weighting function is an indication of where in the atmosphere the majority of the radiation for a given spectral band comes from.

Earth emitted spectra overlaid on Planck function envelopes

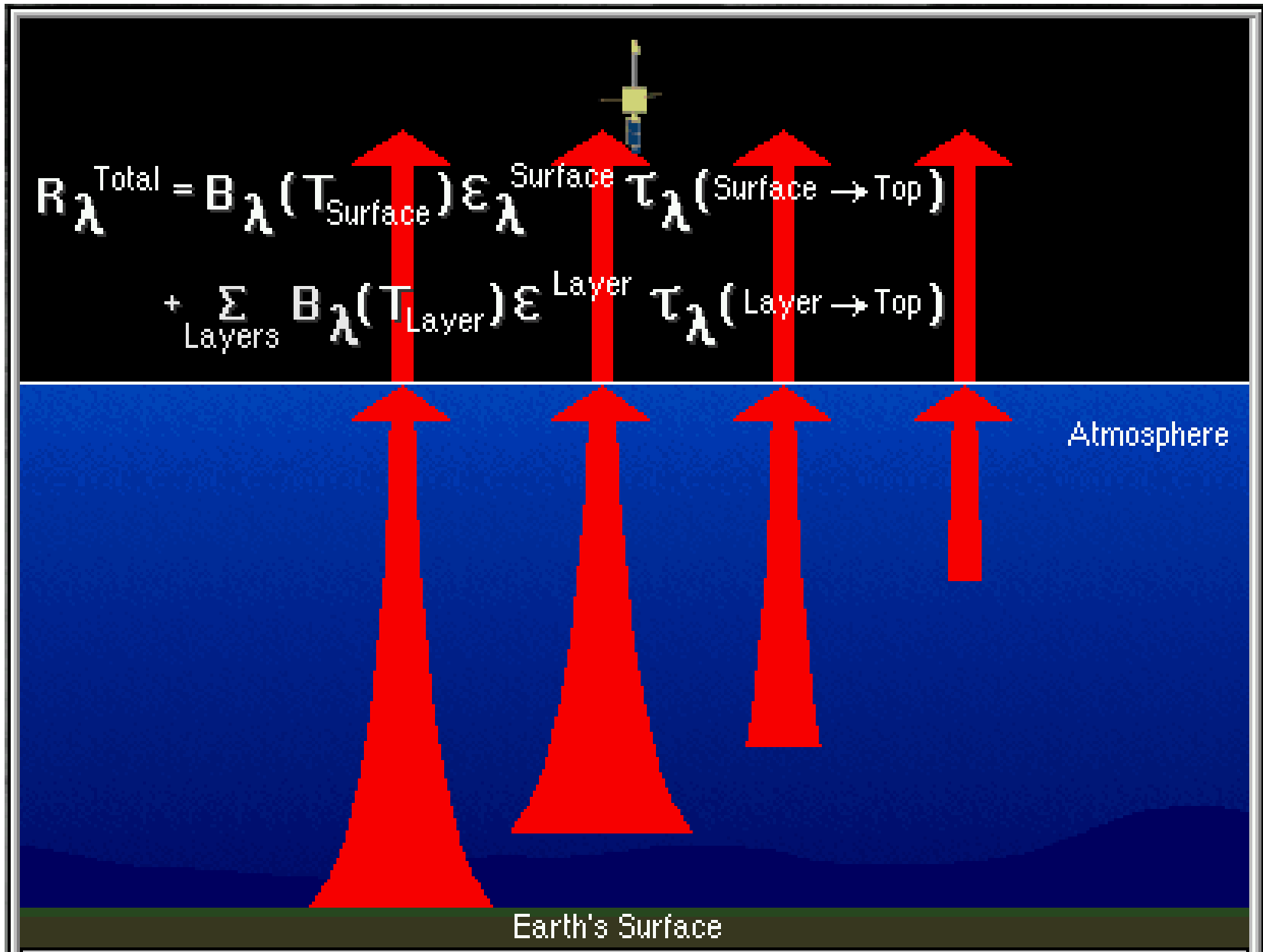
High resolution atmospheric absorption spectrum and comparative blackbody curves.



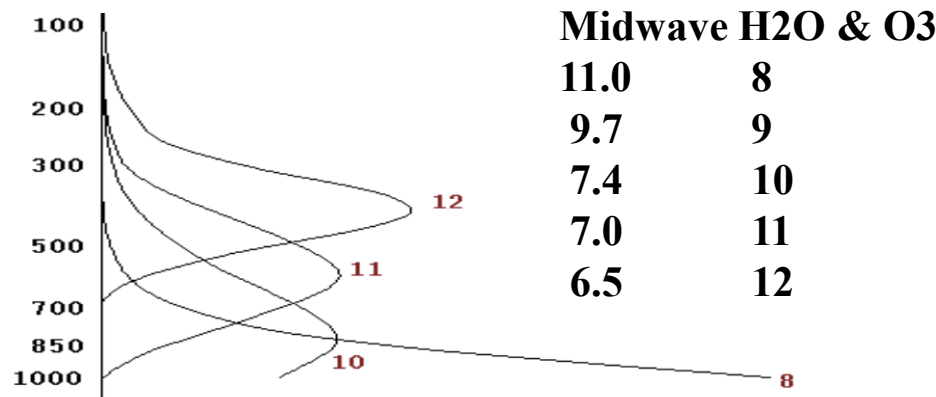
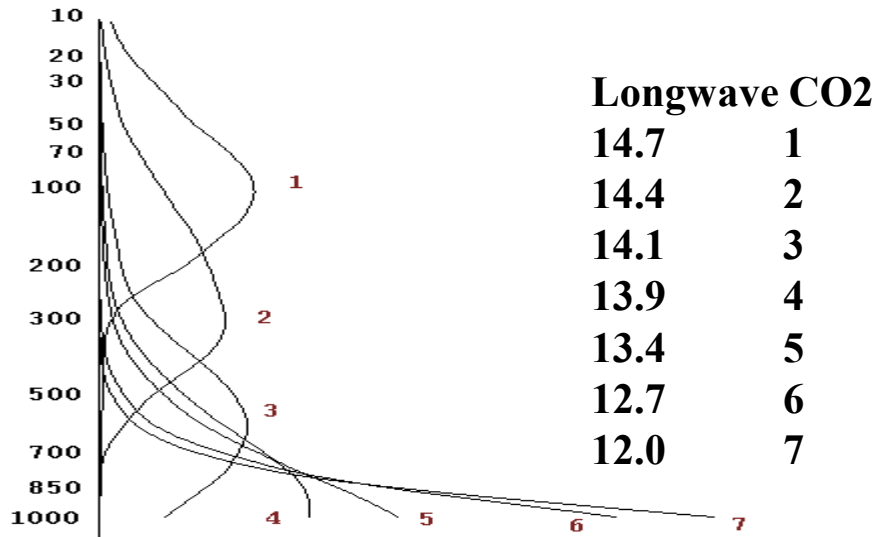
Re-emission of Infrared Radiation



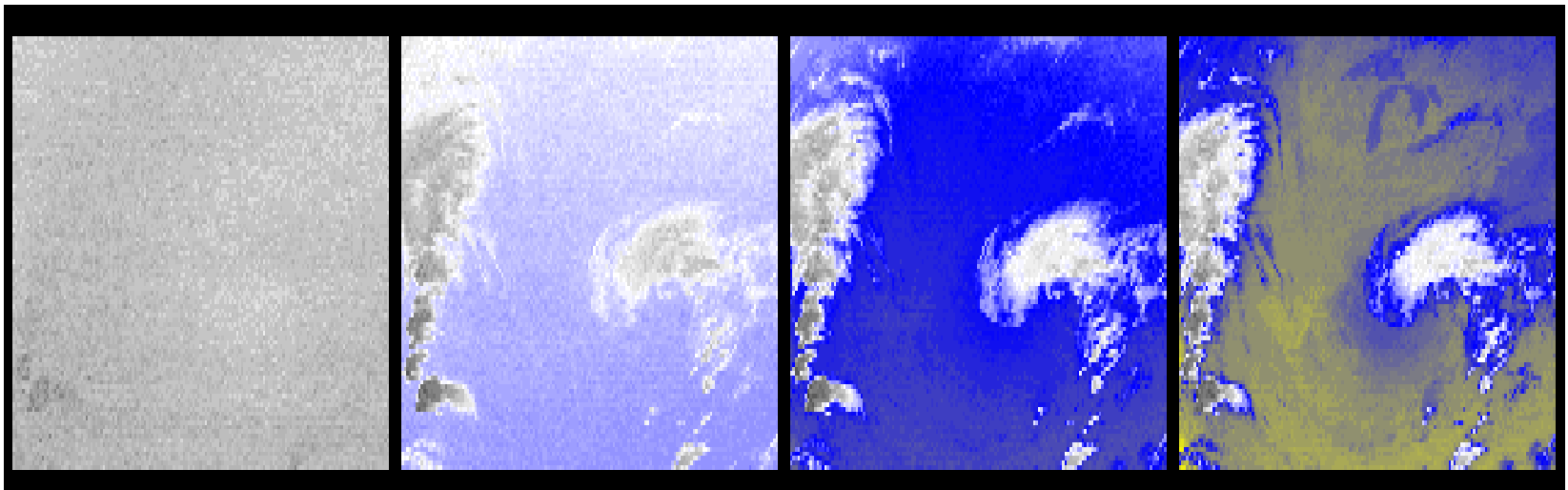
Radiative Transfer through the Atmosphere



Weighting Functions



CO₂ channels see to different levels in the atmosphere



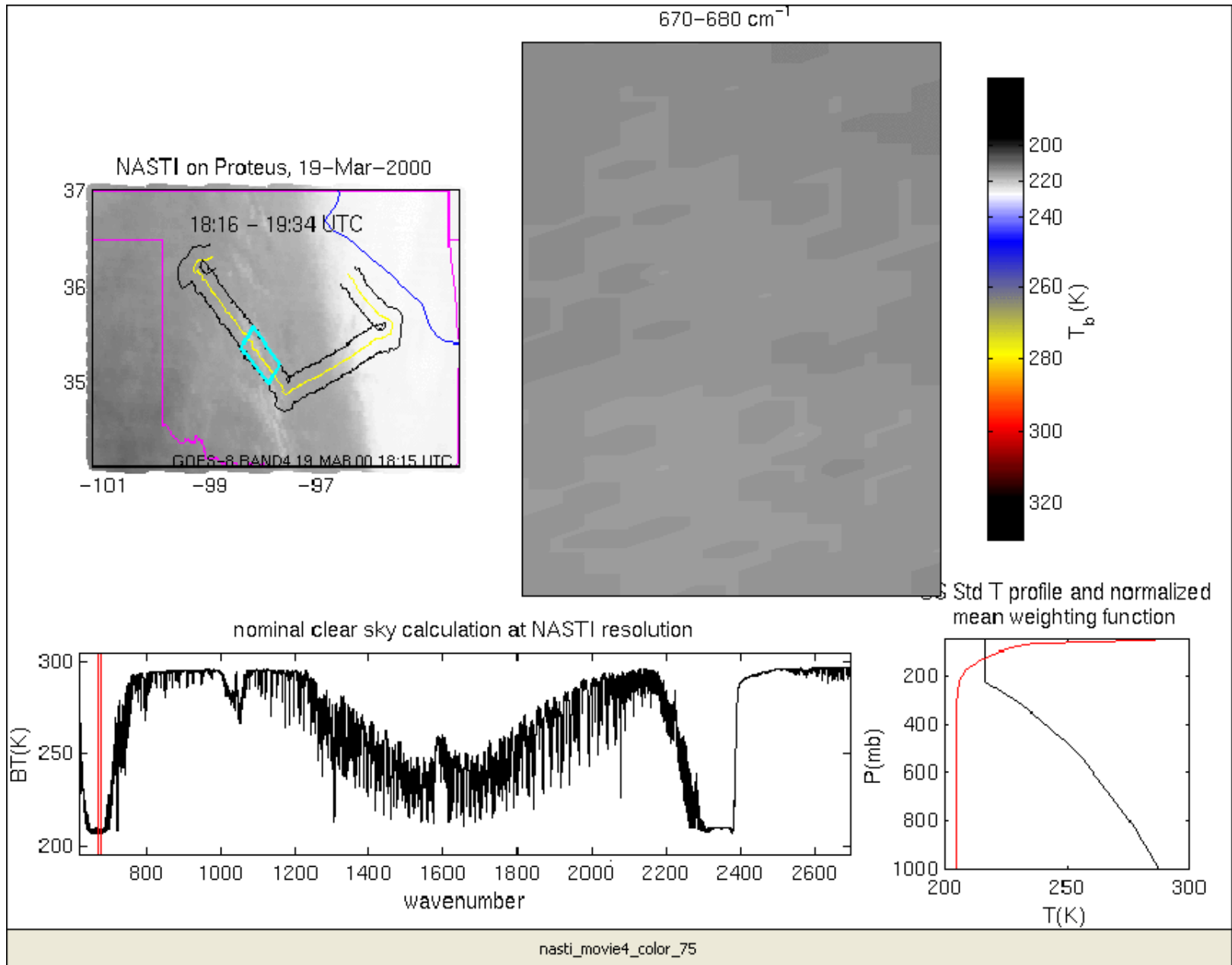
14.2 um

13.9 um

13.6 um

13.3 um

Improvements with Hyperspectral IR Data



Characteristics of RTE

- * Radiance arises from deep and overlapping layers
- * The radiance observations are not independent
- * There is no unique relation between the spectrum of the outgoing radiance and $T(p)$ or $Q(p)$
- * $T(p)$ is buried in an exponent in the denominator in the integral
- * $Q(p)$ is implicit in the transmittance
- * Boundary conditions are necessary for a solution; the better the first guess the better the final solution

To investigate the RTE further consider the atmospheric contribution to the radiance to space of an infinitesimal layer of the atmosphere at height z , $dI_\lambda(z) = B_\lambda(T(z)) d\tau_\lambda(z)$.

Assume a well-mixed isothermal atmosphere where the density drops off exponentially with height $\rho = \rho_0 \exp(-\gamma z)$, and assume k_λ is independent of height, so that the optical depth can be written for normal incidence

$$\sigma_\lambda = \int_z^\infty k_\lambda \rho dz = \gamma^{-1} k_\lambda \rho_0 \exp(-\gamma z)$$

and the derivative with respect to height

$$\frac{d\sigma_\lambda}{dz} = -k_\lambda \rho_0 \exp(-\gamma z) = -\gamma \sigma_\lambda.$$

Therefore, we may obtain an expression for the detected radiance per unit thickness of the layer as a function of optical depth,

$$\frac{dI_\lambda(z)}{dz} = B_\lambda(T_{\text{const}}) \frac{d\tau_\lambda(z)}{dz} = B_\lambda(T_{\text{const}}) \gamma \sigma_\lambda \exp(-\sigma_\lambda).$$

The level which is emitting the most detected radiance is given by

$$\frac{d}{dz} \left\{ \frac{dI_\lambda(z)}{dz} \right\} = 0, \text{ or where } \sigma_\lambda = 1.$$

Most of monochromatic radiance detected is emitted by layers near level of unit optical depth.

Profile Retrieval from Sounder Radiances

$$I_{\lambda} = \varepsilon_{\lambda}^{\text{sfc}} B_{\lambda}(T(p_s)) \tau_{\lambda}(p_s) - \int_0^{p_s} B_{\lambda}(T(p)) F_{\lambda}(p) [d\tau_{\lambda}(p) / dp] dp .$$

$I_1, I_2, I_3, \dots, I_n$ are measured with the sounder

$P(\text{sfc})$ and $T(\text{sfc})$ come from ground based conventional observations

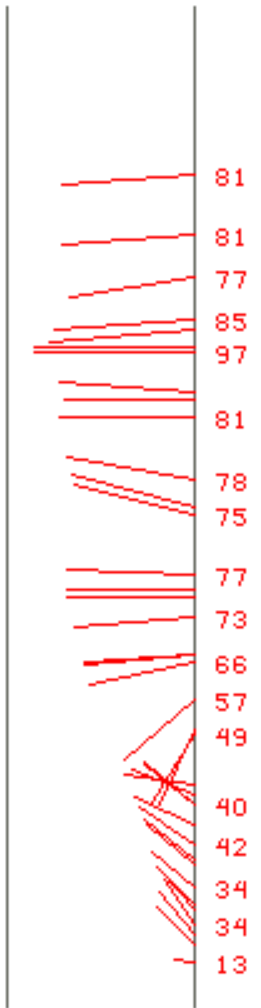
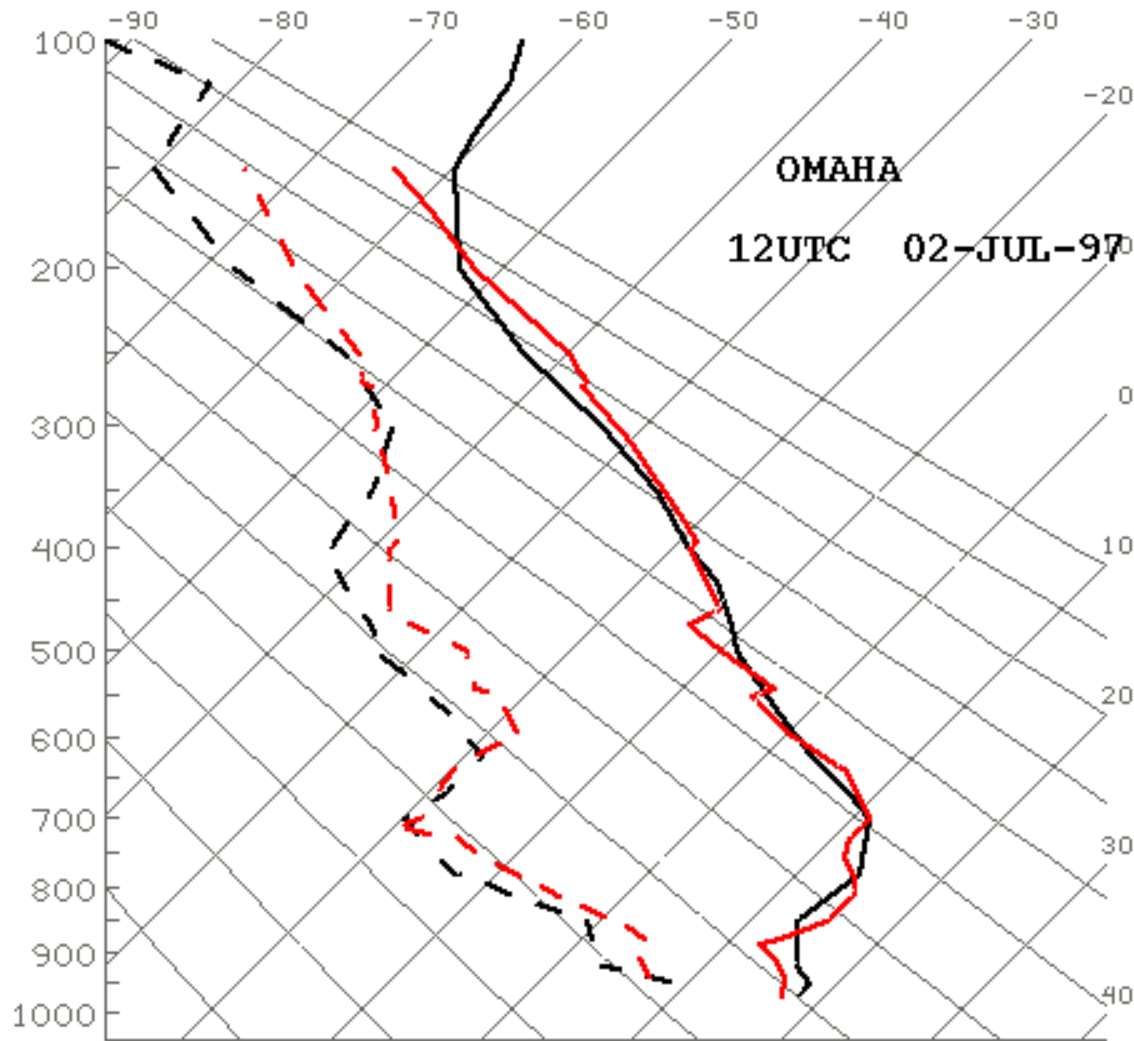
$\tau_{\lambda}(p)$ are calculated with physics models (using for CO₂ and O₃)

$\varepsilon_{\lambda}^{\text{sfc}}$ is estimated from a priori information (or regression guess)

First guess solution is inferred from (1) in situ radiosonde reports,
(2) model prediction, or (3) blending of (1) and (2)

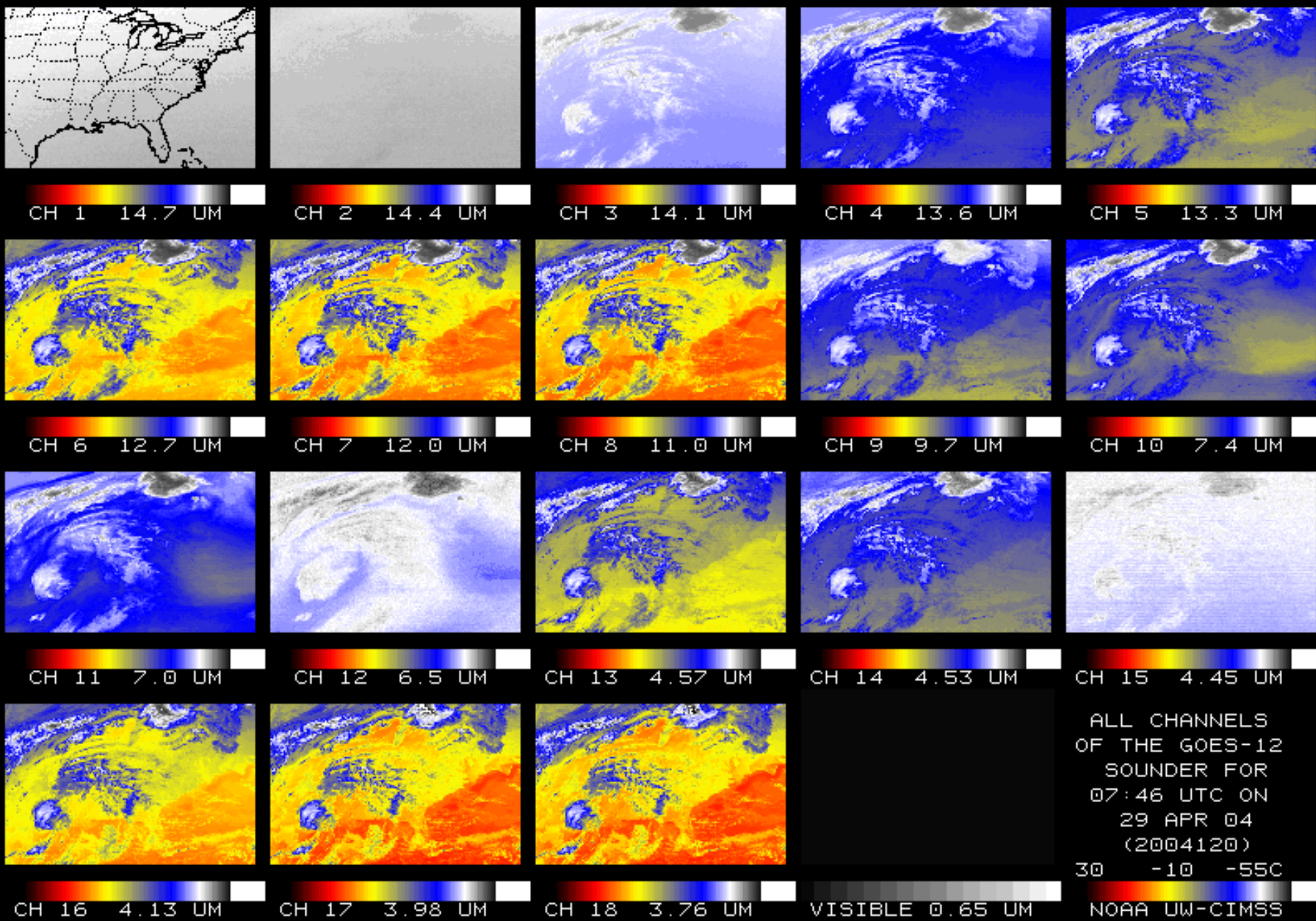
Profile retrieval from perturbing guess to match measured sounder radiances

Example GOES Sounding

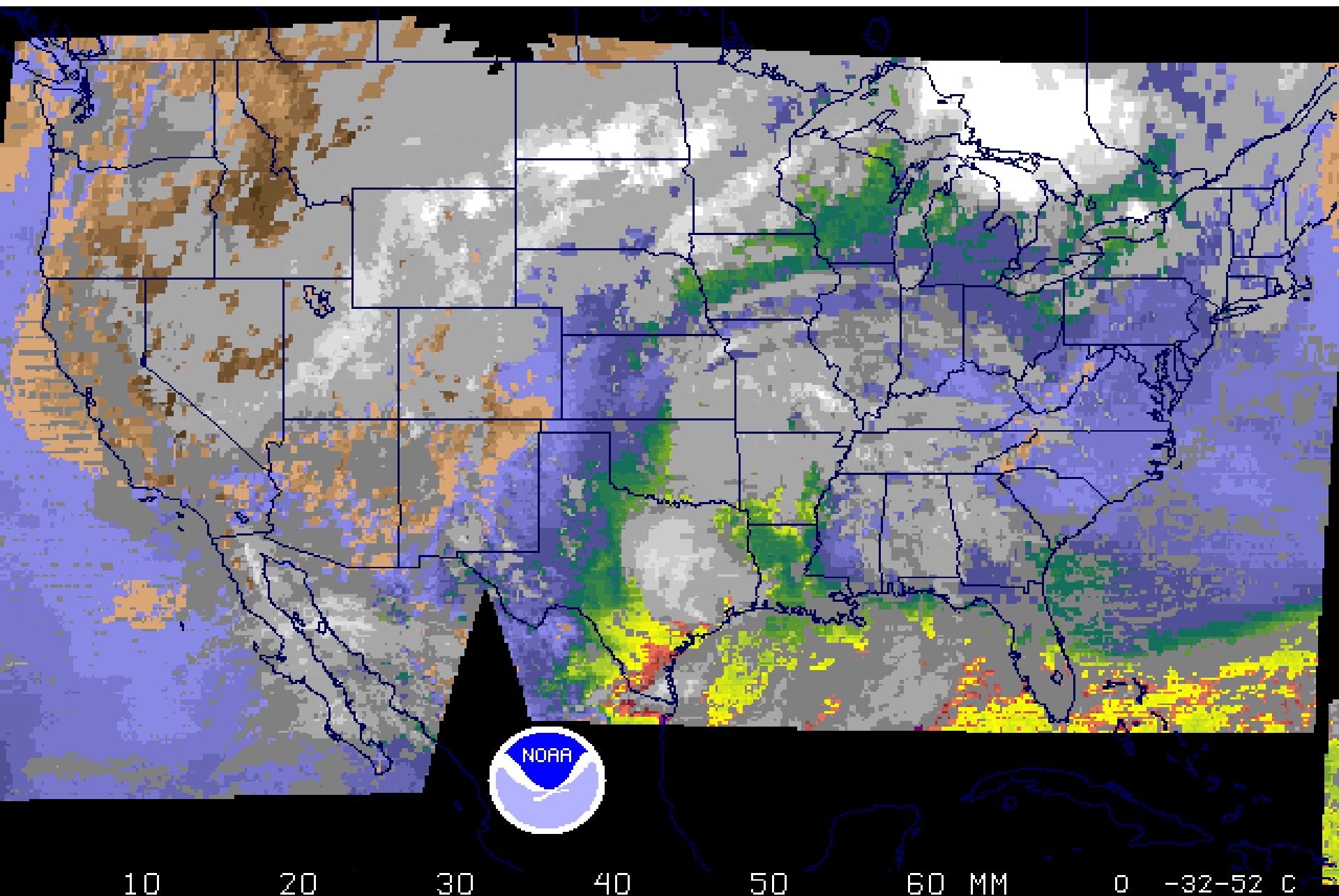


GMT	ID	TOTAL	EQUIP	FMAX	CVT	L. I.	KINX	PW	GOES-8 RTVL
021153	267	30				11	-10	12	
021200	72558	36				10	-4	14	RAOB

GOES-12 Sounder – Brightness Temperature (Radiances) – 12 bands



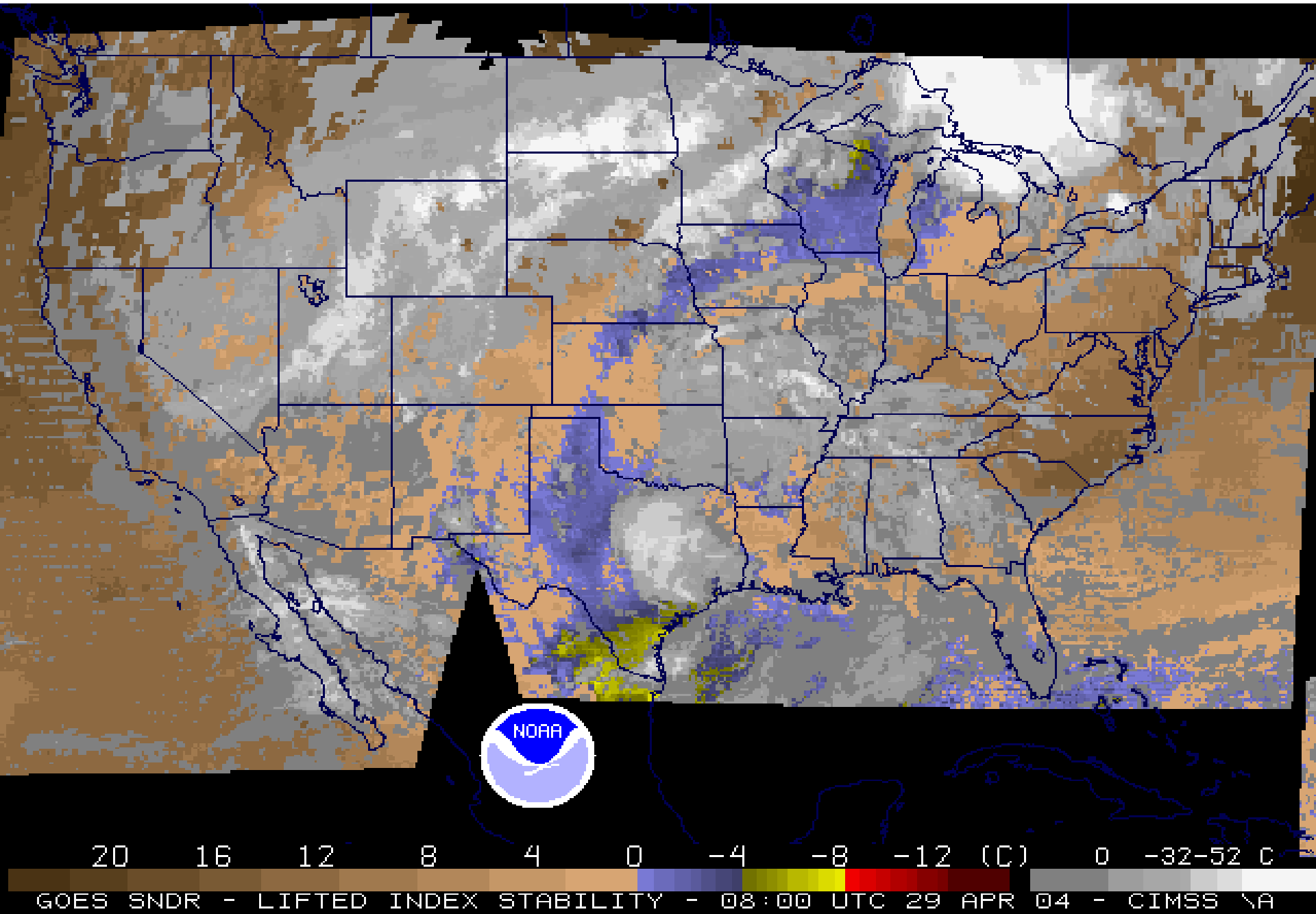
GOES Sounders – Total Precipitable Water

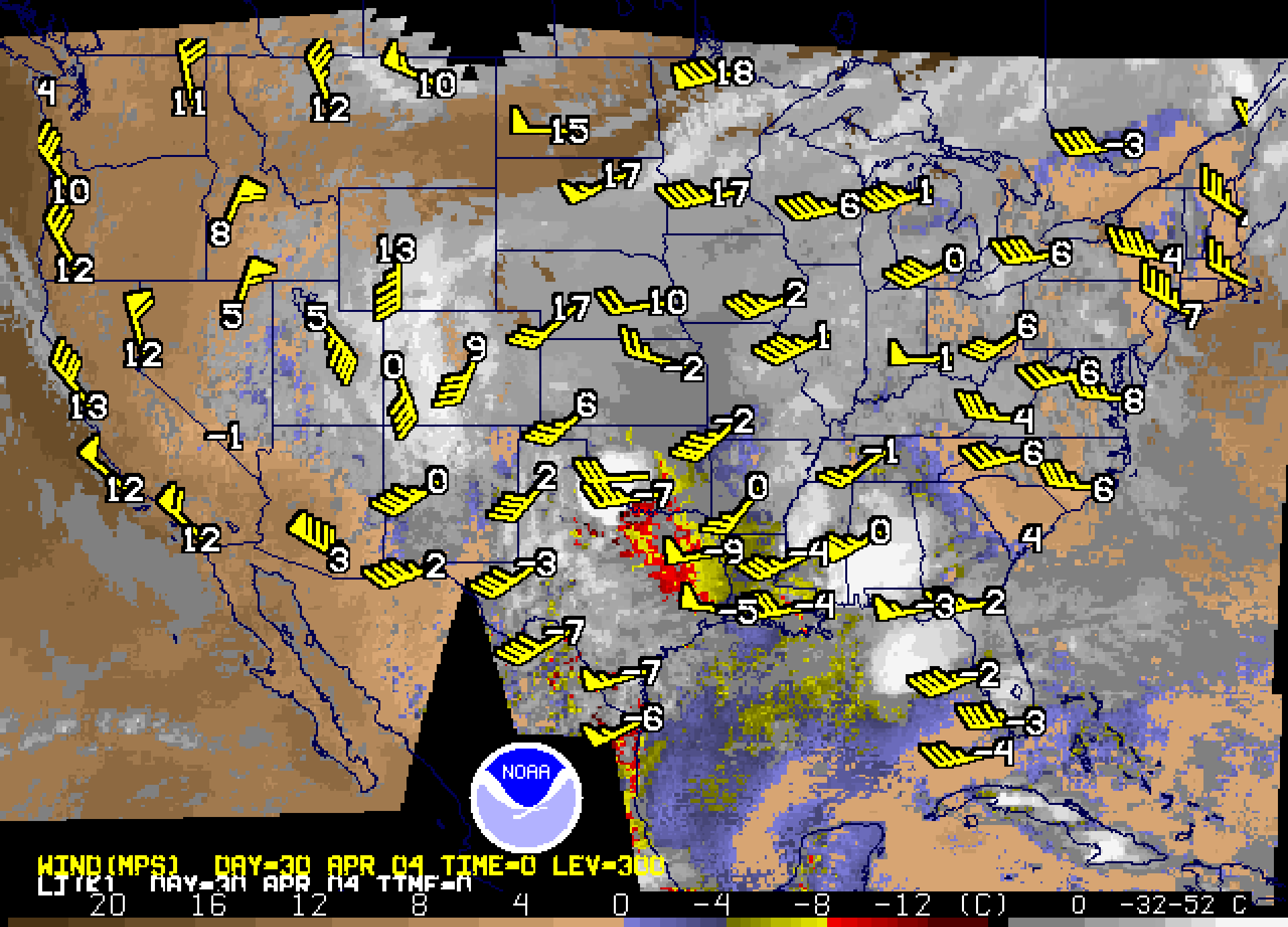


10 20 30 40 50 60 MM 0 -32-52 C

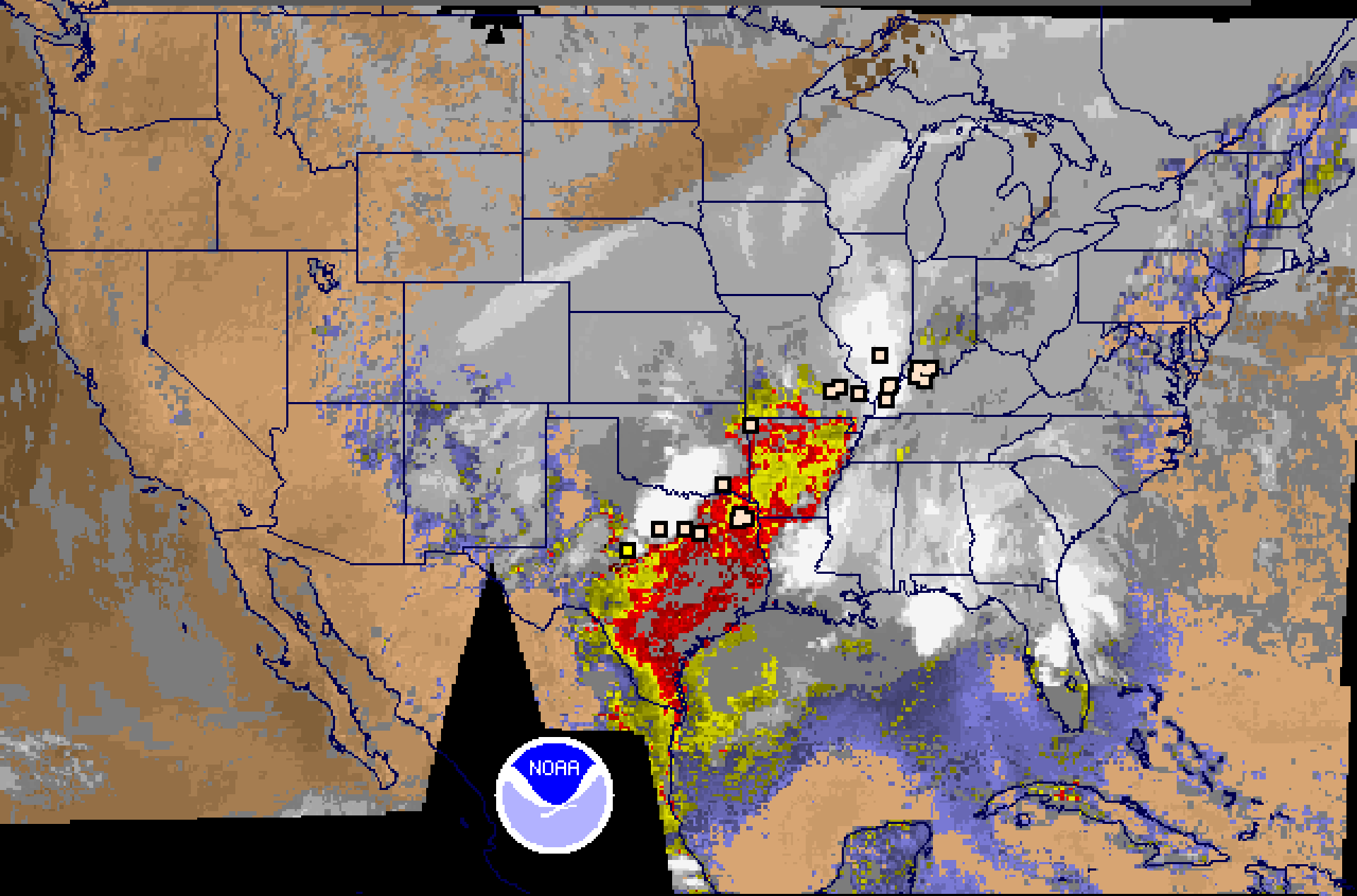
GOES SDR - TOTAL PRECIP WATER VAPOR - 08:00 UTC 29 APR 04 - CIMSS NA

GOES Sounders – Lifted Index Stability





DAILY ■ **TORNADO** ■ HAIL AND ■ WIND DAMAGE REPORTS

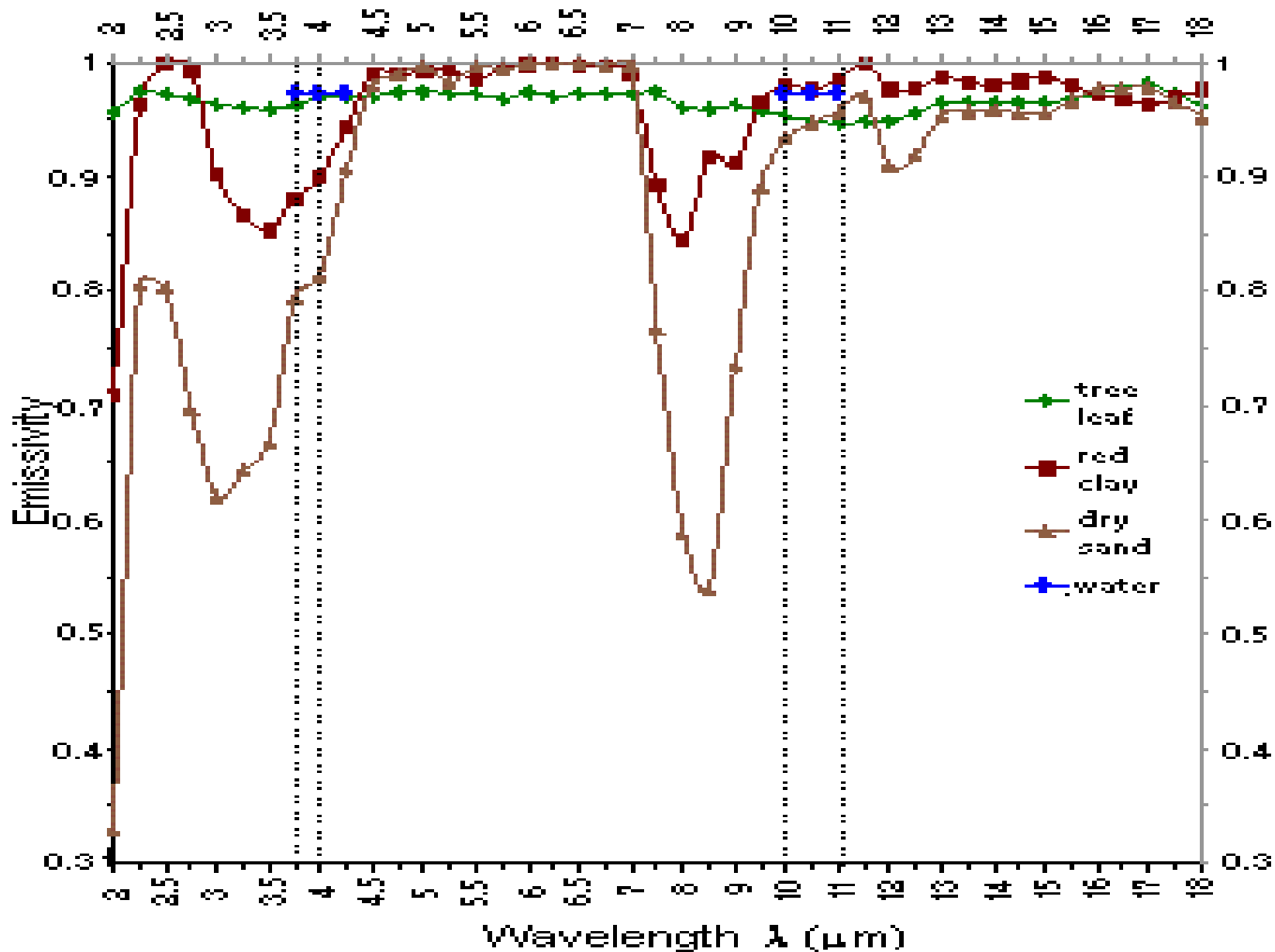


20 16 12 8 4 0 -4 -8 -12 (C) 0 -32-52 C

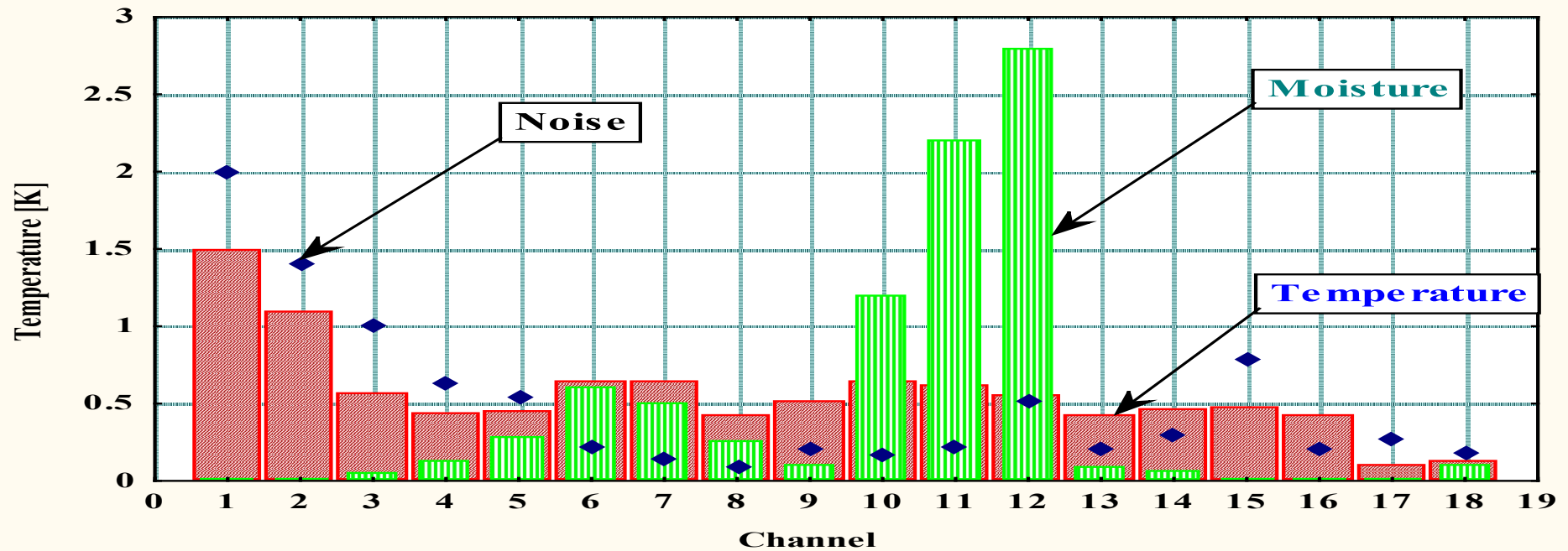
GOES SOUNDER - LIFTED INDEX STABILITY - 22:00 UTC 30 APR 04 - CIMSS

Extra slides on Surface Emissivity

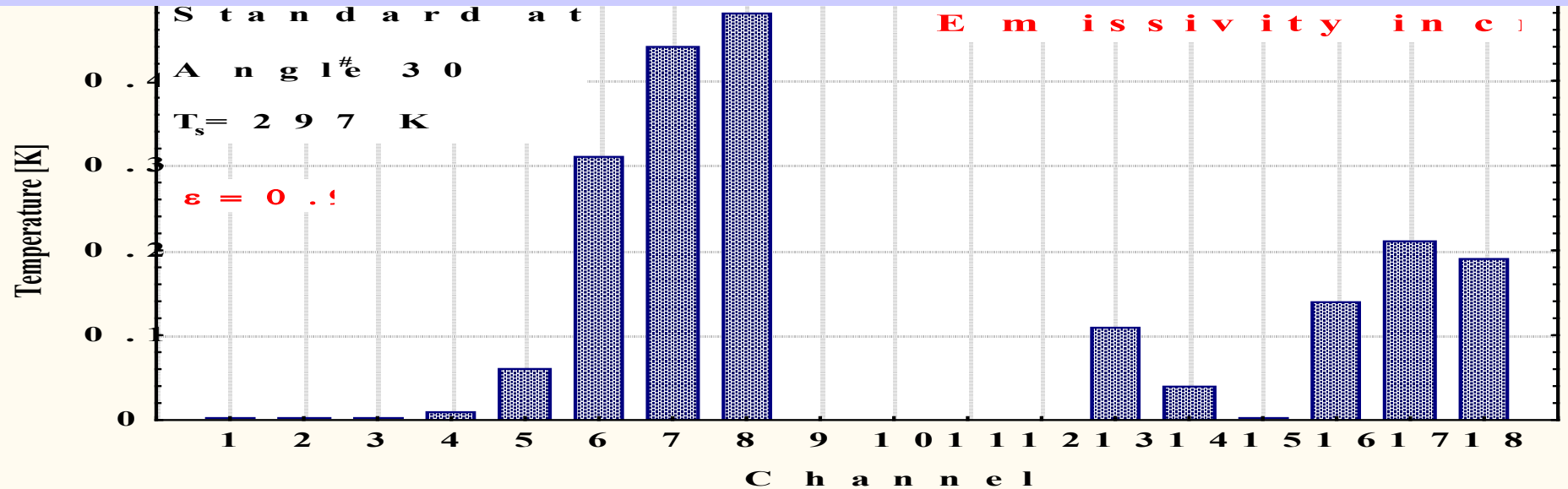
Infrared Emissivity vs. Wavelength



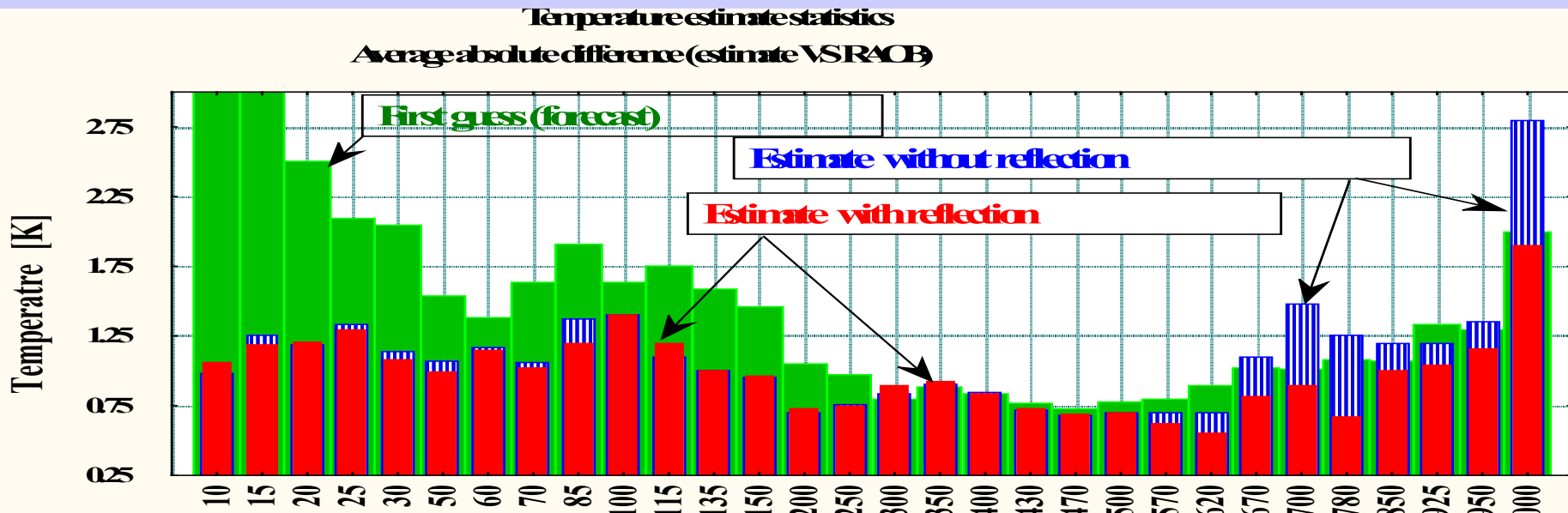
Spectral distribution of sensor noise vs NWP profile uncertainty contributions



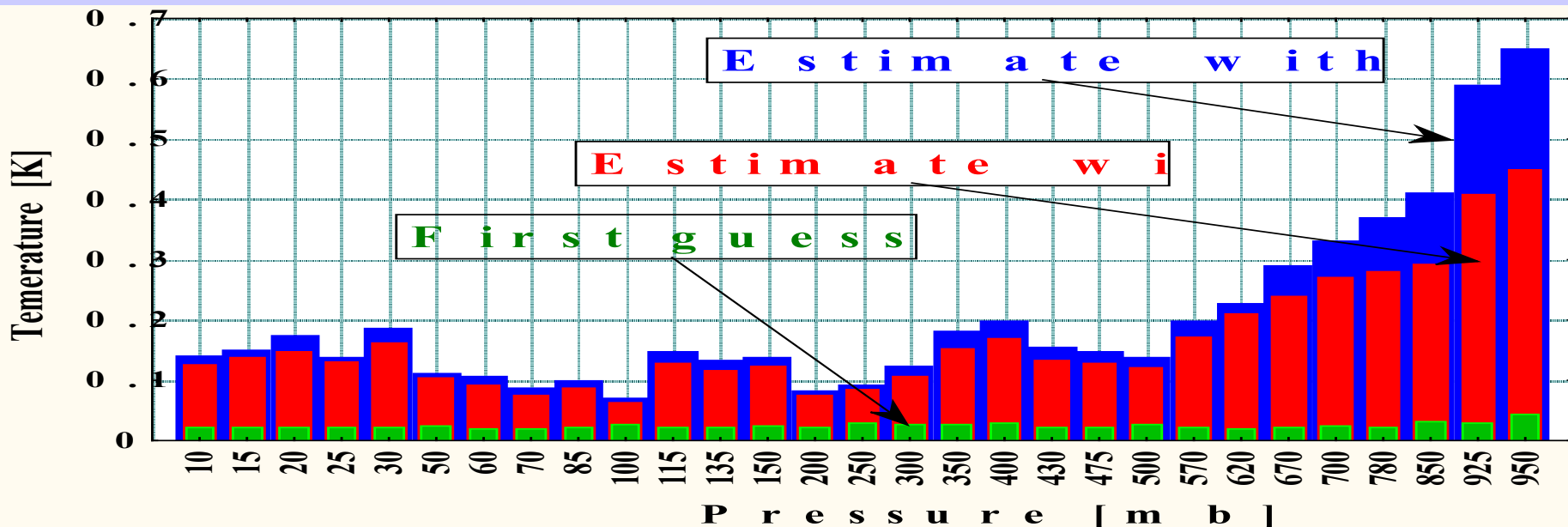
Spectral distribution of reflective changes for emissivity increments of 0.01



Average absolute temp diff (solution with and wo sfc reflection vs raobs)



Spatial smoothness of temperature solution with and wo sfc reflection standard deviation of second spatial derivative (multiplied by 100 * km * km)



Microwave RTE and spectral bands

WAVELENGTH			FREQUENCY		WAVENUMBER
cm	μm	\AA	Hz	GHz	cm^{-1}
10^{-5} Near Ultraviolet (UV)	0.1	1,000	3×10^{15}		
4×10^{-5} Visible	0.4	4,000	7.5×10^{14}		
7.5×10^{-5} Near Infrared (IR)	0.75	7,500	4×10^{14}		13,333
2×10^{-3} Far Infrared (IR)	20	2×10^5	1.5×10^{13}		500
0.1 Microwave (MW)	10^3		3×10^{11}	300	10

Radiation is governed by Planck's Law

$$B(\lambda, T) = \frac{c_1}{\lambda^5} \left[e^{-\frac{c_2}{\lambda T}} - 1 \right]^{-1}$$

In microwave region $c_2/\lambda T \ll 1$ so that

$$e^{-\frac{c_2}{\lambda T}} \approx 1 - \frac{c_2}{\lambda T} + \text{second order}$$

And classical Rayleigh Jeans radiation equation emerges

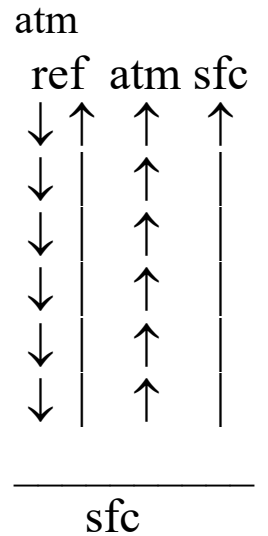
$$B_\lambda(T) \approx \left[\frac{c_1}{c_2} \right] \left[\frac{T}{\lambda^4} \right]$$

Radiance is linear function of brightness temperature.

Microwave Form of RTE

$$I_{\lambda}^{\text{sfc}} = \varepsilon_{\lambda} B_{\lambda}(T_s) \tau_{\lambda}(p_s) + (1-\varepsilon_{\lambda}) \tau_{\lambda}(p_s) \int_0^{p_s} B_{\lambda}(T(p)) \frac{\partial \tau'_{\lambda}(p)}{\partial \ln p} d \ln p$$

$$I_{\lambda} = \varepsilon_{\lambda} B_{\lambda}(T_s) \tau_{\lambda}(p_s) + (1-\varepsilon_{\lambda}) \tau_{\lambda}(p_s) \int_0^{p_s} B_{\lambda}(T(p)) \frac{\partial \tau'_{\lambda}(p)}{\partial \ln p} d \ln p + \int_{p_s}^0 B_{\lambda}(T(p)) \frac{\partial \tau_{\lambda}(p)}{\partial \ln p} d \ln p$$



In the microwave region $c_2/\lambda T \ll 1$, so the Planck radiance is linearly proportional to the temperature

$$B_{\lambda}(T) \approx [c_1 / c_2] [T / \lambda^4]$$

So

$$T_{b\lambda} = \varepsilon_{\lambda} T_s(p_s) \tau_{\lambda}(p_s) + \int_{p_s}^0 T(p) F_{\lambda}(p) \frac{\partial \tau_{\lambda}(p)}{\partial \ln p} d \ln p$$

where

$$F_{\lambda}(p) = \left\{ 1 + (1 - \varepsilon_{\lambda}) \left[\frac{\tau_{\lambda}(p_s)}{\tau_{\lambda}(p)} \right]^2 \right\} .$$

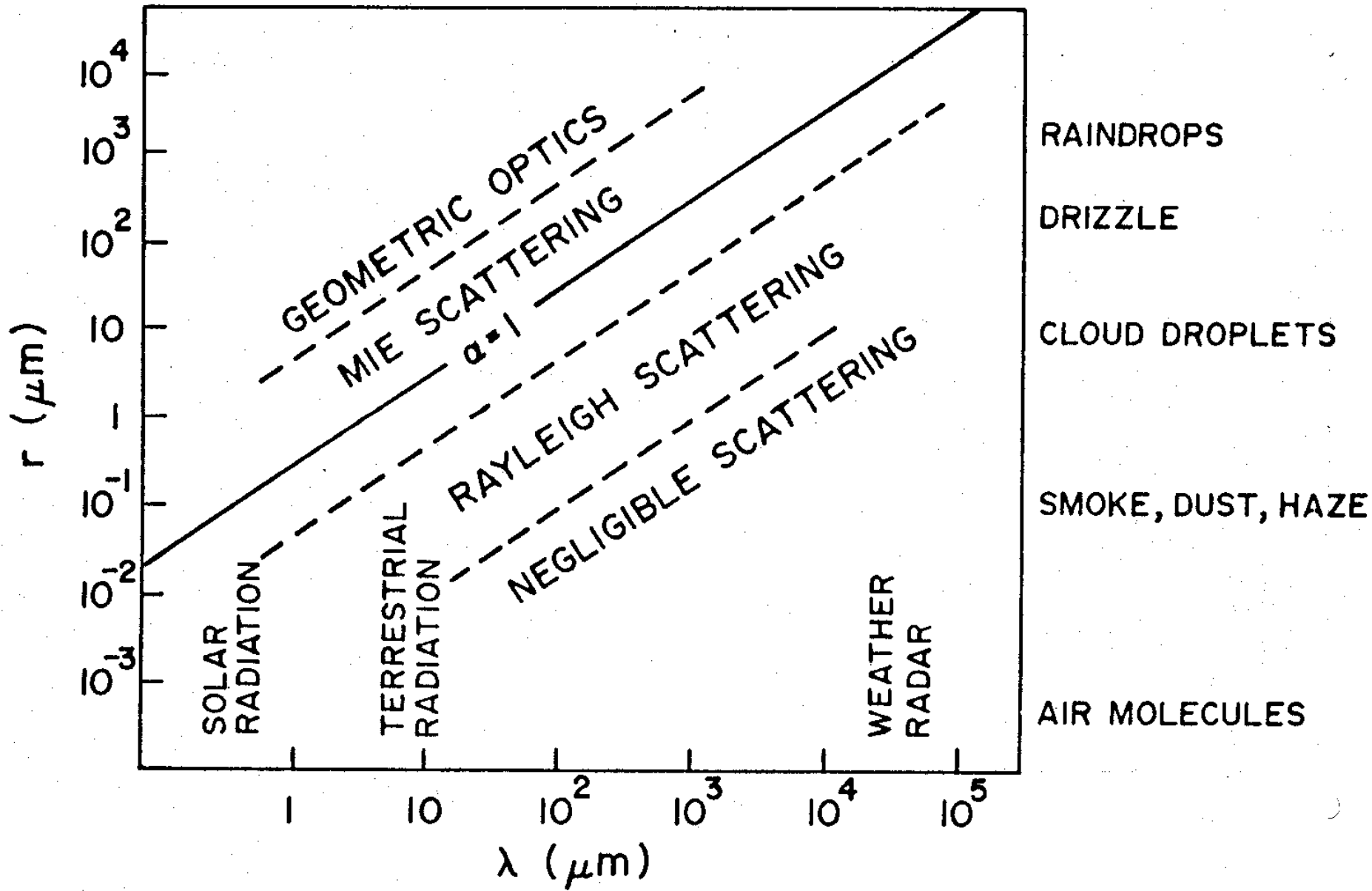
The transmittance to the surface can be expressed in terms of transmittance to the top of the atmosphere by remembering

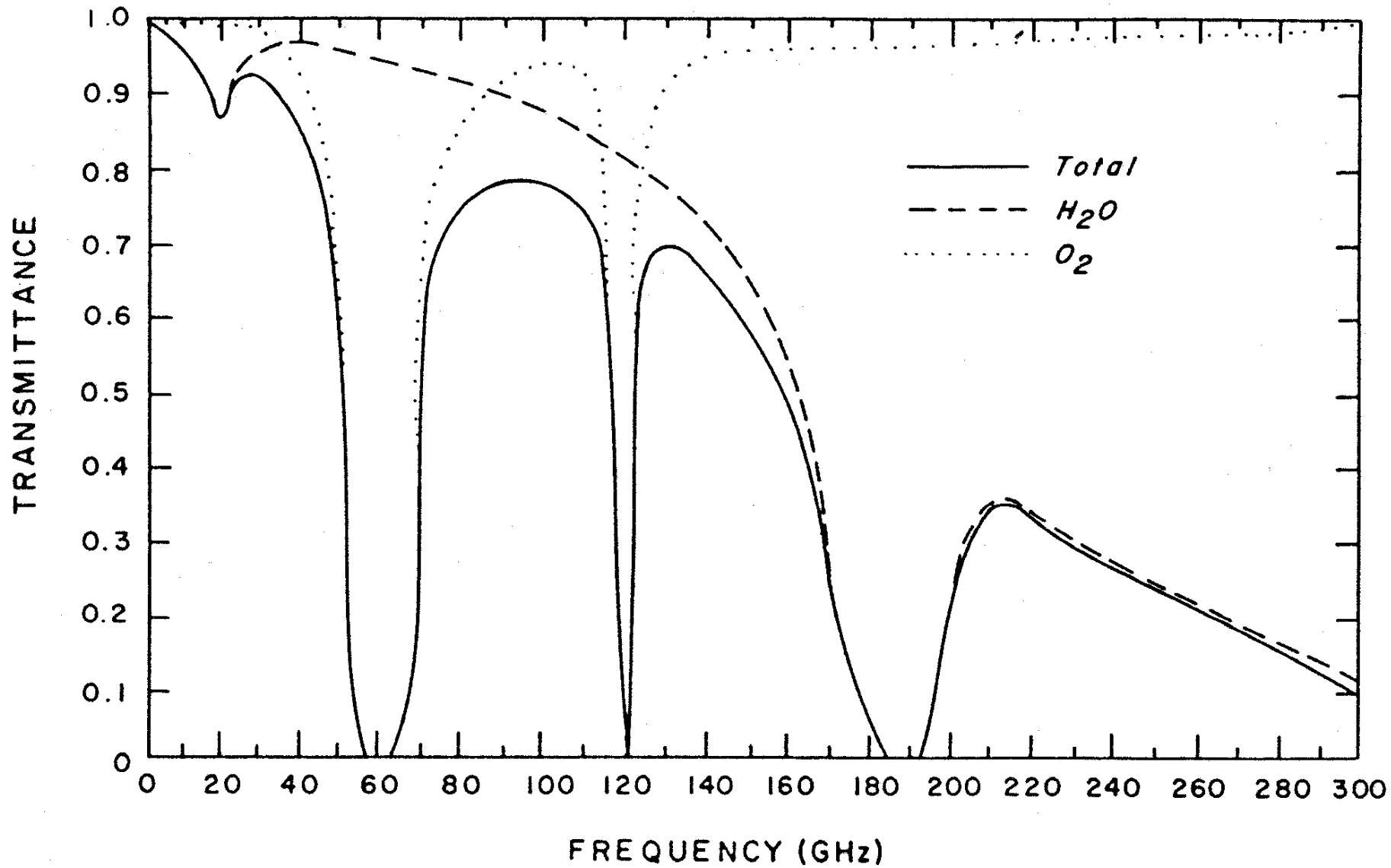
$$\begin{aligned} \tau'_\lambda(p) &= \exp \left[- \frac{1}{g} \int_0^{p_s} k_\lambda(p) g(p) dp \right] \\ &= \exp \left[- \int_0^{p_s} + \int_0^p \right] \\ &= \tau_\lambda(p_s) / \tau_\lambda(p) . \end{aligned}$$

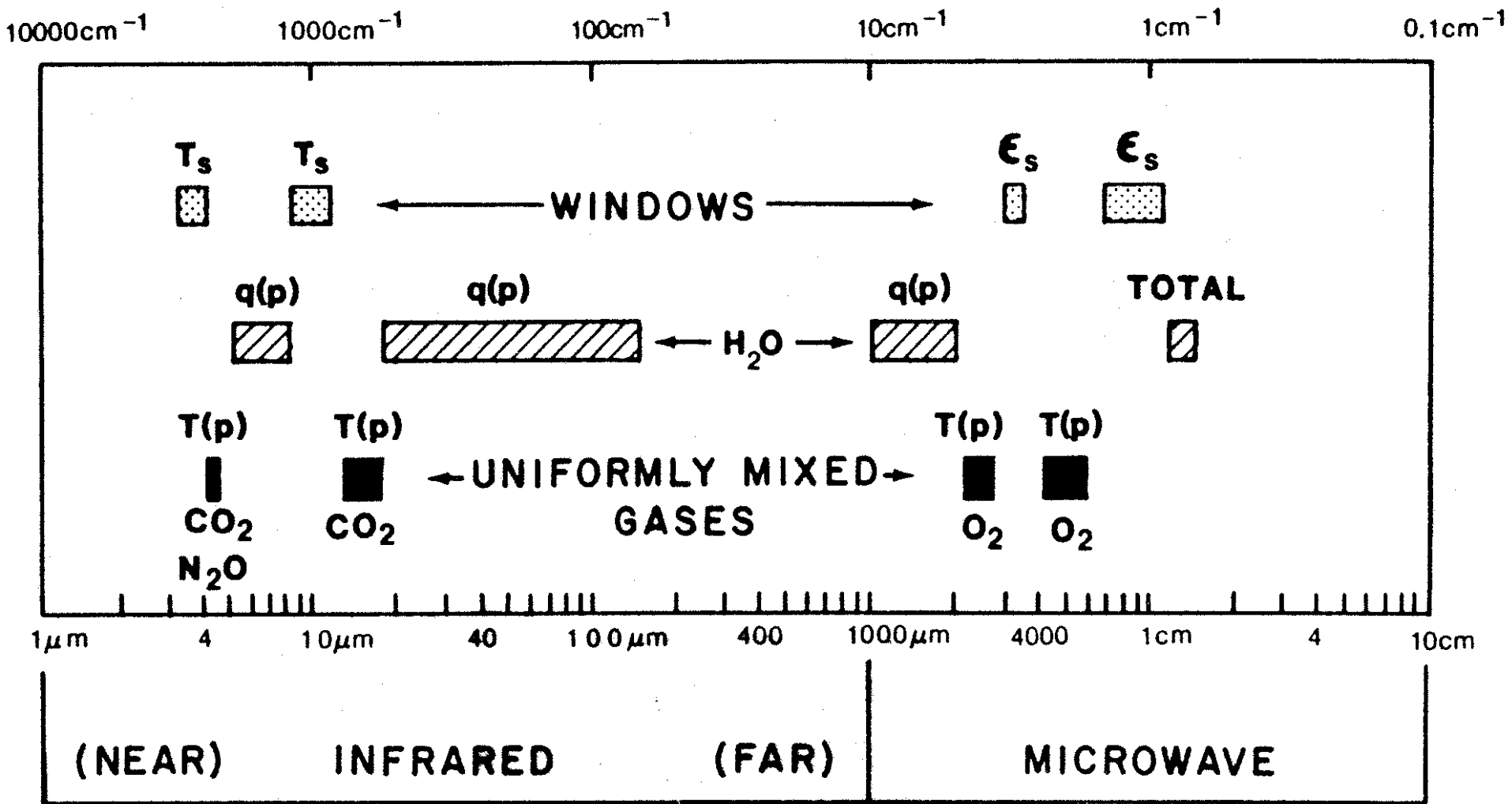
So

$$\frac{\partial \tau'_\lambda(p)}{\partial \ln p} = - \frac{\tau_\lambda(p_s)}{(\tau_\lambda(p))^2} \frac{\partial \tau_\lambda(p)}{\partial \ln p} .$$

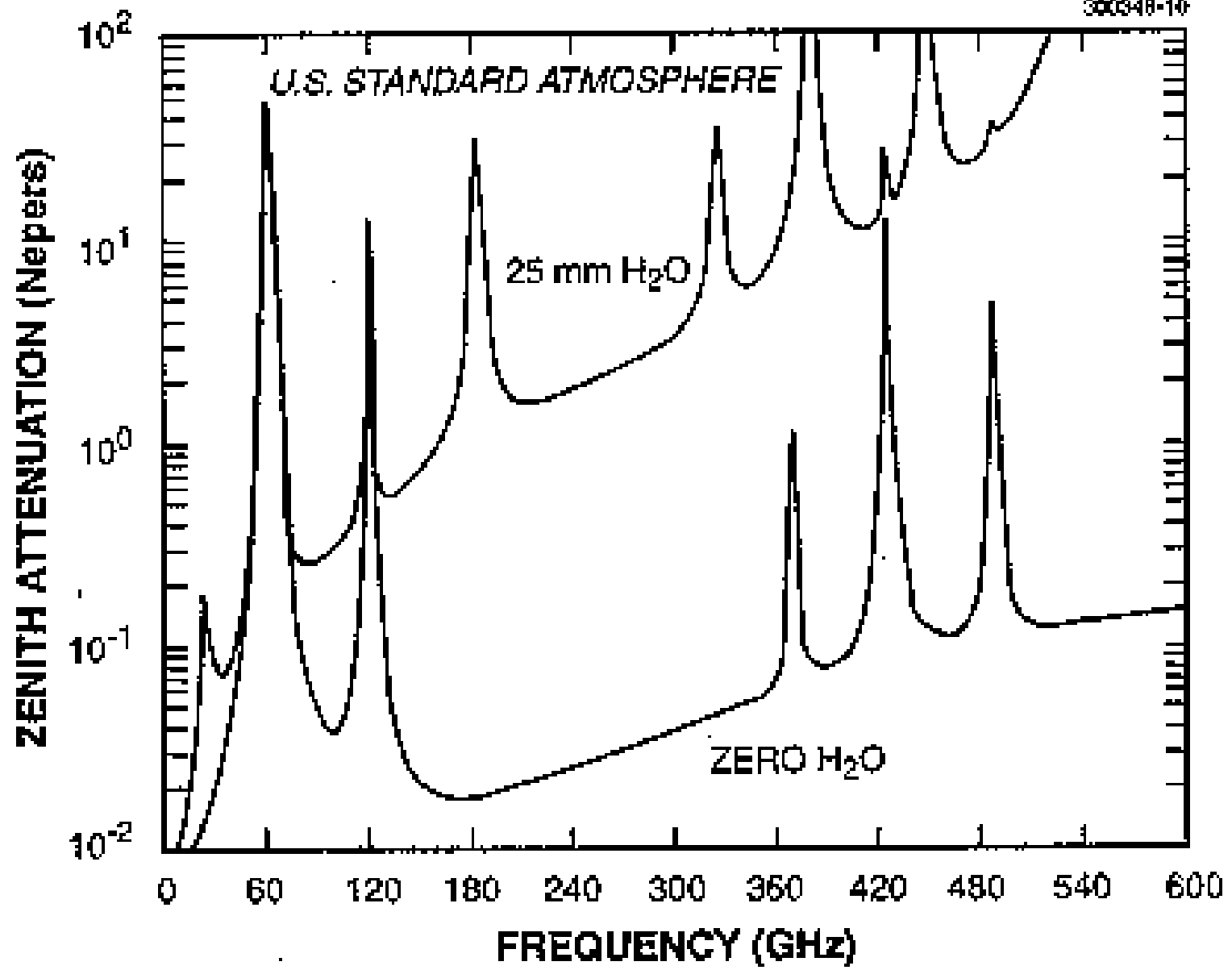
[remember that $\tau_\lambda(p_s, p) \tau_\lambda(p, 0) = \tau_\lambda(p_s, 0)$ and $\tau_\lambda(p_s, p) = \tau_\lambda(p, p_s)$]

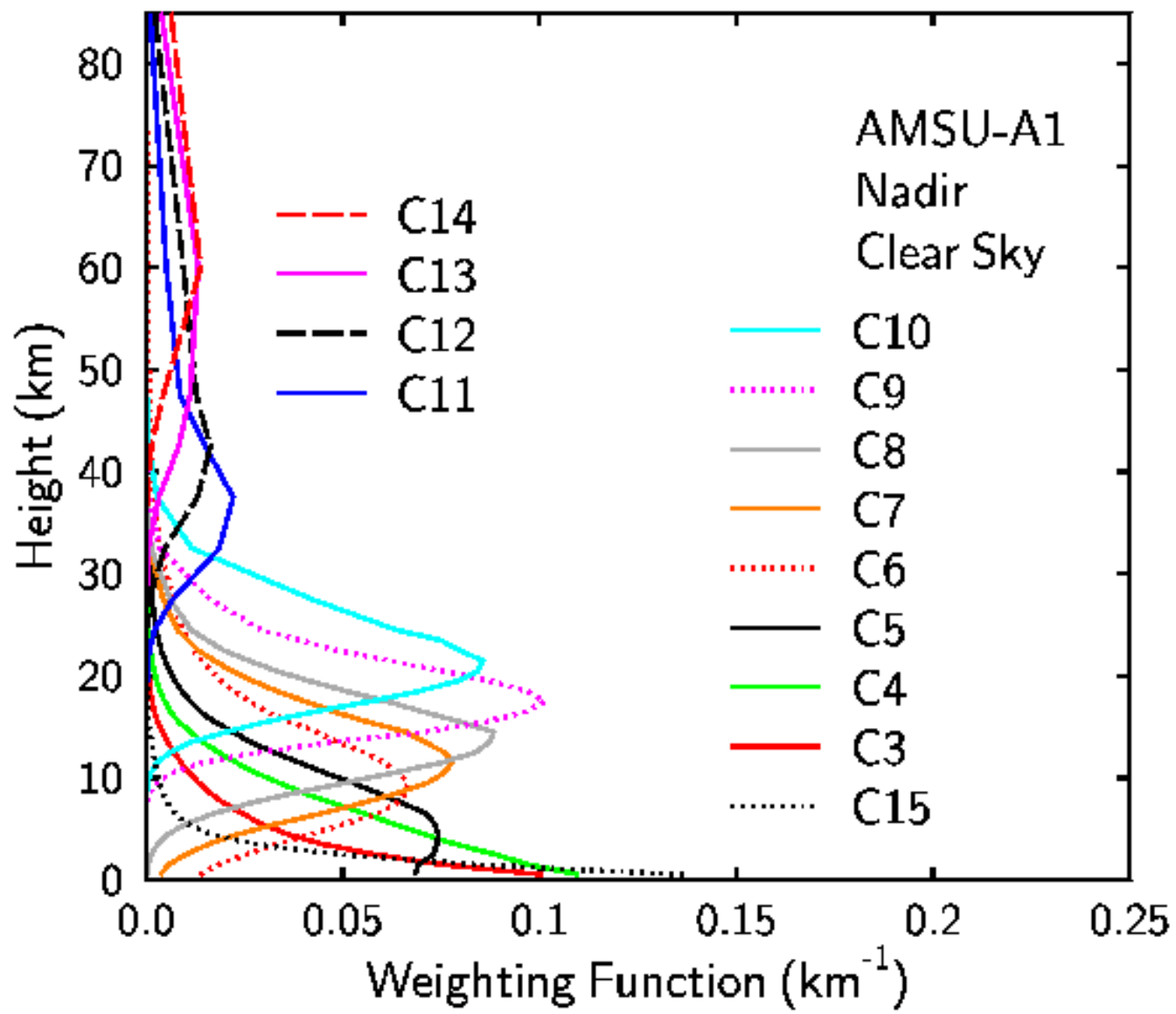


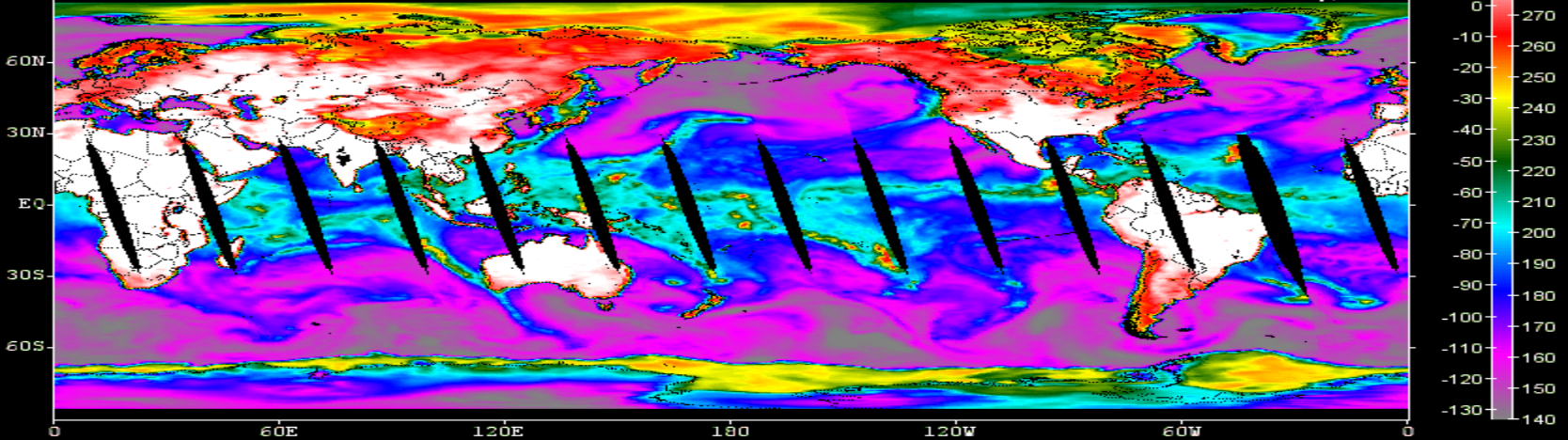




Spectral regions used for remote sensing of the earth atmosphere and surface from satellites. ϵ indicates emissivity, q denotes water vapour, and T represents temperature.

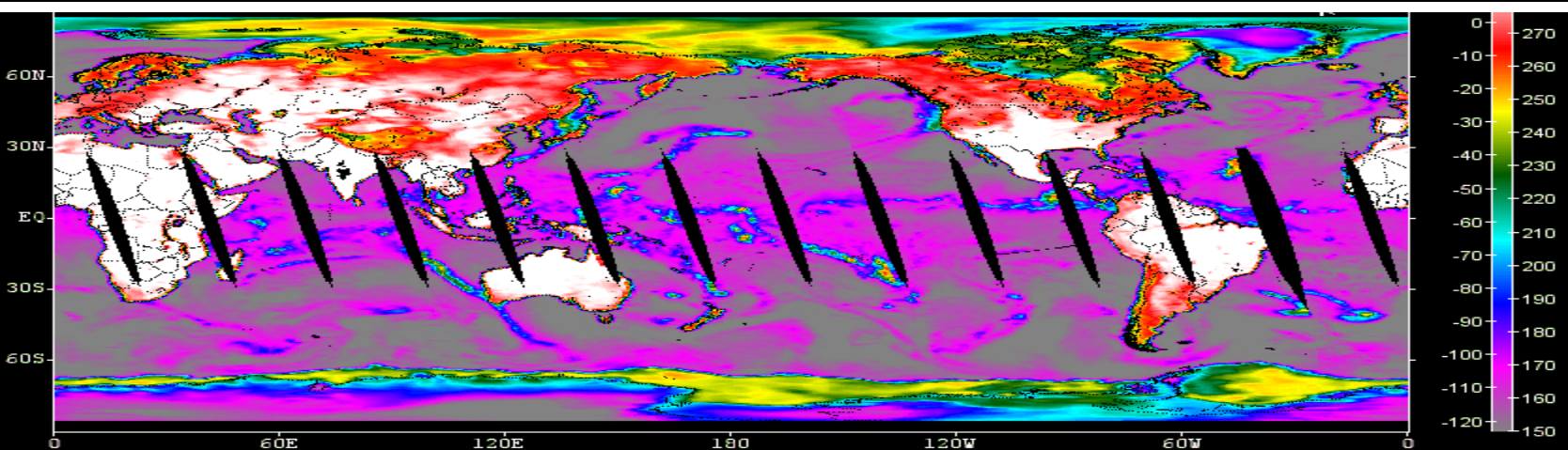




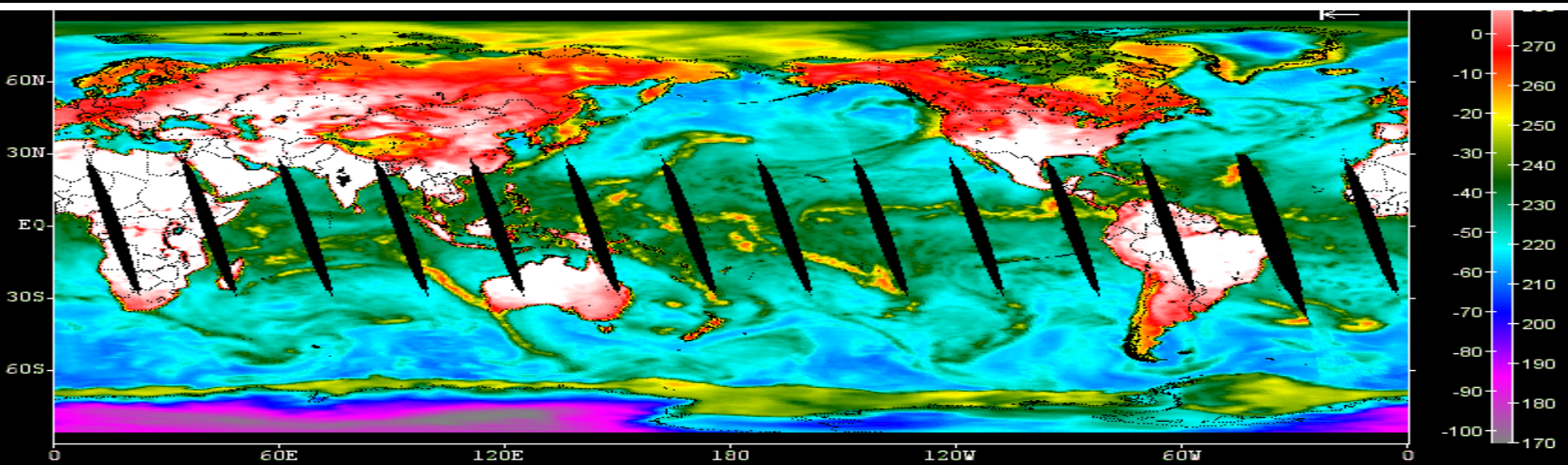


AMSU

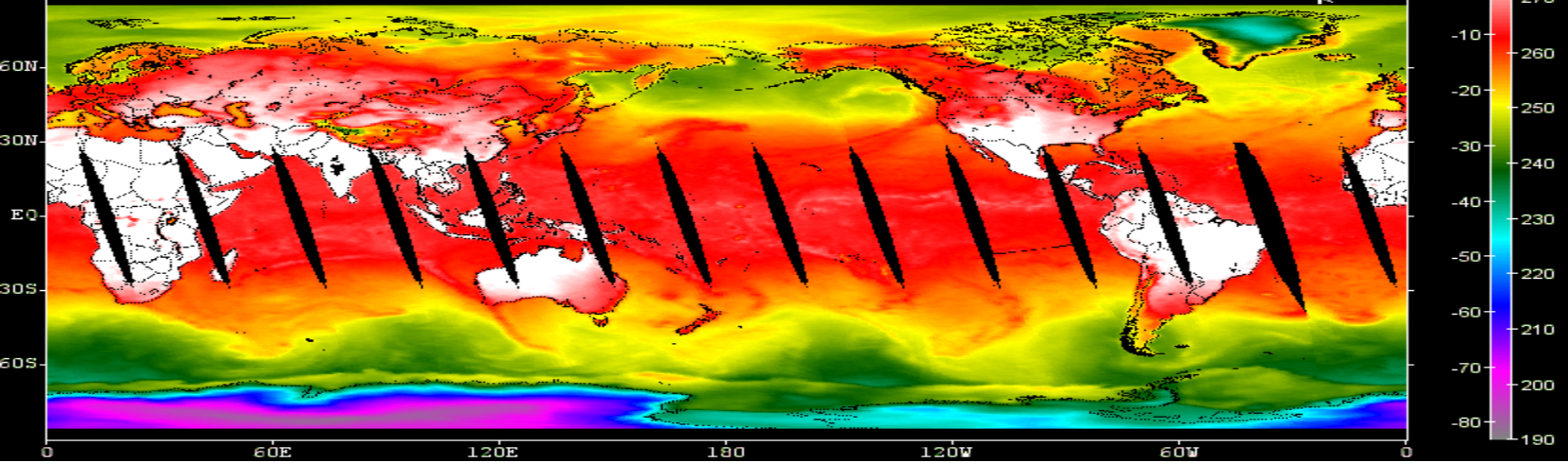
23.8



31.4

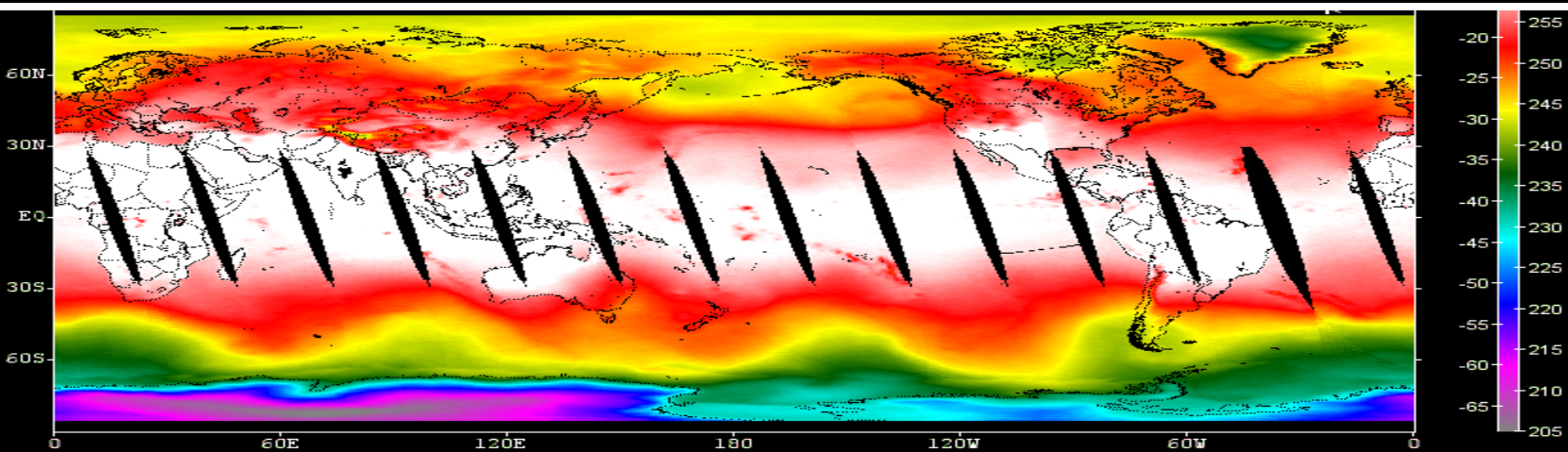


**50.3
GHz**

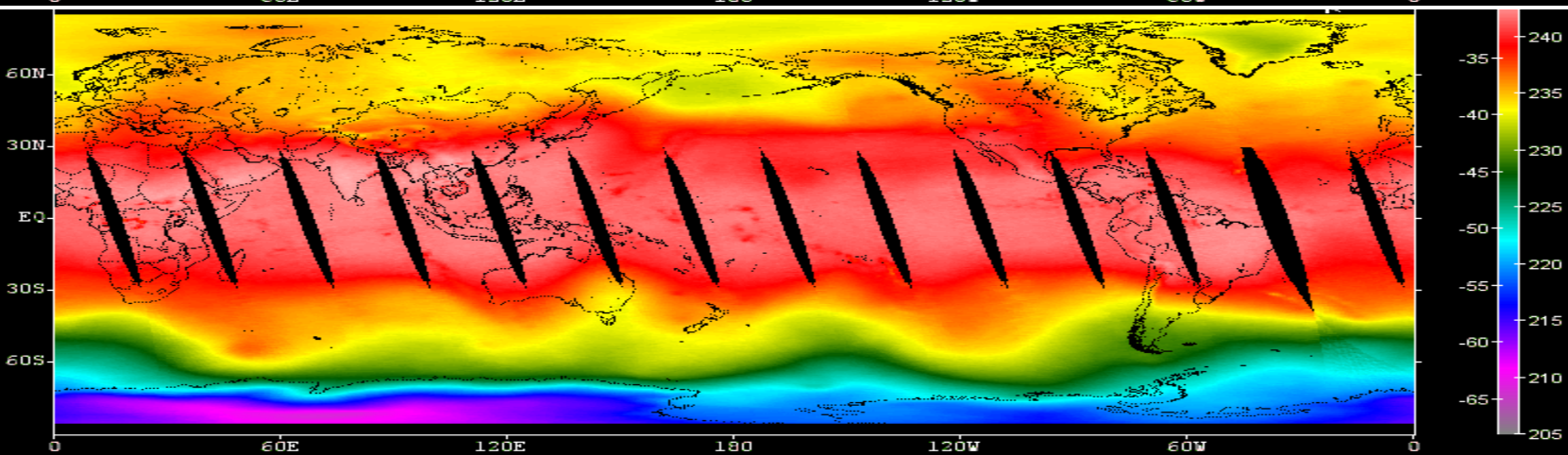


AMSU

52.8



53.6



**54.4
GHz**