Quick Review of Remote Sensing Basic Theory

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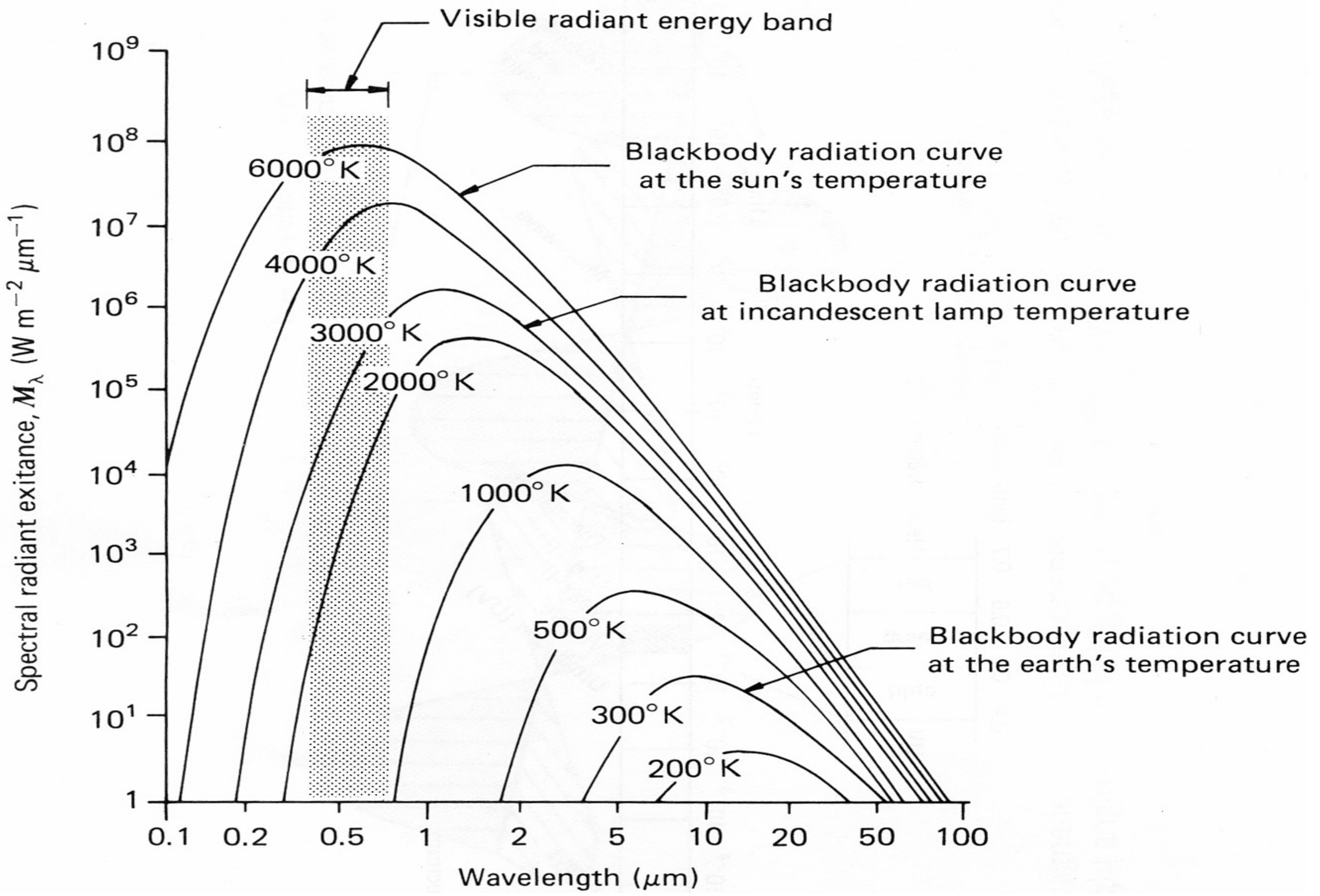
Benevento, June 2007

Outline

- Planck Function
- Infrared: Thermal Sensitivity
- Bit Depth

Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures



Radiation is governed by Planck's Law

In wavelenght:

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c2/\lambda T} - 1] \} \ (mW/m^2/ster/cm)$$

where λ = wavelength (cm)

T = temperature of emitting surface (deg K)

c_1 = 1.191044 x 10-8 ($W/m^2/ster/cm^{-4}$)

c_2 = 1.438769 (cm deg K)

In wavenumber:

$$B(\nu,T) = c_1 \nu^3 / [e^{c2\nu/T} - 1] \quad (mW/m^2/ster/cm^{-1})$$

where ν = # wavelengths in one centimeter (cm-1)

T = temperature of emitting surface (deg K)

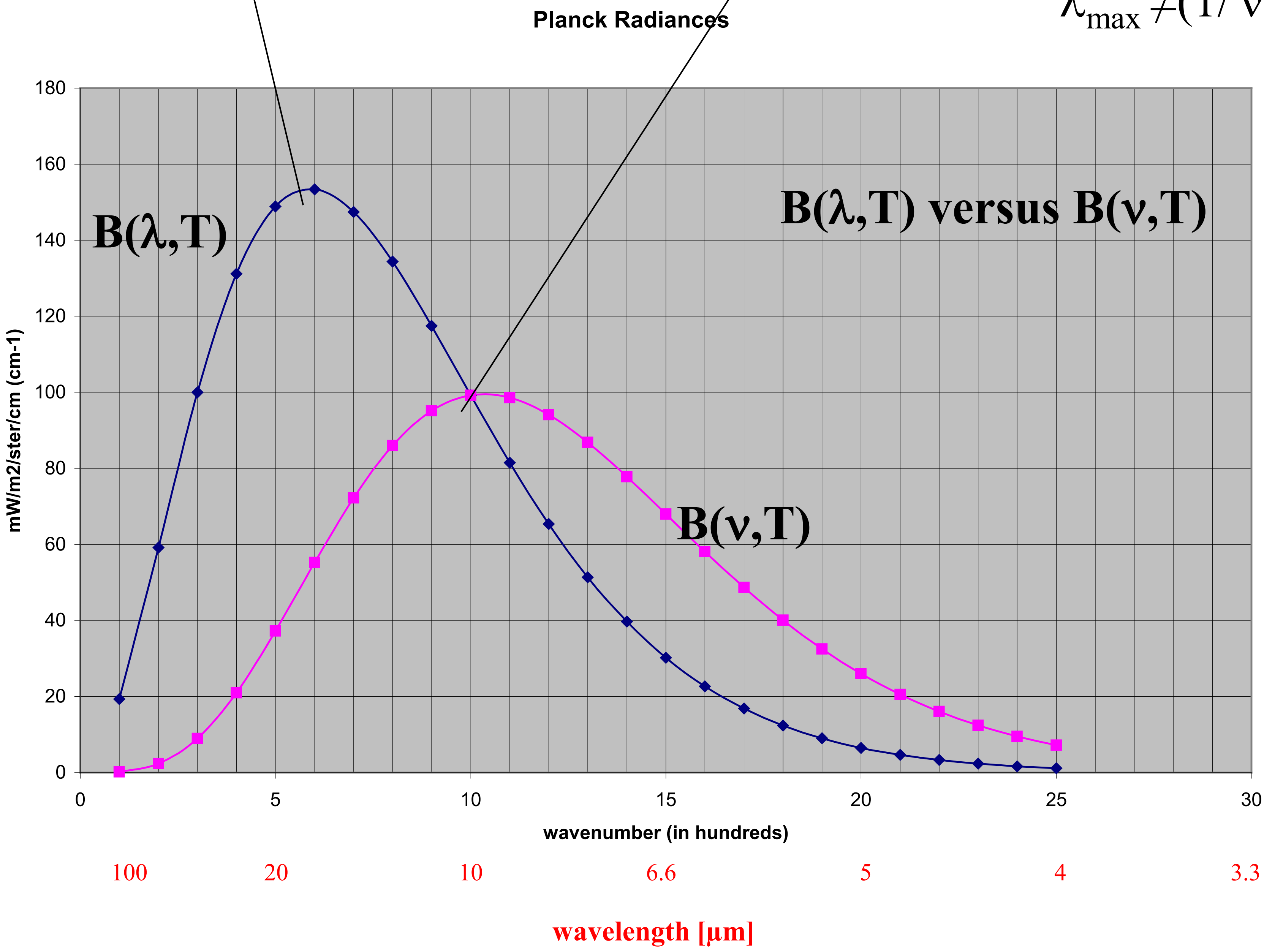
c_1 = 1.191044 x 10-5 ($mW/m^2/ster/cm^{-4}$)

c_2 = 1.438769 (cm deg K)

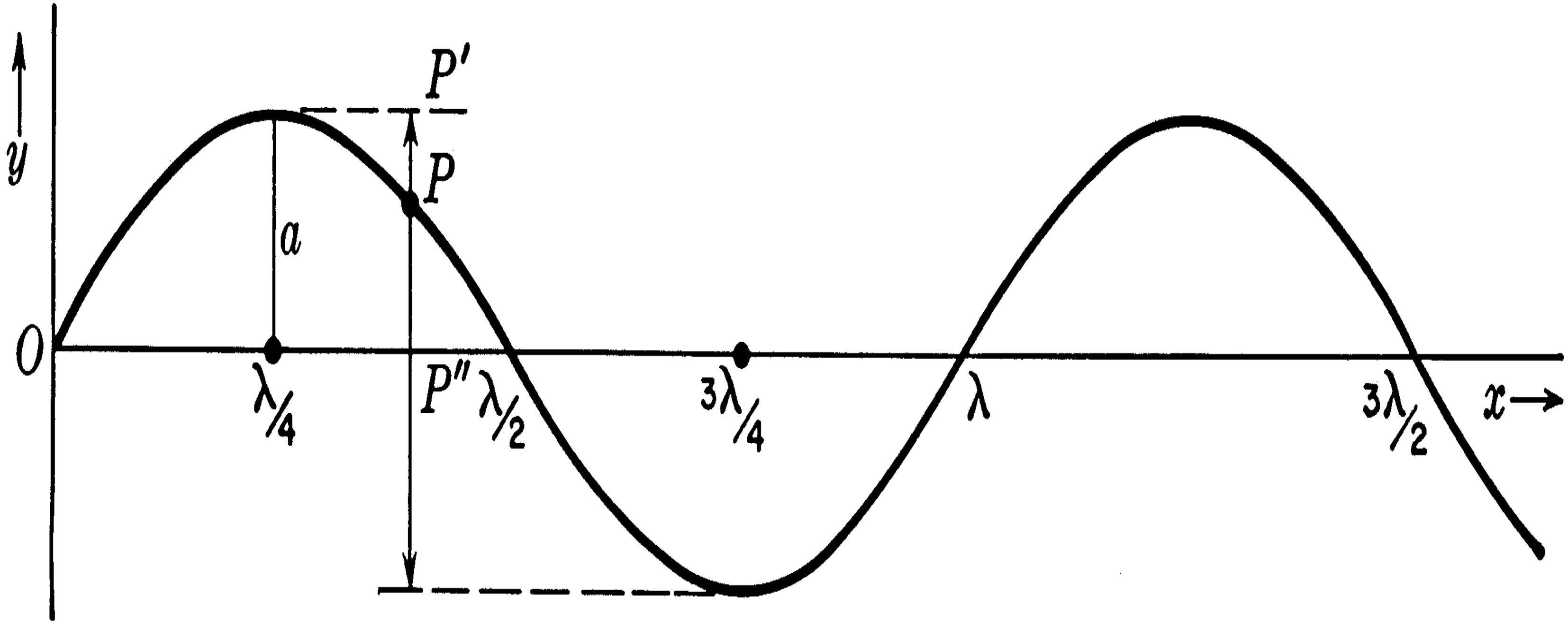
$B(\lambda max,T) \sim T^5$

$B(\nu max,T) \sim T^3$

$\lambda_{max} \neq (1/\nu_{max})$



wavelength λ : distance between peaks (μm)



wavenumber ν : number of waves per unit distance (cm)

$$\lambda = 1/\nu$$

$$d\lambda = -1/\nu^2 \, d\nu$$

Using wavenumbers

Wien's Law $dB(\nu_{max},T) / dT = 0$ where $\nu(max) = 1.95T$
indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\nu_{max},T) \sim T^{**}3$.

Stefan-Boltzmann Law $E = \pi \int_{o}^{\infty} B(\nu,T)\, d\nu = \sigma T^4$, where σ = 5.67 x 10-8 W/m^2/deg^4.

states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

Brightness Temperature

$$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$ is determined by inverting Planck function

Brightness temperature is uniquely related to radiance for a given wavelength by the Planck function.

Using wavenumbers

$$B(\nu,T) = c_1\nu^3 / [e^{c_2\nu/T} - 1]$$

(mW/m^2/ster/cm^{-1})

ν(max in cm-1) = 1.95T

$B(\nu_{max},T) \sim T^{**}3.$

$$E = \pi \int_o^\infty B(\nu,T)\, d\nu = \sigma T^4,$$

$$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$

Using wavelengths

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

(mW/m^2/ster/cm)

λ(max in cm)T = 0.2897

$B(\lambda_{max},T) \sim T^{**}5.$

$$E = \pi \int_o^\infty B(\lambda,T)\, d\lambda = \sigma T^4,$$

$$T = c_2/[\lambda \ln(\frac{c_1}{\lambda^5 B_\lambda} + 1)]$$

Temperature sensitivity

$$dB/B = \alpha \, dT/T$$

The Temperature Sensitivity α is the percentage change in radiance corresponding to a percentage change in temperature

Substituting the Planck Expression, the equation can be solved in α:

$$\alpha = c_2 \nu / T$$

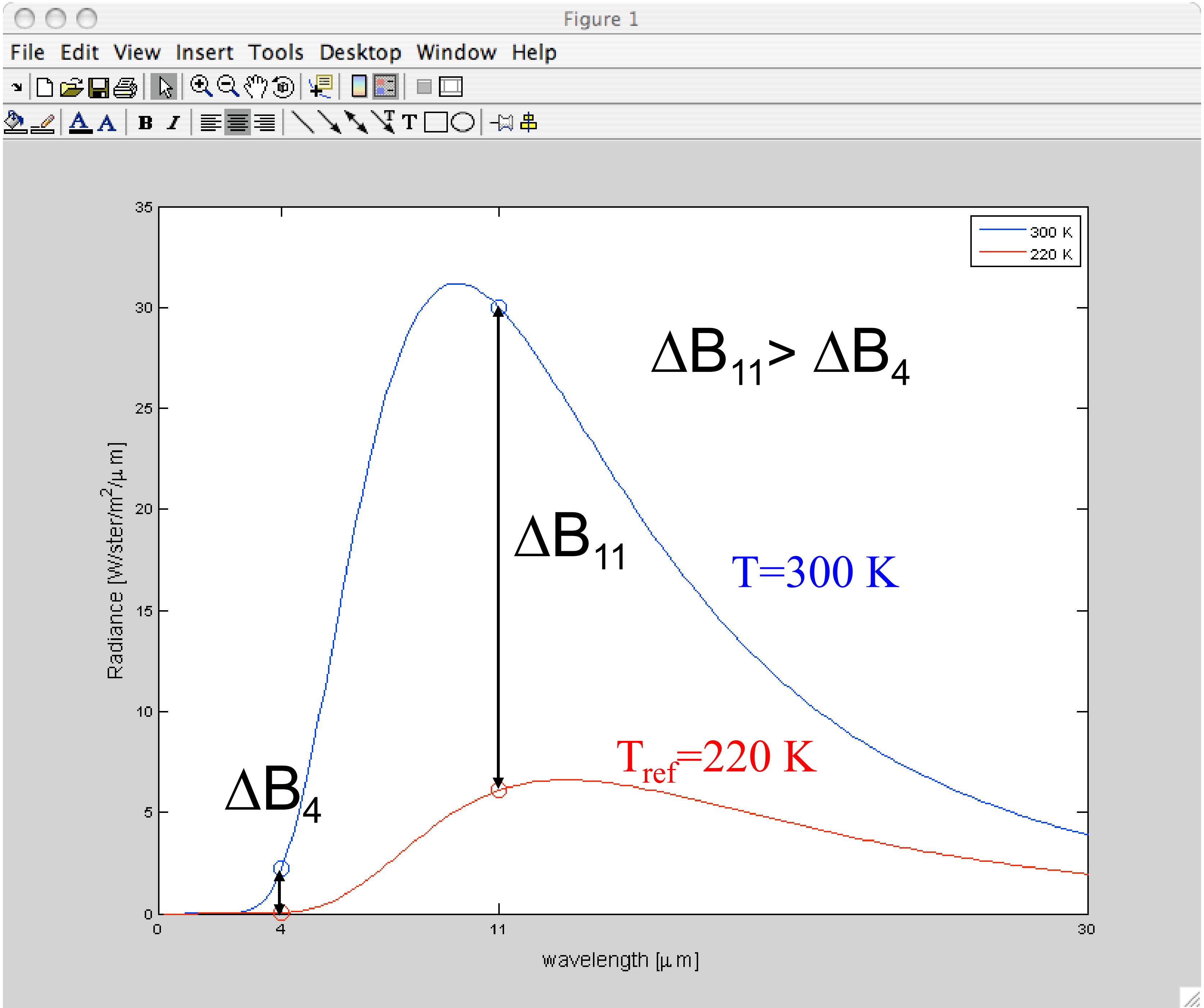
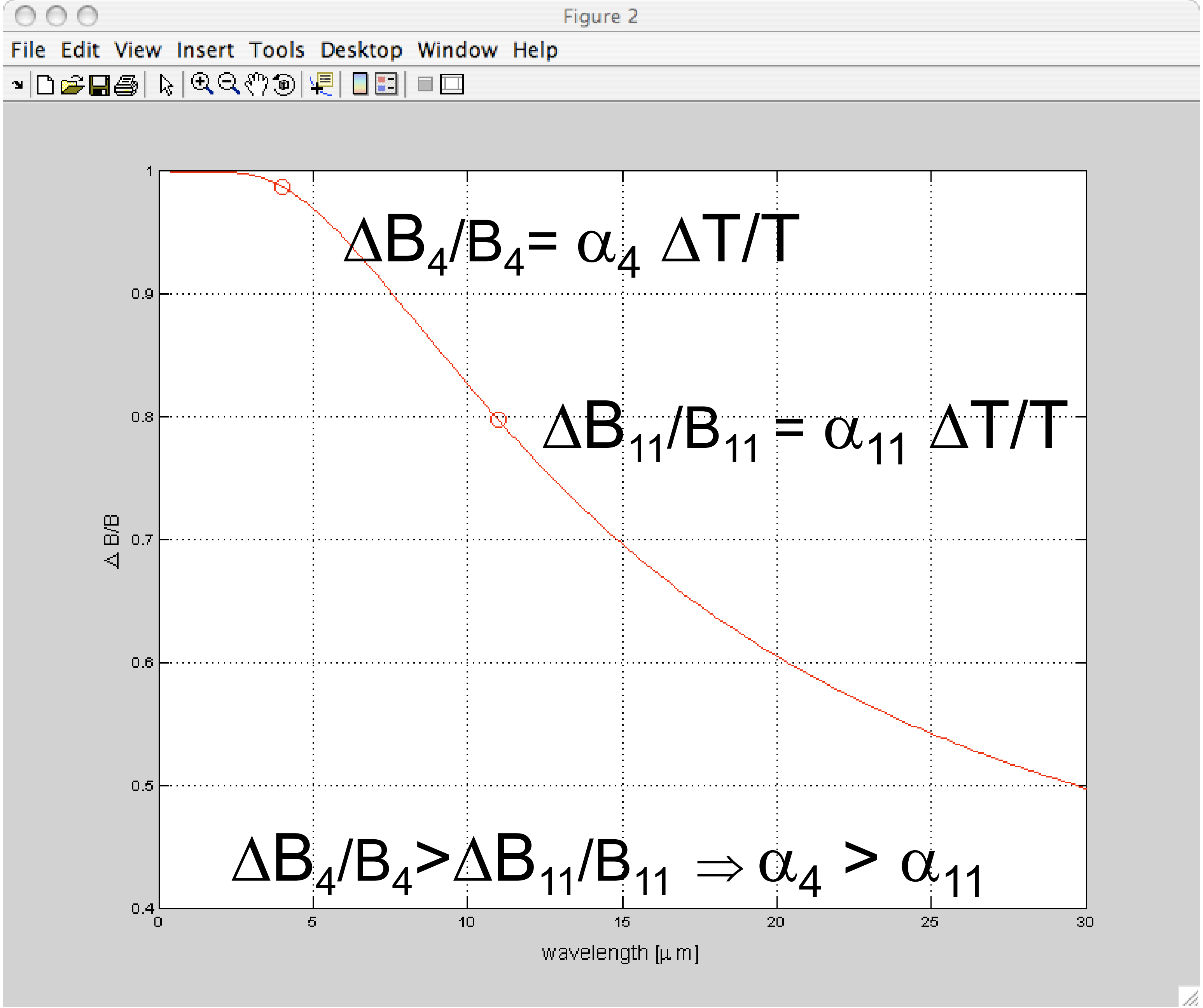


Figure 1
File Edit View Insert Tools Desktop Window Help
35
30
25
20
15
10
5
0
0
4
11
30
Radiance [W/ster/m^2/µ m]
wavelength [µ m]
300 K
220 K
$\Delta B_{11} > \Delta B_4$
ΔB_{11}
ΔB_4
T=300 K
T_{ref}=220 K



$\Delta B_4/B_4 = \alpha_4 \ \Delta T/T$

$\Delta B_{11}/B_{11} = \alpha_{11} \ \Delta T/T$

$\Delta B_4/B_4 > \Delta B_{11}/B_{11} \Rightarrow \alpha_4 > \alpha_{11}$

(values in plot are referred to wavelength)

(Approximation of) B as function of α and T

$\Delta B/B=\alpha\ \Delta T/T$

Integrating the Temperature Sensitivity Equation Between T_{ref} and T (B_{ref} and B):

$B=B_{ref}(T/T_{ref})^{\alpha}$

Where $\alpha=c_2\nu/T_{ref}$ (in wavenumber space)

$$B = B_{ref}(T/T_{ref})^{\alpha}$$

$$\Downarrow$$

$$B = (B_{ref}/\ T_{ref}^{\alpha})\ T^{\alpha}$$

$$\Downarrow$$

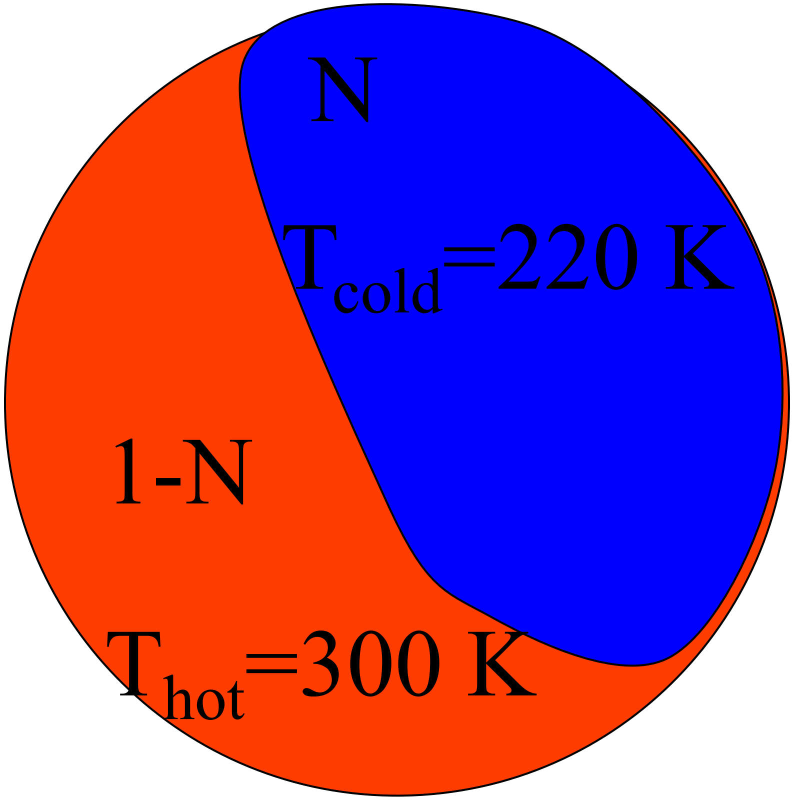
$$B \propto T^{\alpha}$$

The temperature sensitivity indicates the power to which the Planck radiance depends on temperature, since B proportional to T^{α} satisfies the equation. For infrared wavelengths,

$\alpha = c_2\nu/T = c_2/\lambda T$.

Wavenumber	Typical Scene Temperature	Temperature Sensitivity
900	300	4.32
2500	300	11.99

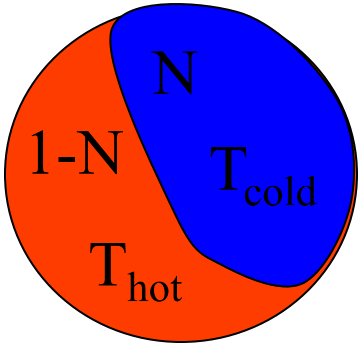
Non-Homogeneous FOV



$$B=NB(T_{cold})+(1-N)B(T_{hot})$$

$$\cancel{BT=NBT_{hot}+(1-N)BT_{cold}}$$

For NON-UNIFORM FOVs:



$$B_{obs}=NB_{cold}+(1-N)B_{hot}$$

$$B_{obs}=N\ B_{ref}(T_{cold}/T_{ref})^{\alpha}+(1-N)\ B_{ref}(T_{hot}/T_{ref})^{\alpha}$$

$$B_{obs}=B_{ref}(1/T_{ref})^{\alpha}\ (N\ T_{cold}^{\alpha}+(1-N)T_{hot}^{\alpha})$$

For N=.5

$$B_{obs}/B_{ref}=.5\ (1/T_{ref})^{\alpha}\ (\ T_{cold}^{\alpha}+T_{hot}^{\alpha})$$

$$B_{obs}/B_{ref}=.5\ (1/T_{ref}T_{cold})^{\alpha}\ (1+(T_{hot}/\ T_{cold})^{\alpha})$$

The greater α the more predominant the hot term

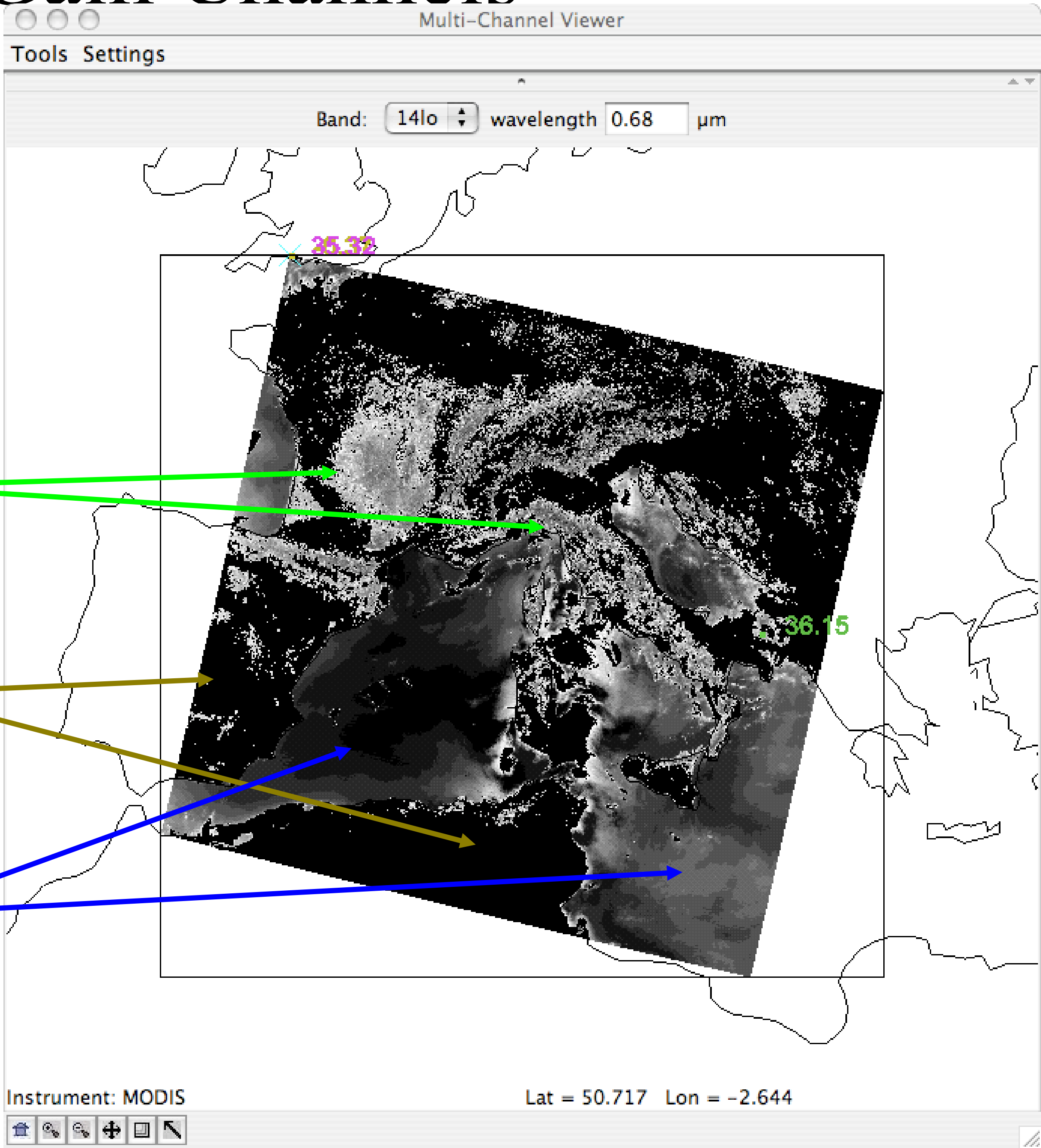
At 4 μm (α=12) the hot term more dominating than at 11 μm (α=4)

Consequences

- At 4 µm (α=12) clouds look smaller than at 11 µm (α=4)
- In presence of fires the difference BT_4-BT_{11} is larger than the solar contribution
- The different response in these 2 windows allow for cloud detection and for fire detection

Low Gain Channels

Band 14 low
0.68 µm



Band: 14lo wavelength 0.68 µm

Instrument: MODIS

Lat = 50.717 Lon = -2.644

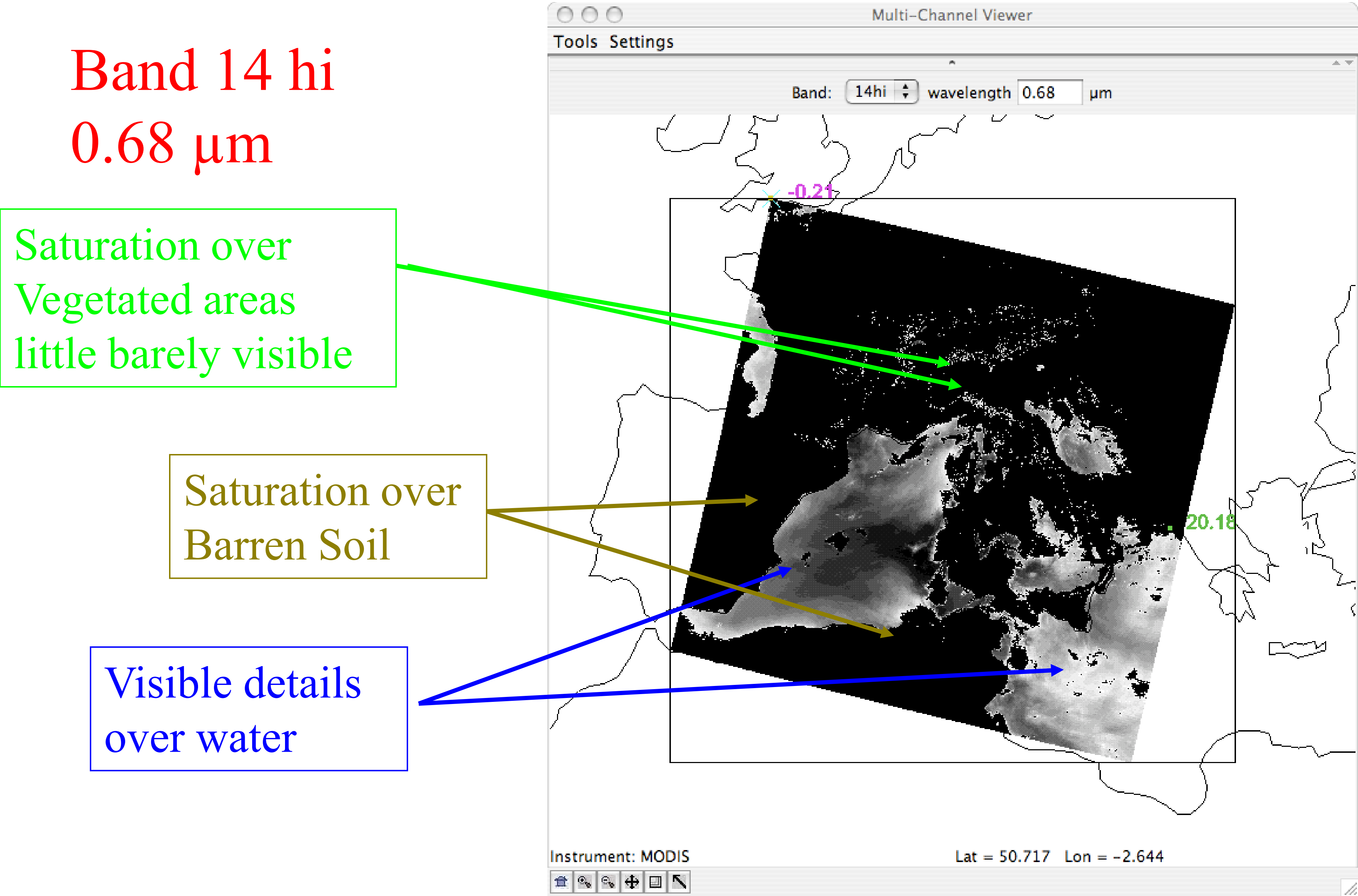
Vegetated areas
Are visible

Saturation over
Barren Soil

Visible details
over water

High Gain Channels

Band 14 hi
0.68 µm

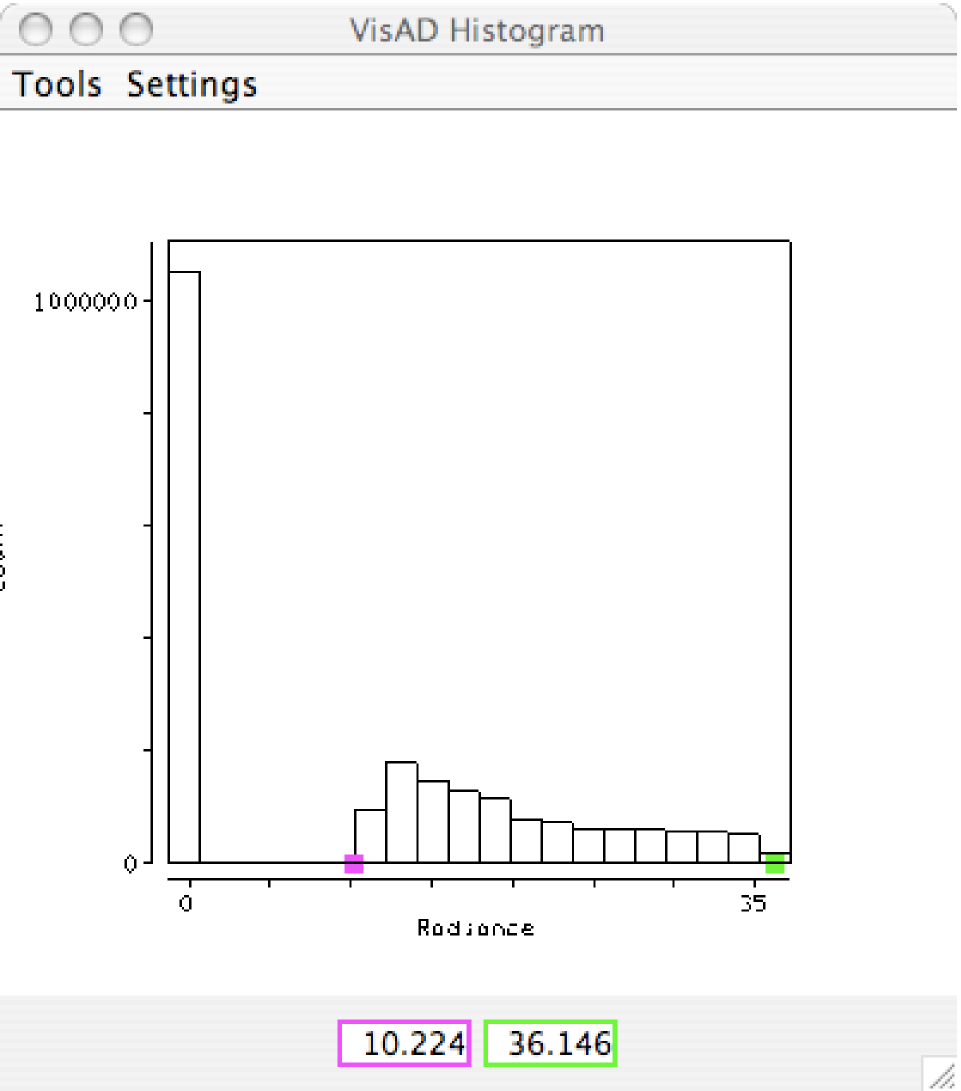


Saturation over Vegetated areas little barely visible

Saturation over Barren Soil

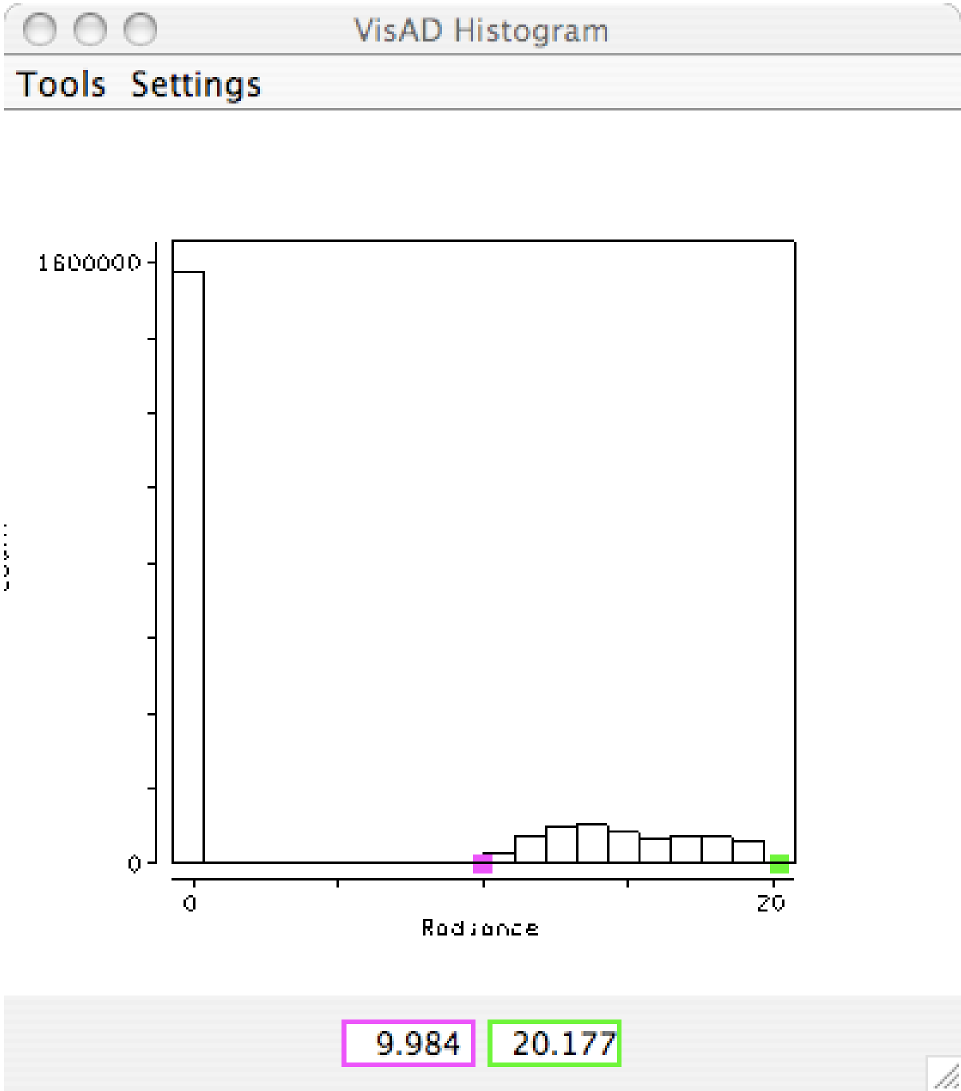
Visible details over water

Range for Band 14 low
0.68 µm



10.224 36.146

Range for Band 14 high
0.68 µm

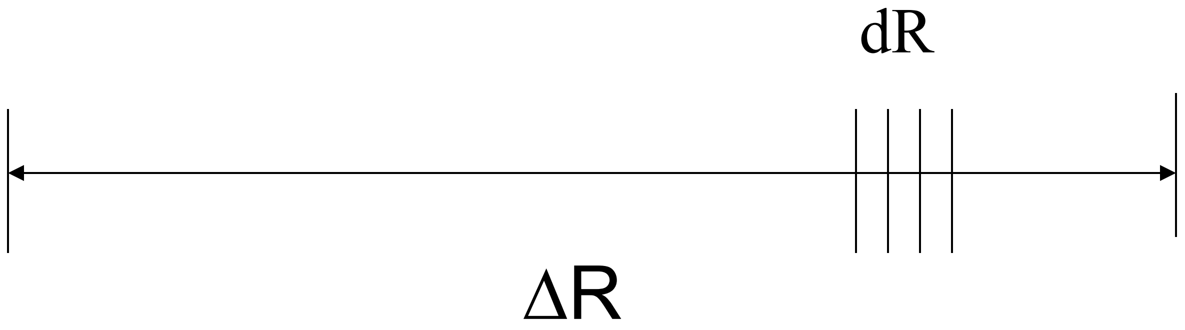


9.984 20.177

Bit Depth and Value Range

- With 12 bits 2^{12} integer numbers can be represented
  - Given ΔR, the range of radiances we want to observe, the smallest observable variation is $\Delta R / 2^{12}$
  - Given dR smallest observable variation, the range of observable radiances is $dR * 2^{12}$

For this reason Band 14low (larger range) is used for cloud detection and Band 14hi (smaller range) is used for ocean products



Conclusions

- Planck Function: at any wavenumber/wavelength relates the temperature of the observed target to its radiance (for Blackbodies);

- Thermal Sensitivity: different emissive channels respond differently to target temperature variations. Thermal Sensitivity helps in explaining why, and allows for cloud and fire detection;

- Bit Depth: key concepts are spectral range and number of bits.