Radiative Transfer in the Atmosphere

Lectures in Brienza

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Outline

Radiation Definitions

Planck Function

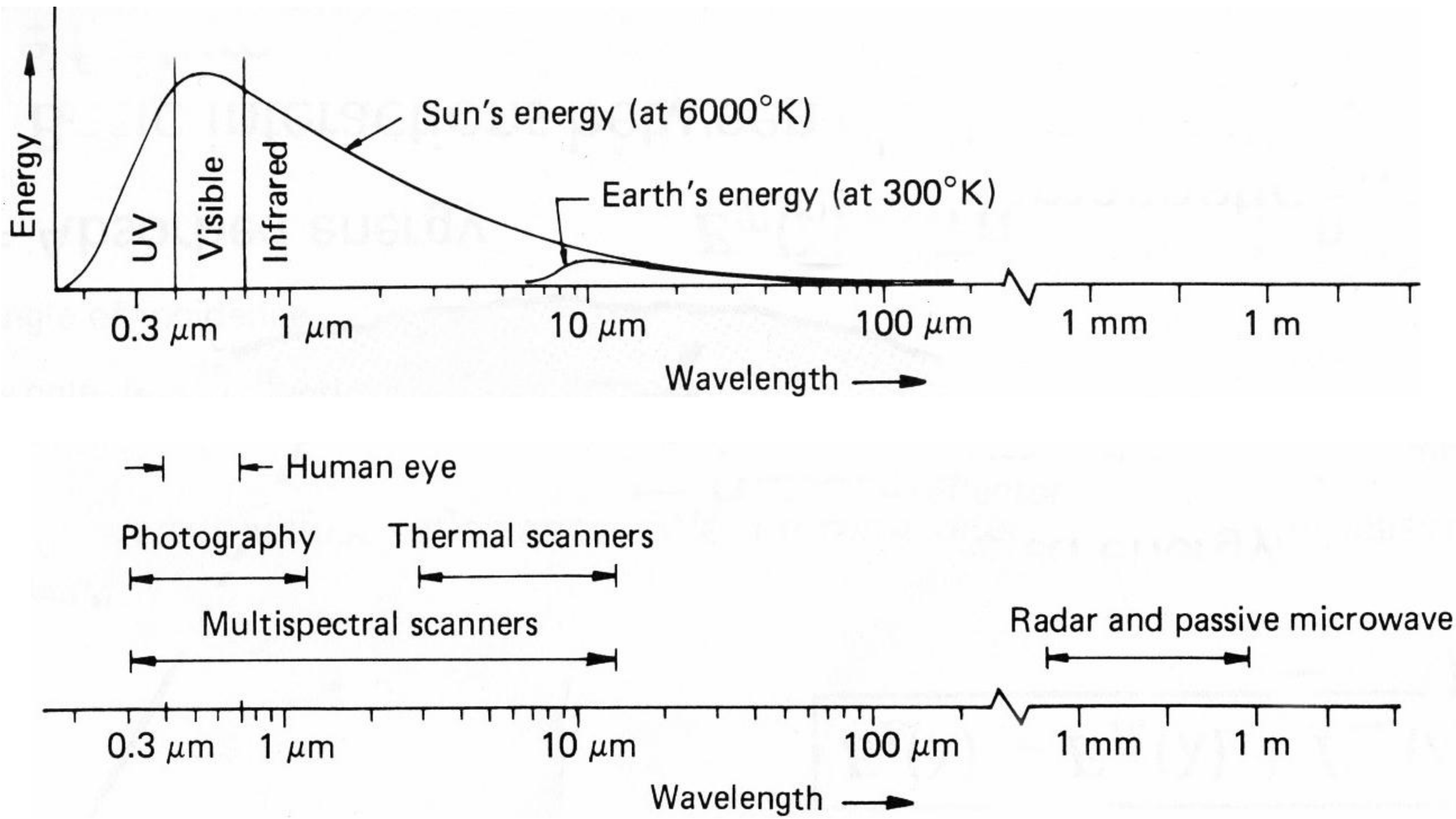
Emission, Absorption, Scattering

Radiative Transfer Equation

Satellite Derived Met Parameters

Microwave Considerations

Spectral Characteristics of Energy Sources and Sensing Systems



Terminology of radiant energy

Energy from the Earth Atmosphere

over time is

Flux

which strikes the detector area

Irradiance

at a given wavelength interval

Monochromatic Irradiance

over a solid angle on the Earth

Radiance observed by satellite radiometer

is described by

The Planck function

can be inverted to

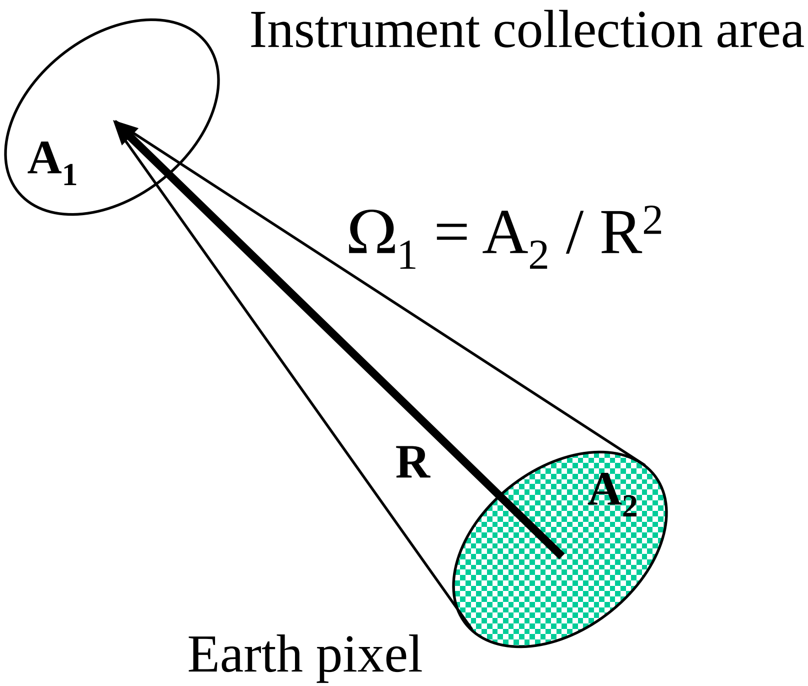
Brightness temperature

Definitions of Radiation

QUANTITY	SYMBOL	UNITS
Energy	**dQ**	**Joules**
Flux	**dQ/dt**	**Joules/sec = Watts**
Irradiance	**dQ/dt/dA**	**Watts/meter^2**
Monochromatic Irradiance	**dQ/dt/dA/dλ**	**W/m^2/micron**
	or	
	dQ/dt/dA/dν	**W/m^2/cm^{-1}**
Radiance	**dQ/dt/dA/dλ/dΩ**	**W/m^2/micron/ster**
	or	
	dQ/dt/dA/dν/dΩ	**W/m^2/cm^{-1}/ster**

Telescope Radiative Power Capture proportional to $A\Omega$

Spectral Power radiated from A_2 to A_1 = $L(\lambda)\, A_1\Omega_1$ W/μm



Radiance from surface = $L(\lambda)$ W /m^2 /sr /μm

{ Note: $A_1 A_2 / R^2 = A_1\Omega_1 = A_2\Omega_2$ }

Using wavelengths

Planck's Law $B(\lambda,T) = c_1 / \lambda^5 / [e^{c_2/\lambda T} - 1]$ (mW/m²/ster/cm)

where λ = wavelengths in cm

T = temperature of emitting surface (deg K)

c_1 = 1.191044 x 10-5 (mW/m²/ster/cm^{-4})

c_2 = 1.438769 (cm deg K)

Wien's Law $dB(\lambda_{max},T) / d\lambda = 0$ where $\lambda(max) = .2897/T$

indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\lambda_{max},T) \sim T^5$.

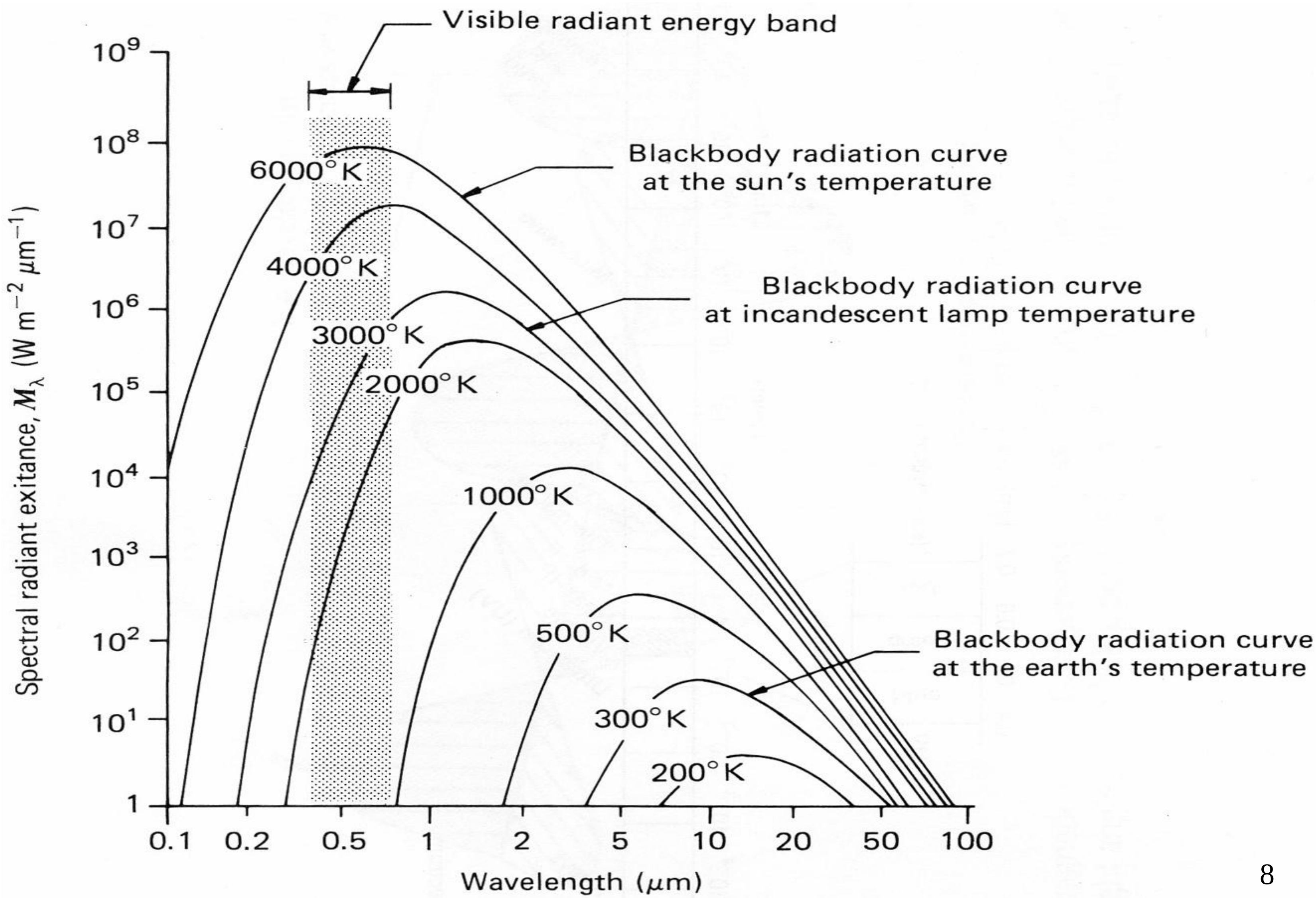
Stefan-Boltzmann Law $E = \pi \int_o^\infty B(\lambda,T)\, d\lambda = \sigma T^4$, where σ = 5.67 x 10-8 W/m2/deg4.

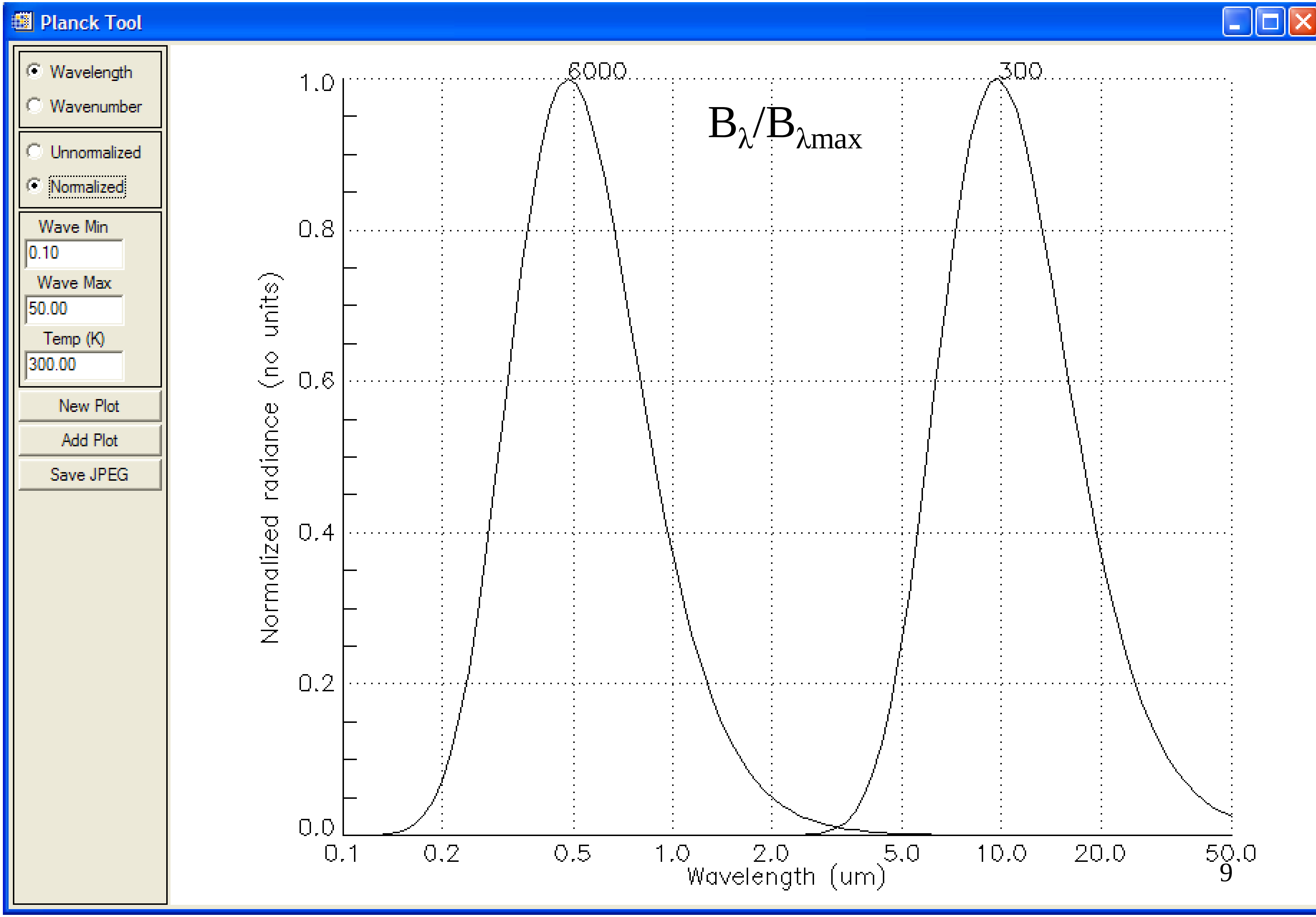
states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

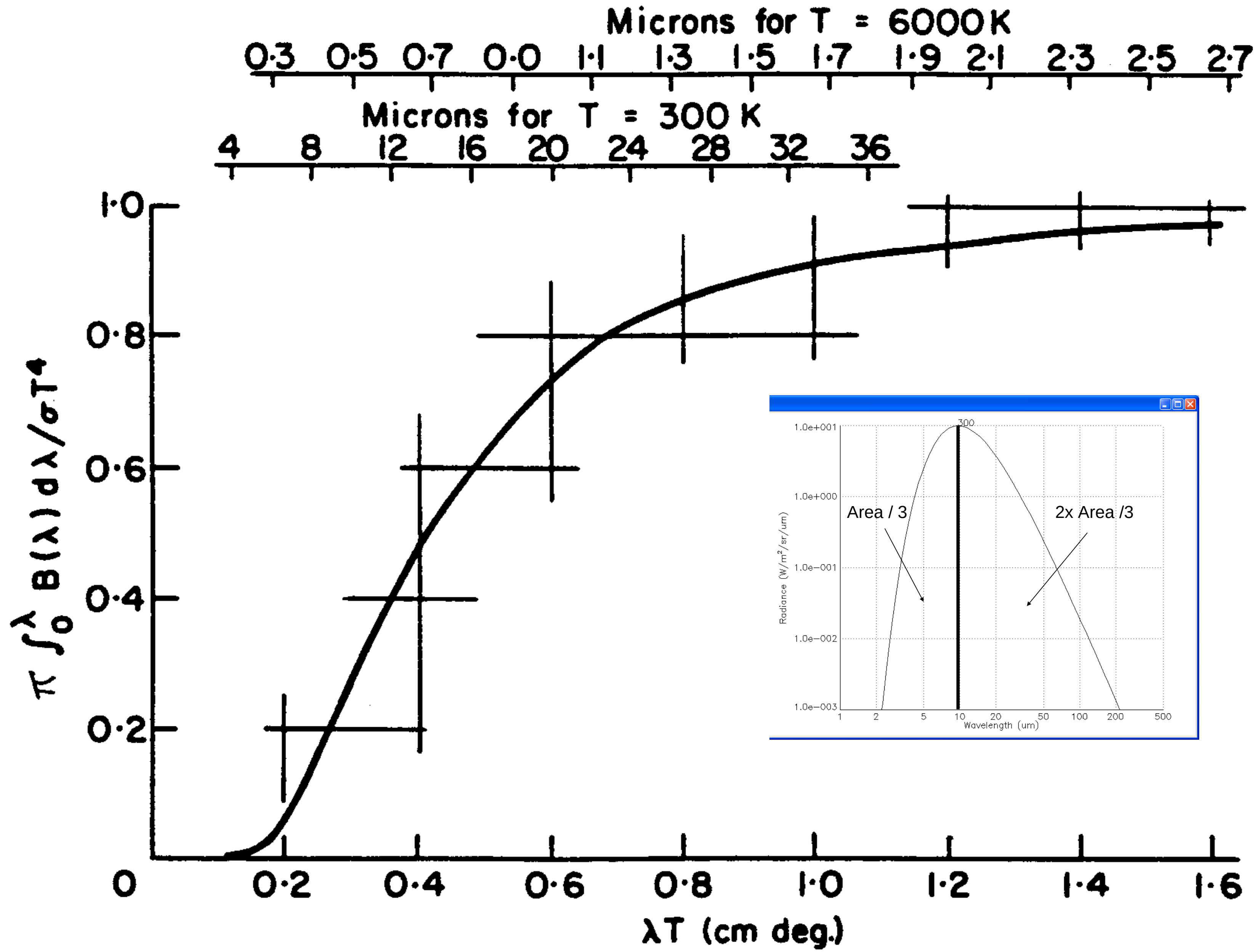
Brightness Temperature

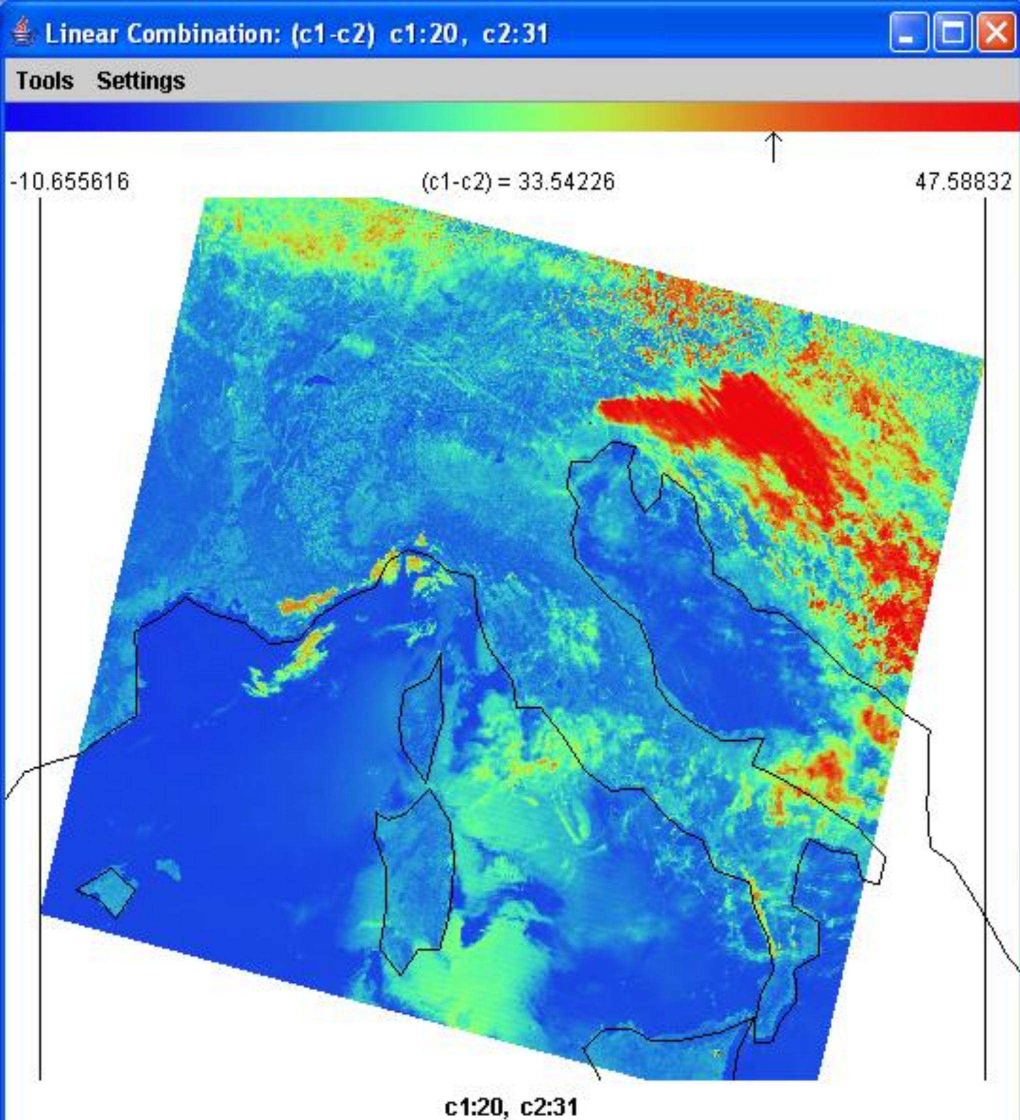
$$T = c_2 / [\lambda \ln\left(\frac{c_1}{\lambda^5 B_\lambda} + 1\right)]$$ is determined by inverting Planck function

Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures

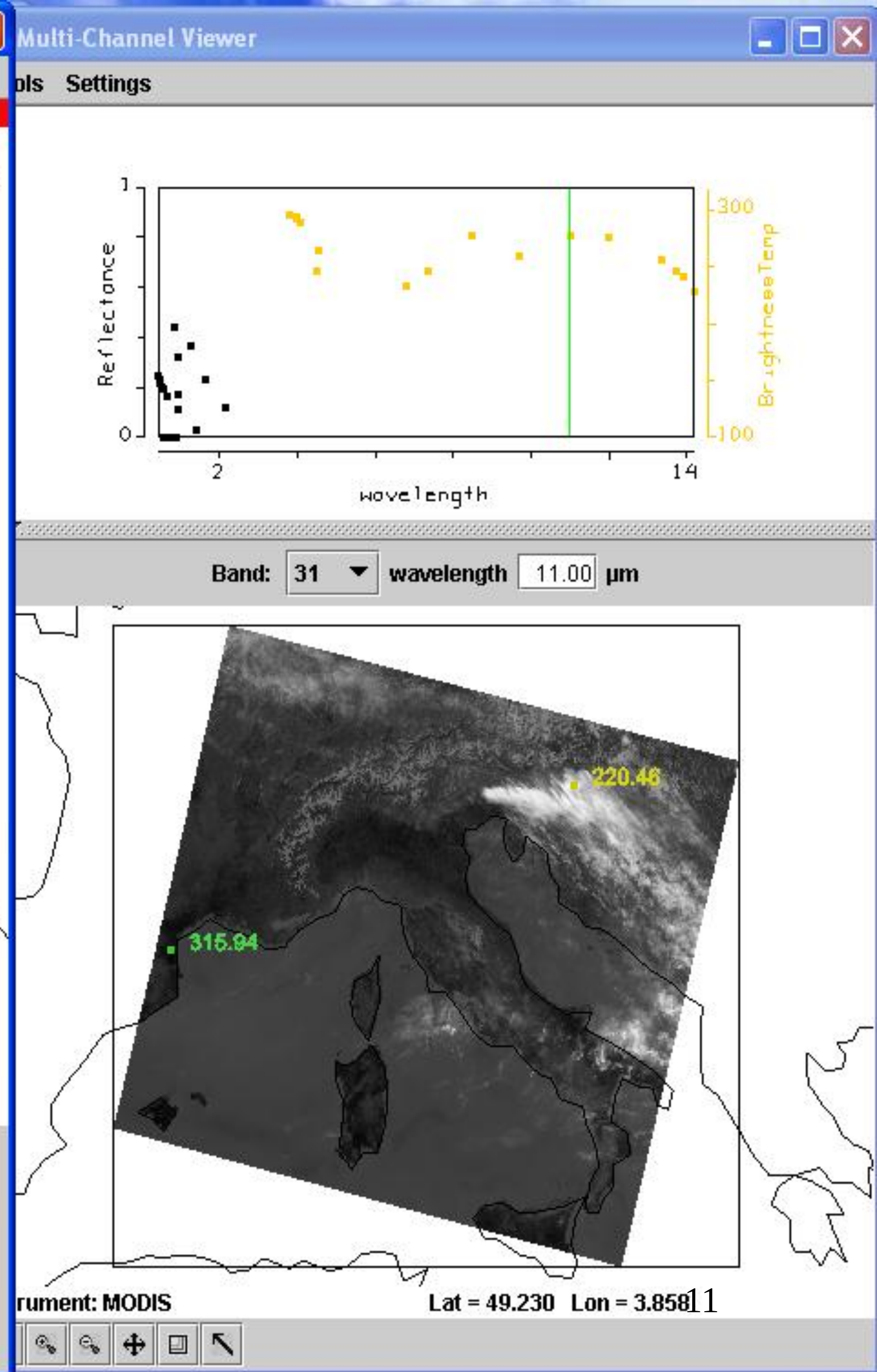



Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
0.10
Wave Max
50.00
Temp (K)
300.00
New Plot
Add Plot
Save JPEG
6000
300
$B_\lambda / B_{\lambda max}$
Normalized radiance (no units)
1.0
0.8
0.6
0.4
0.2
0.0
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0
50.0
Wavelength (um)


Microns for T = 6000K
0·3 0·5 0·7 0·0 1·1 1·3 1·5 1·7 1·9 2·1 2·3 2·5 2·7
Microns for T = 300 K
4 8 12 16 20 24 28 32 36
1·0 0·8 0·6 0·4 0·2
$\pi \int_0^\lambda B(\lambda)\,d\lambda / \sigma T^4$
0 0·2 0·4 0·6 0·8 1·0 1·2 1·4 1·6
λT (cm deg.)
Area / 3
2x Area /3
300
1.0e+001
1.0e+000
1.0e−001
1.0e−002
1.0e−003
Radiance (W/m²/sr/um)
1 2 5 10 20 50 100 200 500
Wavelength (um)


Linear Combination: (c1-c2) c1:20, c2:31
Tools Settings
-10.655616
(c1-c2) = 33.54226
47.58832
c1:20, c2:31
XAxis
YAxis
Mask Is On
Box
Curve

BT4 – BT11


Multi-Channel Viewer
ols Settings
Reflectance
BrightnessTemp
wavelength
1
0
300
100
2
14
Band: 31
wavelength 11.00 µm
220.46
315.94
rument: MODIS
Lat = 49.230 Lon = 3.858

Observed BT at 4 micron

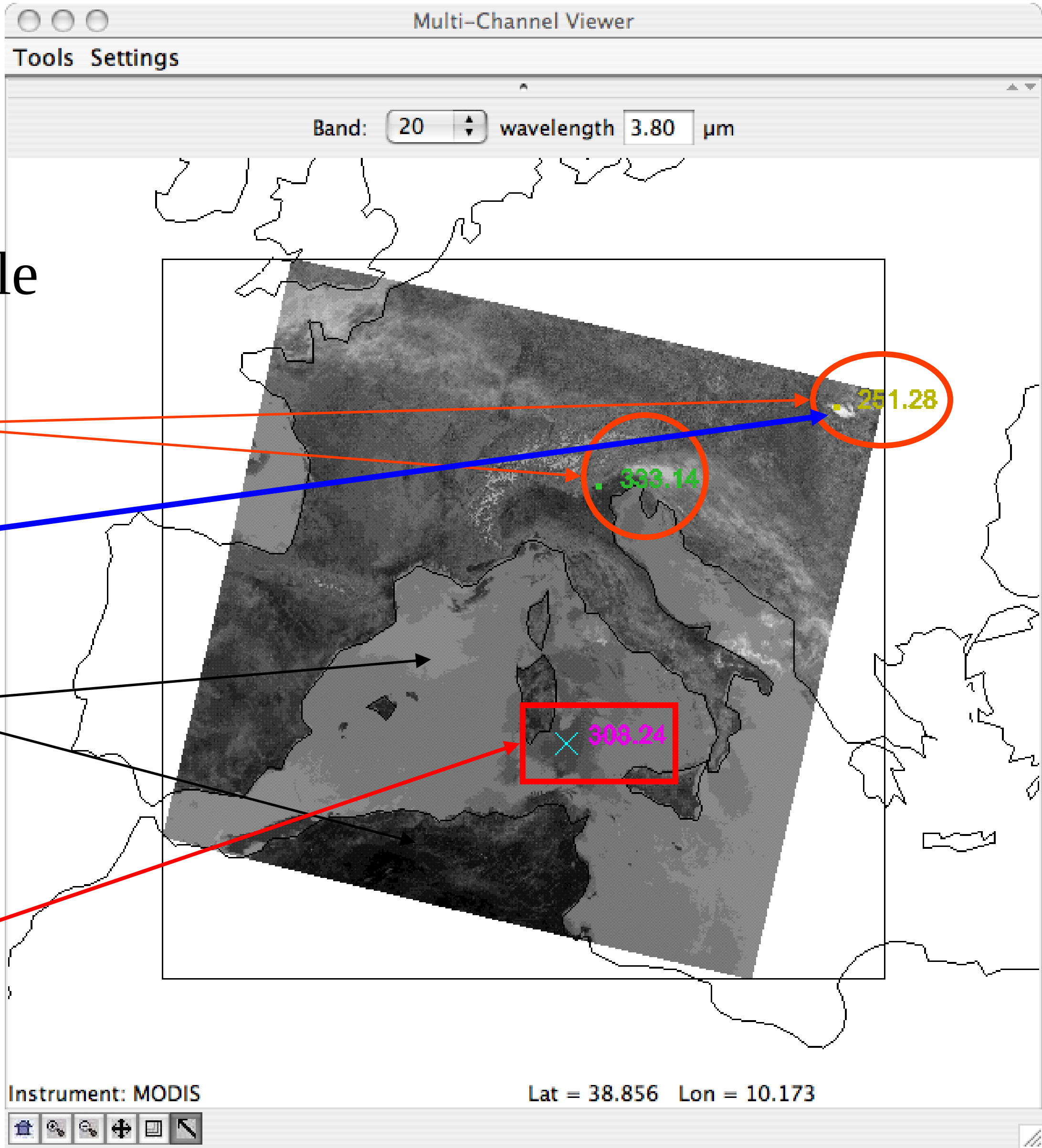
Window Channel:
•little atmospheric absorption
•surface features clearly visible

Range BT[250, 335]
Range R[0.2, 1.7]

Clouds are cold

Values over land
Larger than over water

Reflected Solar everywhere
Stronger over Sunglint



Observed BT at 11 micron

Window Channel:

- little atmospheric absorption
- surface features clearly visible

Range BT [220, 320]

Range R [2.1, 12.4]

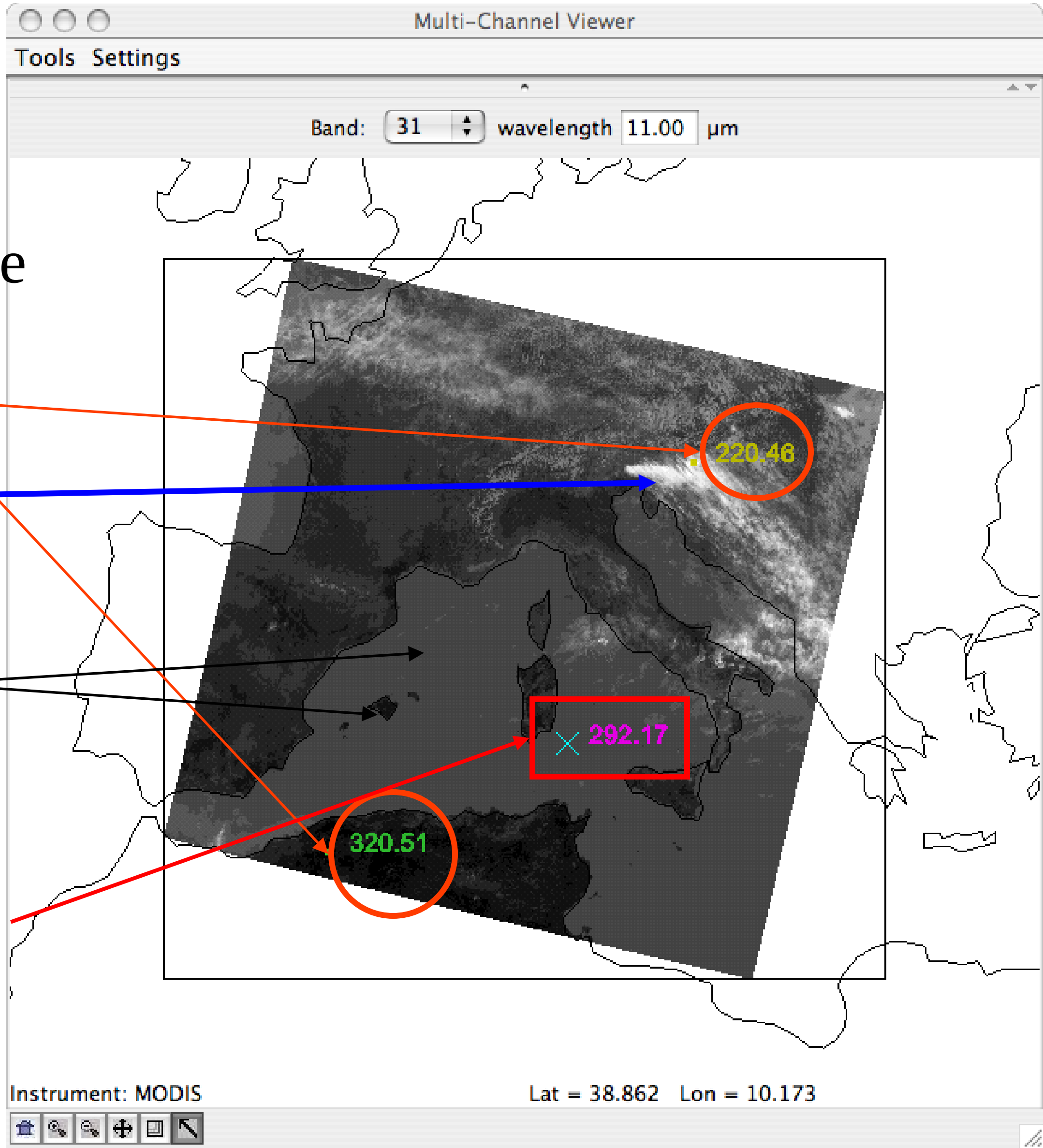
Clouds are cold

Values over land

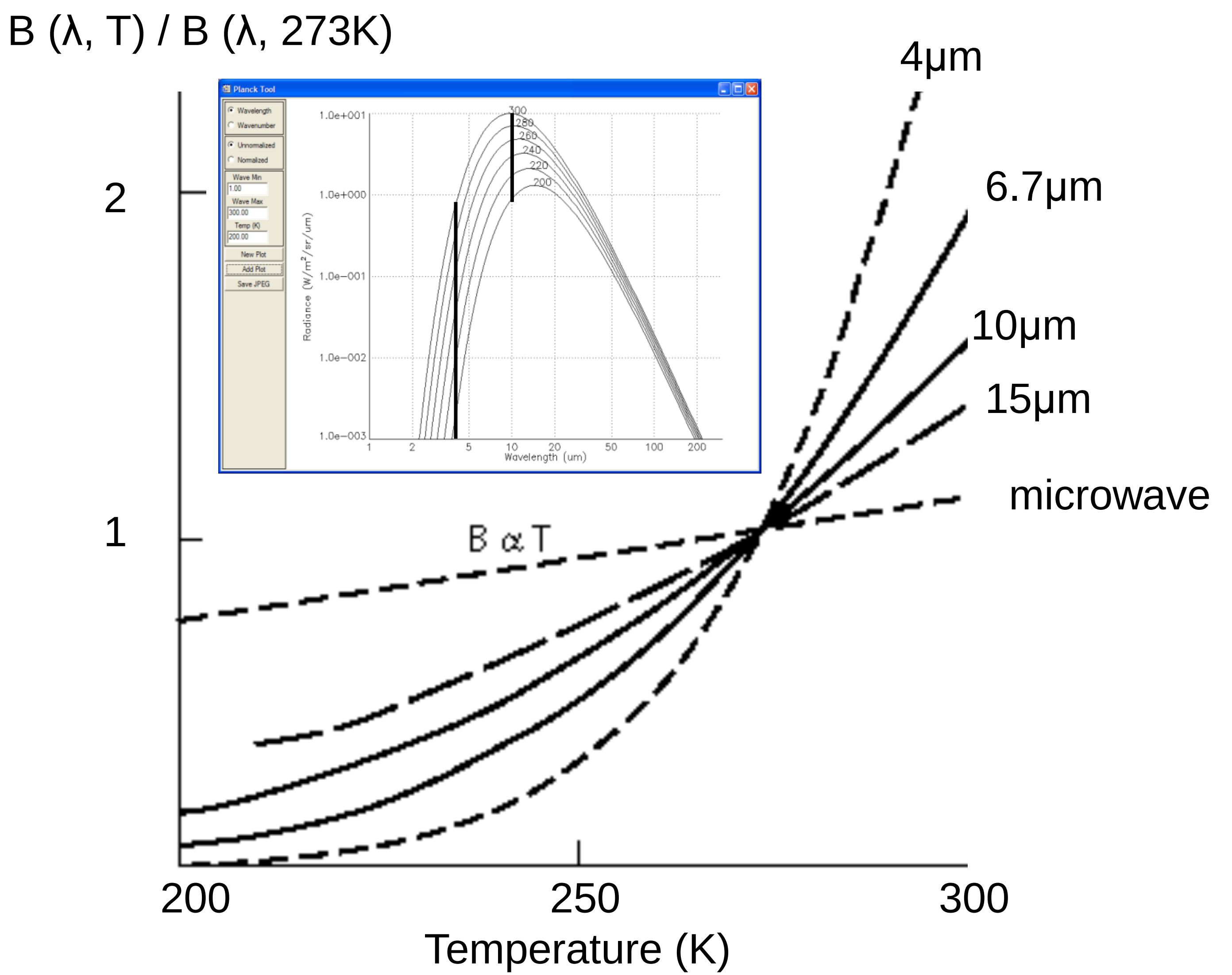
Larger than over water

Undetectable Reflected Solar

Even over Sunglint



Temperature Sensitivity of B(λ,T) for typical earth temperatures



(Approximation of) B as function of α and T

$\Delta B/B = \alpha \, \Delta T/T$

Integrating the Temperature Sensitivity Equation Between T_{ref} and T (B_{ref} and B):

$B = B_{ref}(T/T_{ref})^{\alpha}$

Where $\alpha = c_2 \nu / T_{ref}$ (in wavenumber space)

$$B = B_{ref}(T/T_{ref})^{\alpha}$$

$$\Downarrow$$

$$B = (B_{ref} / T_{ref}^{\alpha})\ T^{\alpha}$$

$$\Downarrow$$

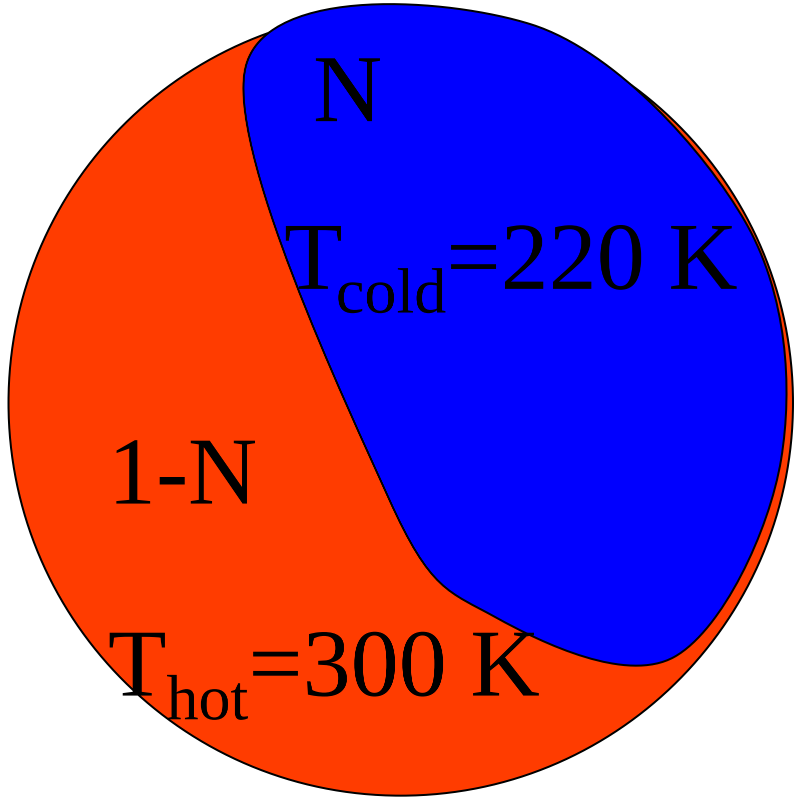
$$B \propto T^{\alpha}$$

The temperature sensitivity indicates the power to which the Planck radiance depends on temperature, since B proportional to T^{α} satisfies the equation. For infrared wavelengths,

$\alpha = c_2\nu/T = c_2/\lambda T.$

Wavenumber	Typical Scene Temperature	Temperature Sensitivity
900	300	4.32
2500	300	11.99

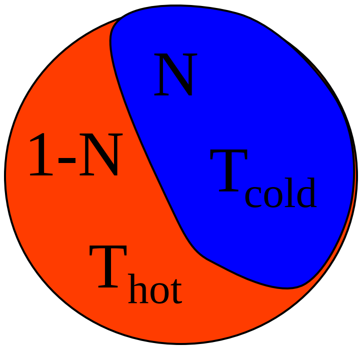
Non-Homogeneous FOV



$$B = N * B(T_{cold}) + (1-N) * B(T_{hot})$$

$$BT = N * T_{cold} + (1-N) * T_{hot}$$

For NON-UNIFORM FOVs:



$B_{obs}=NB_{cold}+(1-N)B_{hot}$

$B_{obs}=N\ B_{ref}(T_{cold}/T_{ref})^{\alpha}+\ (1-N)\ B_{ref}(T_{hot}/T_{ref})^{\alpha}$

$B_{obs}=\ B_{ref}(1/T_{ref})^{\alpha}\ (N\ T_{cold}^{\alpha}\ +\ (1-N)T_{hot}^{\alpha})$

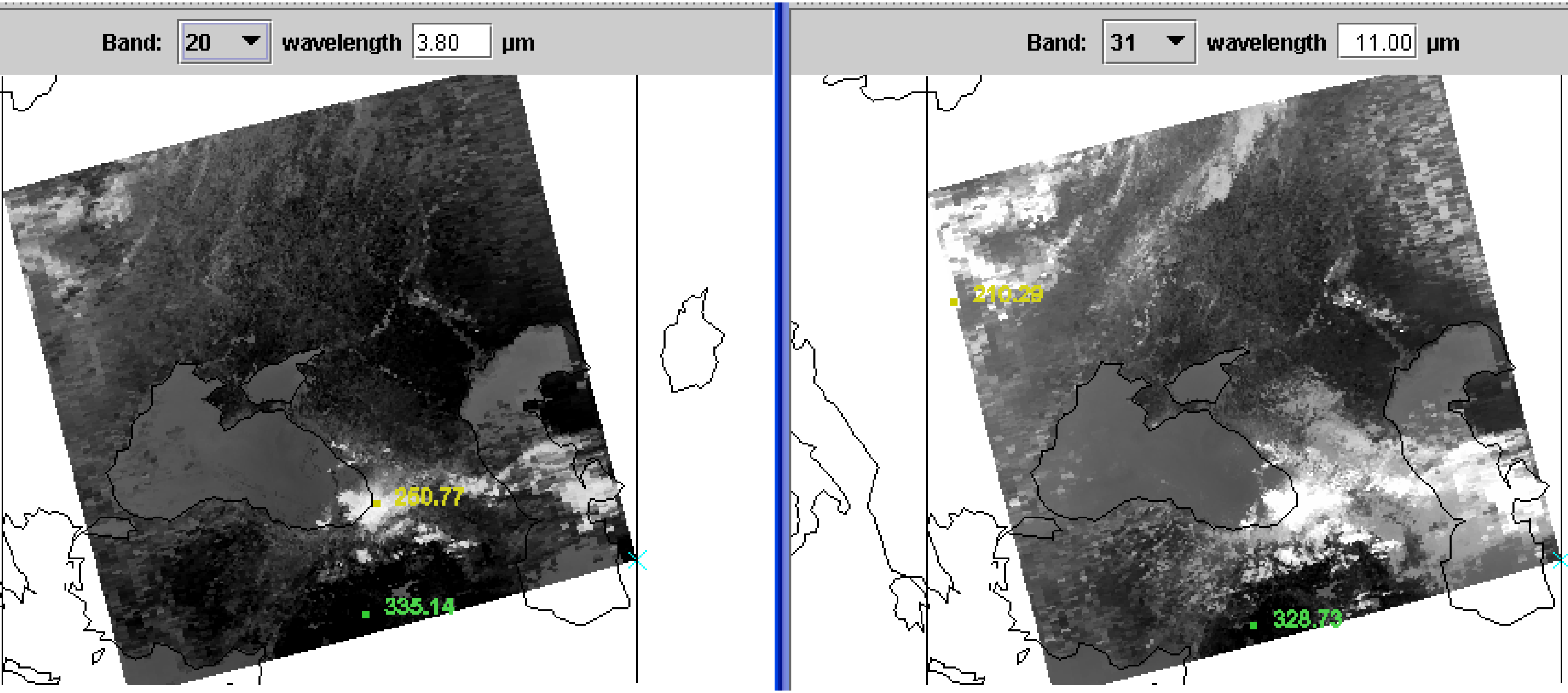
For N=.5

$B_{obs}/B_{ref}=.5\ (1/T_{ref})^{\alpha}\ (\ T_{cold}^{\alpha}\ +\ T_{hot}^{\alpha})$

$B_{obs}/B_{ref}=.5\ (1/T_{ref}T_{cold})^{\alpha}\ (1+\ (T_{hot}/\ T_{cold})^{\ \alpha})$

The greater α the more predominant the hot term

At 4 μm (α=12) the hot term more dominating than at 11 μm (α=4)



Cloud edges and broken clouds appear different in 11 and 4 um images.

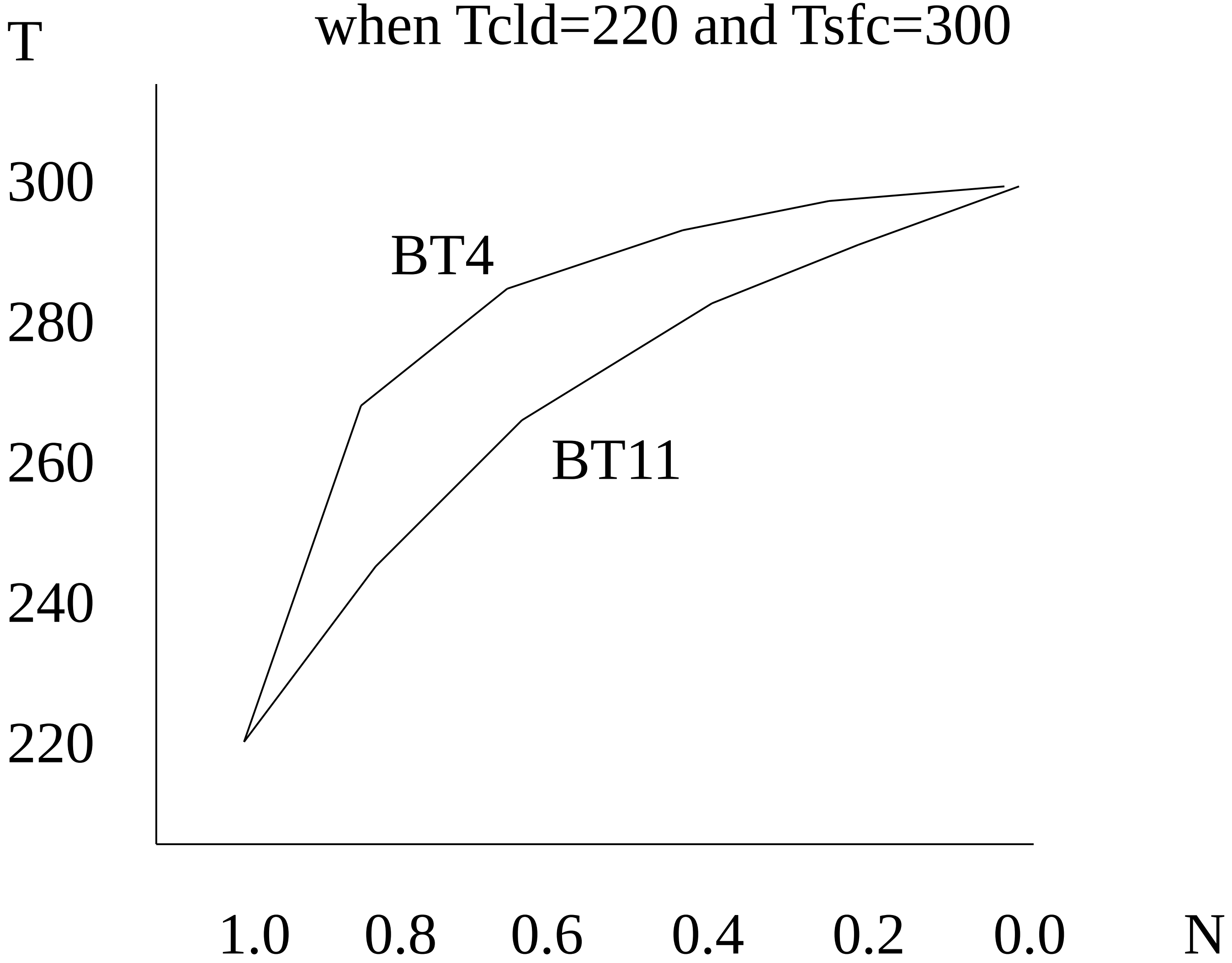
T(11)**4=(1-N)*Tclr**4+N*Tcld**4~(1-N)*300**4+N*200**4
T(4)**12=(1-N)*Tclr**12+N*Tcld**12~(1-N)*300**12+N*200**12

Cold part of pixel has more influence for B(11) than B(4)

Table 6.1 Longwave and Shortwave Window Planck Radiances (mW/m**2/ster/cm-1) and Brightness Temperatures (degrees K) as a function of Fractional Cloud Amount (for cloud of 220 K and surface of 300 K) using $B(T) = (1-N)*B(T_{sfc}) + N*B(T_{cld})$.

Cloud Fraction N	Longwave Window Rad	Longwave Window Temp	Shortwave Window Rad	Shortwave Window Temp	$T_s - T_1$
1.0	23.5	220	.005	220	0
.8	42.0	244	.114	267	23
.6	60.5	261	.223	280	19
.4	79.0	276	.332	289	13
.2	97.5	289	.441	295	6
.0	116.0	300	.550	300	0

SW and LW BTs for different cloud amounts when Tcld=220 and Tsfc=300



Using wavenumbers

Planck's Law

$$B(\nu,T) = c_1\nu^3 / [e^{c_2\nu/T} - 1] \quad (mW/m^2/ster/cm^{-1})$$

where ν = # wavelengths in one centimeter (cm-1)

T = temperature of emitting surface (deg K)

c_1 = 1.191044 x 10-5 ($mW/m^2/ster/cm^{-4}$)

c_2 = 1.438769 (cm deg K)

Wien's Law $dB(\nu_{max},T) / d\nu = 0$ where νmax) = 1.95T

indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase.

Stefan-Boltzmann Law $E = \pi \int_o^{\infty} B(\nu,T)\, d\nu = \sigma T^4$, where σ = 5.67 x 10-8 W/m2/deg4.

states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

Brightness Temperature

$$T = c_2\nu / [\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$ is determined by inverting Planck function

Using wavenumbers

$$B(\nu,T) = c_1 \nu^3 / [e^{c_2 \nu / T} - 1]$$

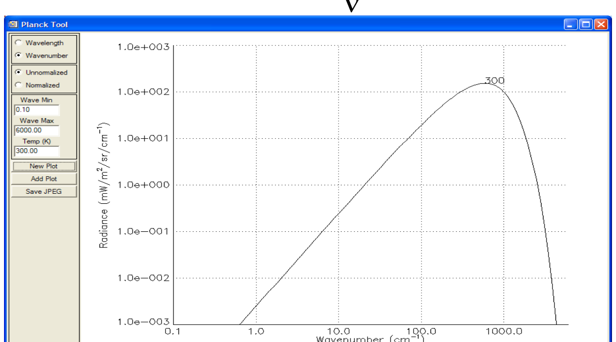
$(mW/m^2/ster/cm^{-1})$

ν(max in cm-1) = 1.95T

$B(\nu_{max},T) \sim T^{**}3.$

$$E = \pi \int_0^\infty B(\nu,T)\, d\nu = \sigma T^4,$$

$$T = c_2 \nu / [\ln(\frac{c_1 \nu^3}{B_\nu} + 1)]$$



Using wavelengths

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2 / \lambda T} - 1] \}$$

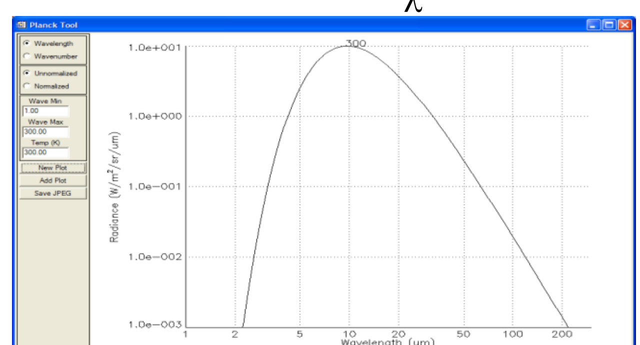
$(mW/m^2/ster/\mu m)$

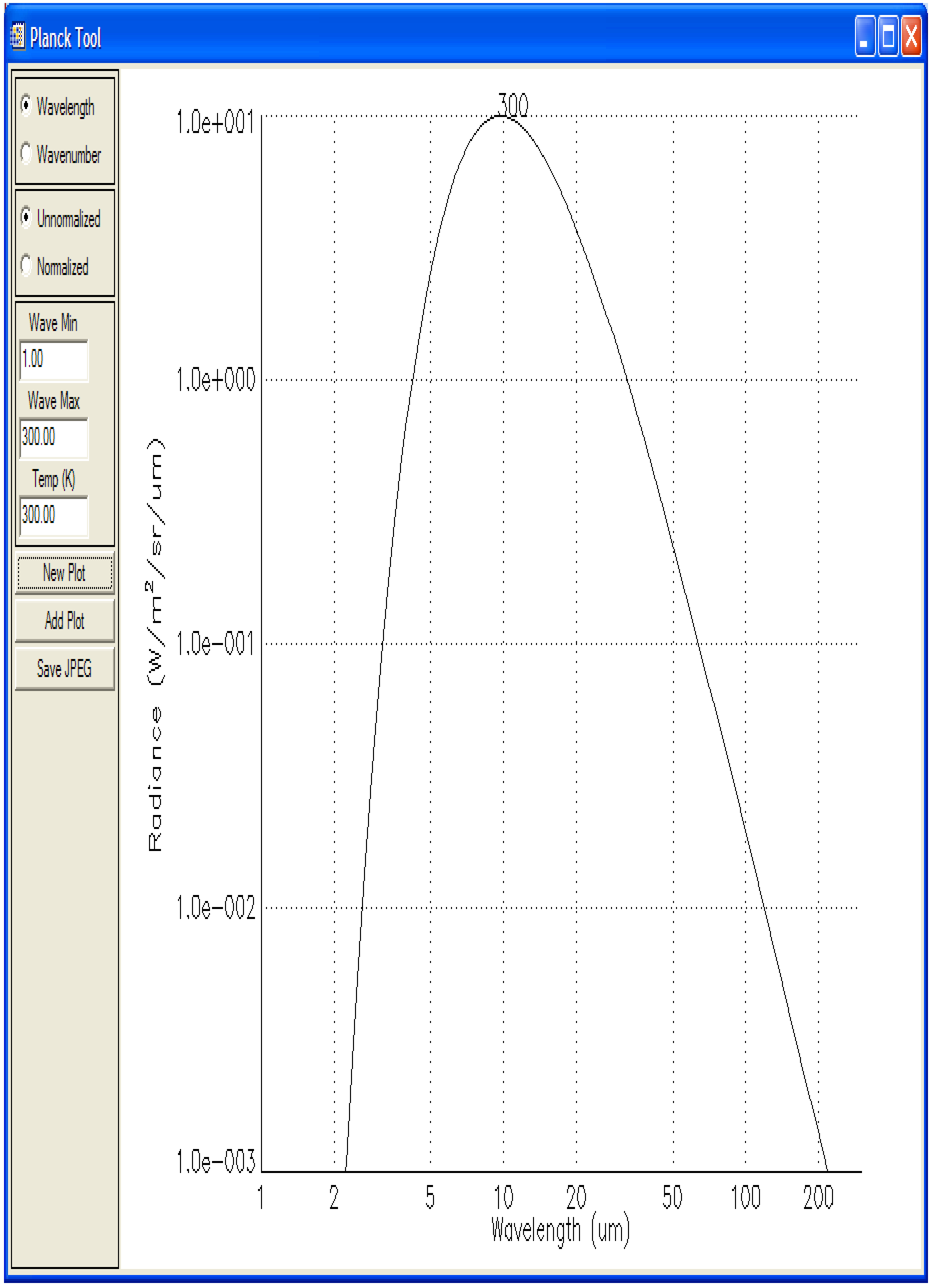
λ(max in cm)T = 0.2897

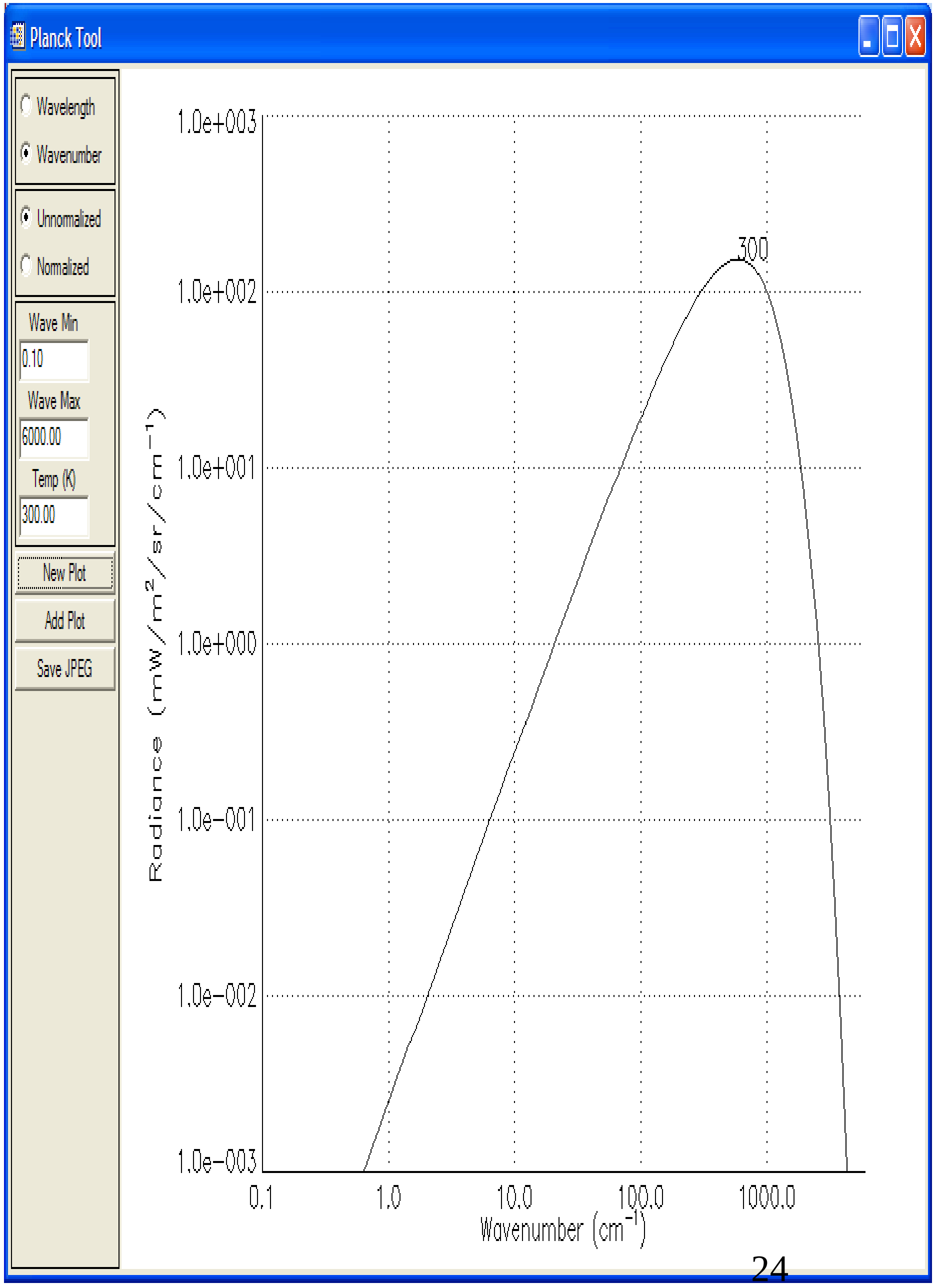
$B(\lambda_{max},T) \sim T^{**}5.$

$$E = \pi \int_0^\infty B(\lambda,T)\, d\lambda = \sigma T^4,$$

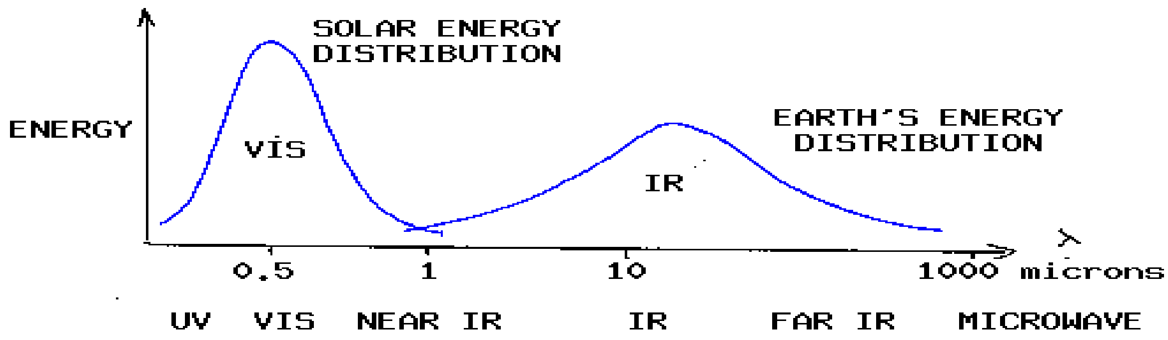
$$T = c_2 / [\lambda \ln(\frac{c_1}{\lambda^5 B_\lambda} + 1)]$$




Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
1.00
Wave Max
300.00
Temp (K)
300.00
New Plot
Add Plot
Save JPEG
300
Radiance (W/m²/sr/um)
1.0e+001
1.0e+000
1.0e-001
1.0e-002
1.0e-003
1
2
5
10
20
50
100
200
Wavelength (um)


Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
0.10
Wave Max
6000.00
Temp (K)
300.00
New Plot
Add Plot
Save JPEG
300
Radiance (mW/m²/sr/cm⁻¹)
1.0e+003
1.0e+002
1.0e+001
1.0e+000
1.0e-001
1.0e-002
1.0e-003
0.1
1.0
10.0
100.0
1000.0
Wavenumber (cm⁻¹)

Solar (visible) and Earth emitted (infrared) energy



Incoming solar radiation (mostly visible) drives the earth-atmosphere (which emits infrared).

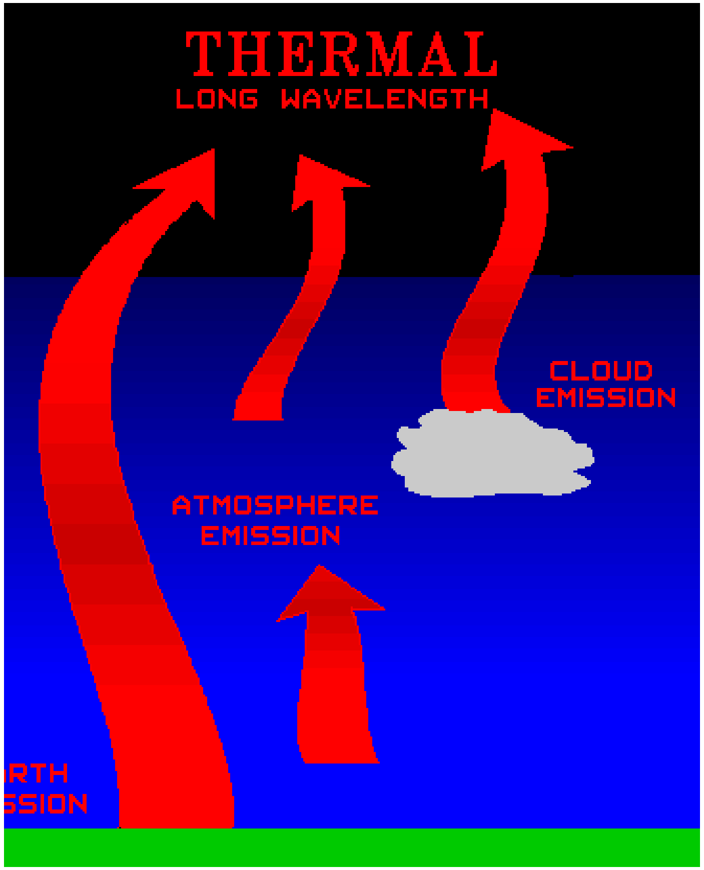
Over the annual cycle, the incoming solar energy that makes it to the earth surface (about 50 %) is balanced by the outgoing thermal infrared energy emitted through the atmosphere.

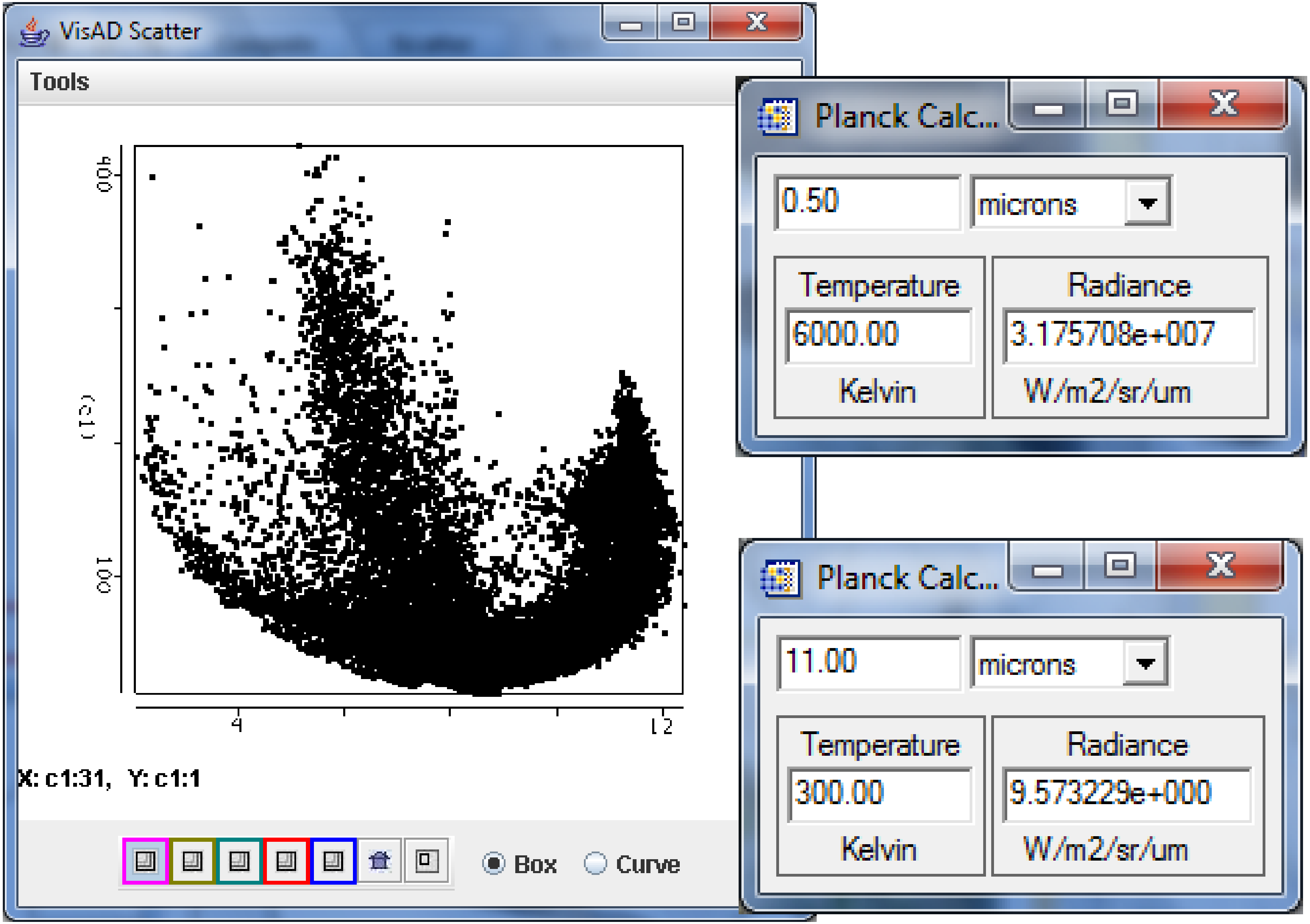
The atmosphere transmits, absorbs (by H2O, O2, O3, dust) reflects (by clouds), and scatters (by aerosols) incoming visible; the earth surface absorbs and reflects the transmitted visible. Atmospheric H2O, CO2, and O3 selectively transmit or absorb the outgoing infrared radiation. The outgoing microwave is primarily affected by H2O and O2.

Infrared
(Emissive Bands)

Radiative Transfer Equation

in the IR




VisAD Scatter
Tools
400
100
4
12
X: c1:31, Y: c1:1
Box
Curve
Planck Calc...
0.50
microns
Temperature
6000.00
Kelvin
Radiance
3.175708e+007
W/m2/sr/um
Planck Calc...
11.00
microns
Temperature
300.00
Kelvin
Radiance
9.573229e+000
W/m2/sr/um

Relevant Material in Applications of Meteorological Satellites

Emission, Absorption, Reflection, and Scattering

Blackbody radiation B_λ represents the upper limit to the amount of radiation that a real substance may emit at a given temperature for a given wavelength.

Emissivity ε_λ is defined as the fraction of emitted radiation R_λ to Blackbody radiation,

$$\varepsilon_\lambda = R_\lambda / B_\lambda \ .$$

In a medium at thermal equilibrium, what is absorbed is emitted (what goes in comes out) so

$$a_\lambda = \varepsilon_\lambda \ .$$

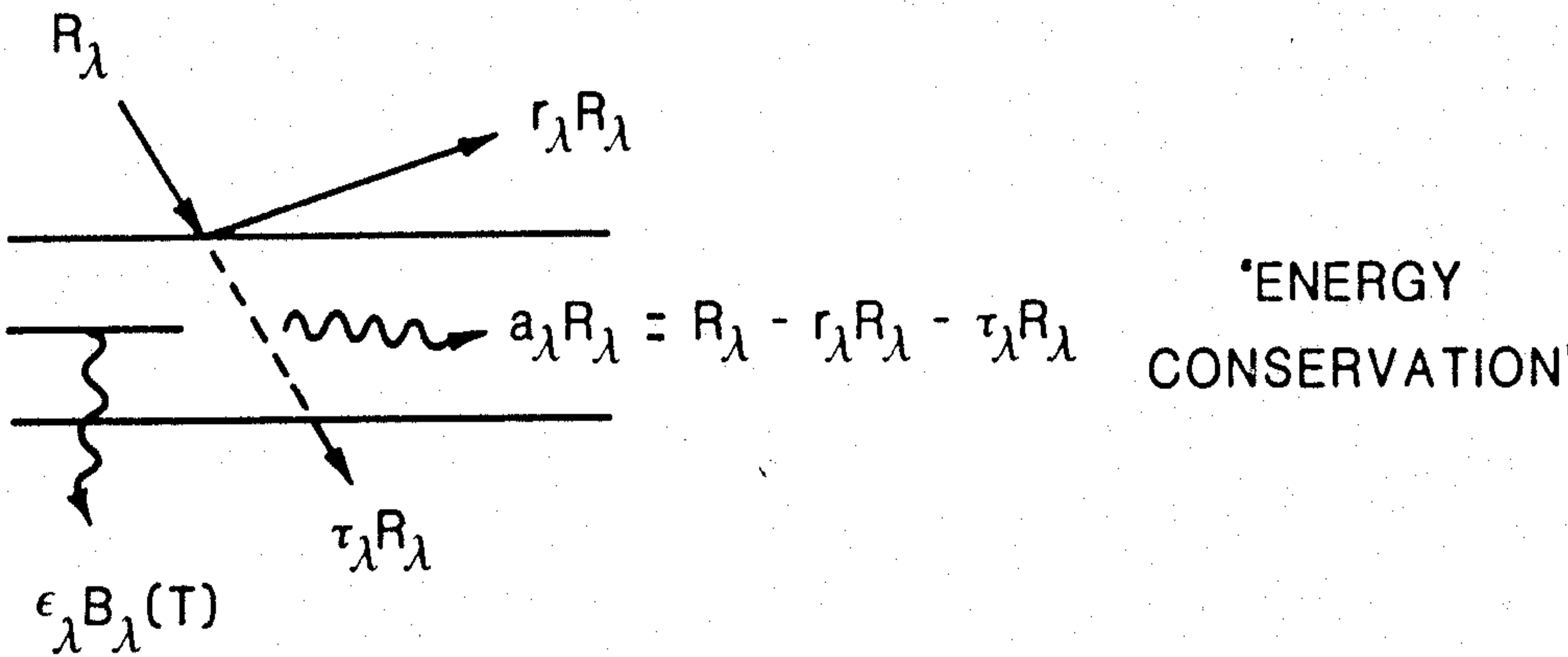
Thus, materials which are strong absorbers at a given wavelength are also strong emitters at that wavelength; similarly weak absorbers are weak emitters.

If a_λ, r_λ, and τ_λ represent the fractional absorption, reflectance, and transmittance, respectively, then conservation of energy says

$$a_\lambda + r_\lambda + \tau_\lambda = 1 \ .$$

For a blackbody $a_\lambda = 1$, it follows that $r_\lambda = 0$ and $\tau_\lambda = 0$ for blackbody radiation. Also, for a perfect window $\tau_\lambda = 1$, $a_\lambda = 0$ and $r_\lambda = 0$. For any opaque surface $\tau_\lambda = 0$, so radiation is either absorbed or reflected $a_\lambda + r_\lambda = 1$.

At any wavelength, strong reflectors are weak absorbers (i.e., snow at visible wavelengths), and weak reflectors are strong absorbers (i.e., asphalt at visible wavelengths).

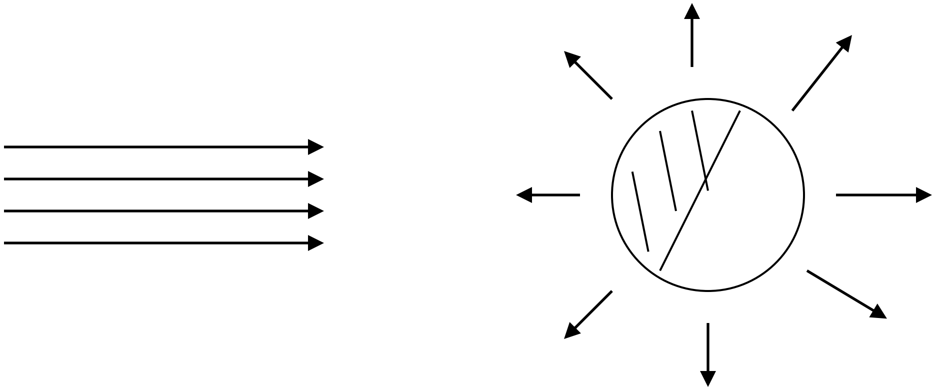

R_λ
$r_\lambda R_\lambda$
$a_\lambda R_\lambda = R_\lambda - r_\lambda R_\lambda - \tau_\lambda R_\lambda$
$\tau_\lambda R_\lambda$
$\epsilon_\lambda B_\lambda(T)$
"ENERGY CONSERVATION"

Planetary Albedo

Planetary albedo is defined as the fraction of the total incident solar irradiance, S, that is reflected back into space. Radiation balance then requires that the absorbed solar irradiance is given by

$$E = (1 - A) S/4.$$

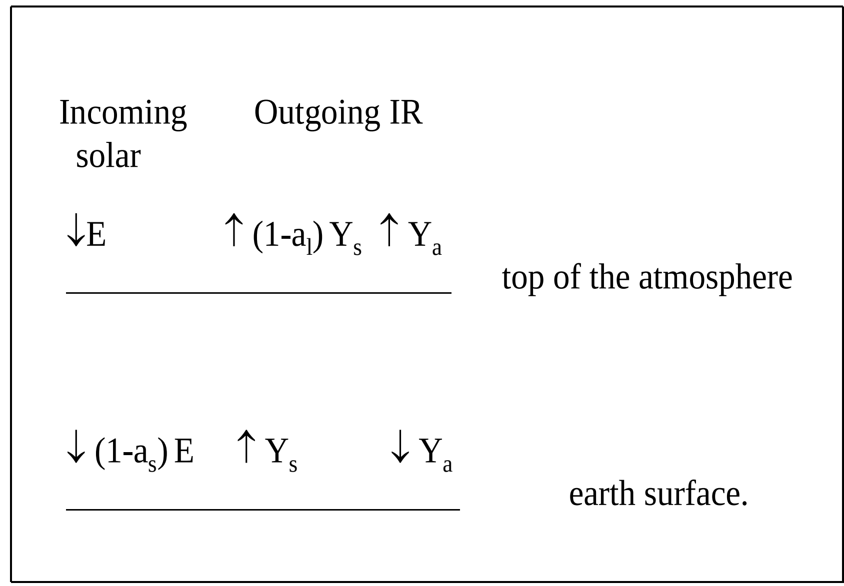
The factor of one-fourth arises because the cross sectional area of the earth disc to solar radiation, πr^2, is one-fourth the earth radiating surface, $4\pi r^2$. Thus recalling that $S = 1380\ Wm^{-2}$, if the earth albedo is 30 percent, then $E = 241\ Wm^{-2}$. For radiative equilibrium $E=\sigma T^4$ or $T=255K$



Selective Absorption and Transmission

Assume that the earth behaves like a blackbody and that the atmosphere has an absorptivity a_S for incoming solar radiation and a_L for outgoing longwave radiation. Let Y_a be the irradiance emitted by the atmosphere (both upward and downward); Y_s the irradiance emitted from the earth's surface; and E the solar irradiance absorbed by the earth-atmosphere system. Then, radiative equilibrium requires

$$E - (1-a_L)\, Y_s - Y_a = 0 \text{ , at top of atm,}$$
$$(1-a_S)\, E - Y_s + Y_a = 0 \text{ , at sfc.}$$



Solving yields

$$Y_s = \frac{(2-a_S)}{(2-a_L)} \, E \text{ , and}$$

$$Y_a = \frac{(2-a_L) - (1-a_L)(2-a_S)}{(2-a_L)} \, E \, .$$

Since $a_L > a_S$, the irradiance and hence the radiative equilibrium temperature at the earth surface is increased by the presence of the atmosphere. With $a_L = 0.8$ and $a_S = 0.1$ and $E = 241$ Wm^{-2}, Stefans Law yields a blackbody temperature at the surface of 286 K, in contrast to 255 K for an atmospheric absorptance independent of wavelength ($a_S = a_L$). The atmospheric temperature in this example is 245 K.

Transmittance

Transmission through an absorbing medium for a given wavelength is governed by the number of intervening absorbing molecules (path length u) and their absorbing power (k_λ) at that wavelength. Beer's law indicates that transmittance decays exponentially with increasing path length

$$\tau_\lambda (z \to \infty) = e^{-k_\lambda u(z)}$$

where the path length is given by $u(z) = \int_z^\infty \rho \, dz$.

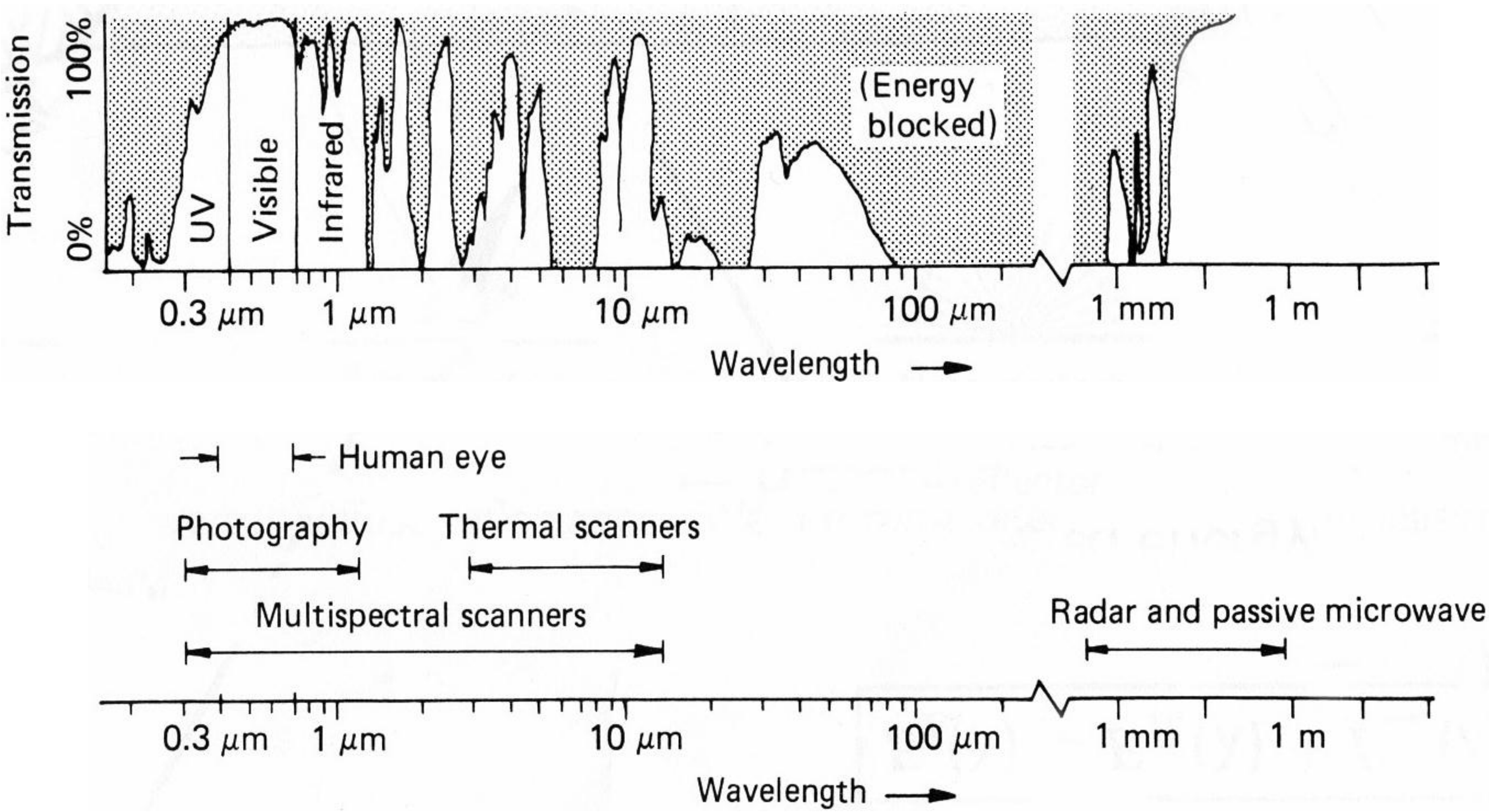
$k_\lambda u$ is a measure of the cumulative depletion that the beam of radiation has experienced as a result of its passage through the layer and is often called the optical depth σ_λ.

Realizing that the hydrostatic equation implies $g \rho \, dz = - q \, dp$

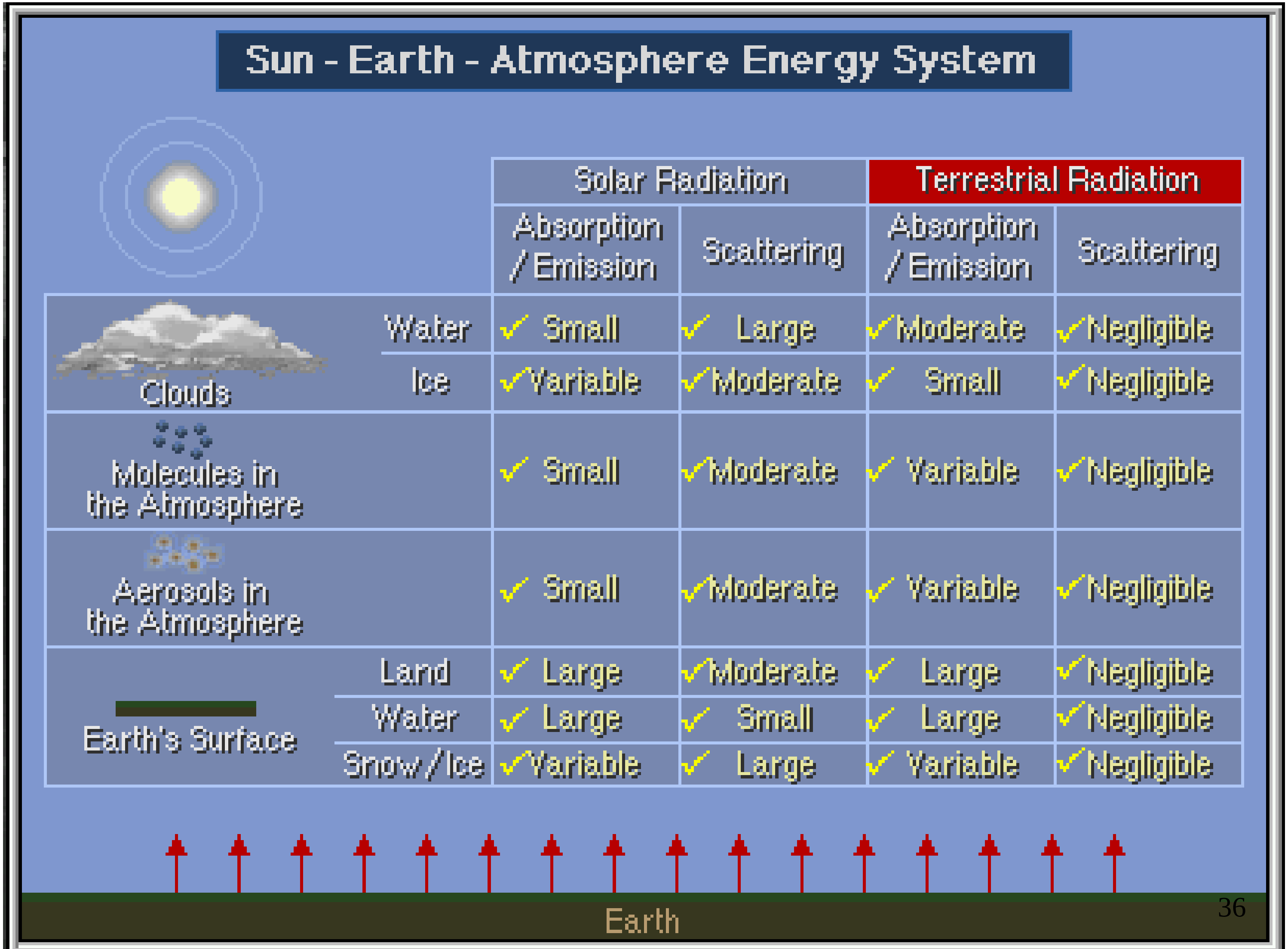
where q is the mixing ratio and ρ is the density of the atmosphere, then

$$u(p) = \int_o^p q \, g^{-1} \, dp \qquad \text{and} \qquad \tau_\lambda (p \to o) = e^{-k_\lambda u(p)} .$$

Spectral Characteristics of Atmospheric Transmission and Sensing Systems

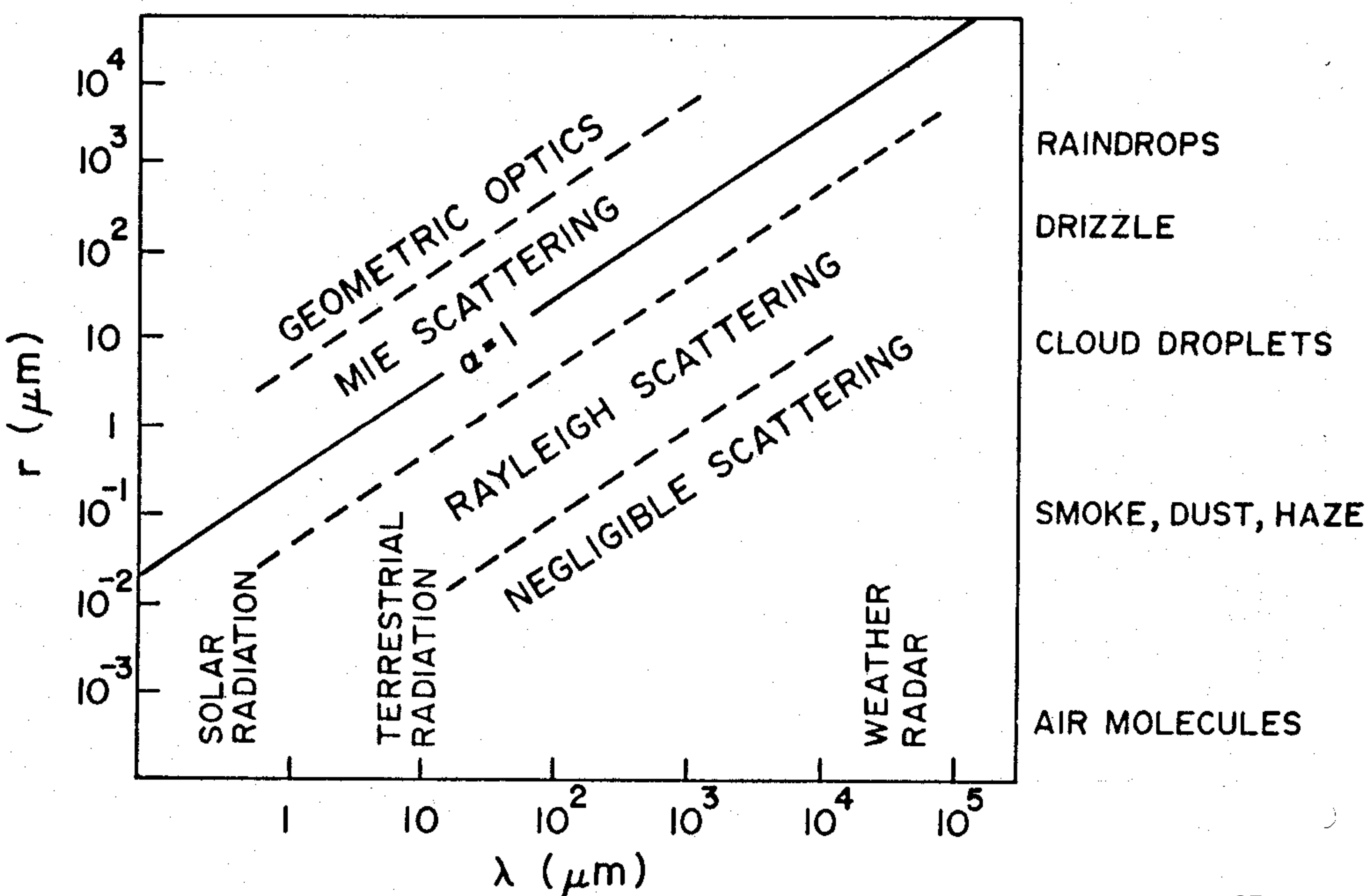


Relative Effects of Radiative Processes



Sun - Earth - Atmosphere Energy System

		Solar Radiation		Terrestrial Radiation	
		Absorption / Emission	Scattering	Absorption / Emission	Scattering
Clouds	Water	✓ Small	✓ Large	✓ Moderate	✓ Negligible
	Ice	✓ Variable	✓ Moderate	✓ Small	✓ Negligible
Molecules in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
Aerosols in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
Earth's Surface	Land	✓ Large	✓ Moderate	✓ Large	✓ Negligible
	Water	✓ Large	✓ Small	✓ Large	✓ Negligible
	Snow / Ice	✓ Variable	✓ Large	✓ Variable	✓ Negligible


GEOMETRIC OPTICS
MIE SCATTERING
α=1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES
r (μm)
λ (μm)
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
1
10
10^2
10^3
10^4
10^5

Aerosol Size Distribution

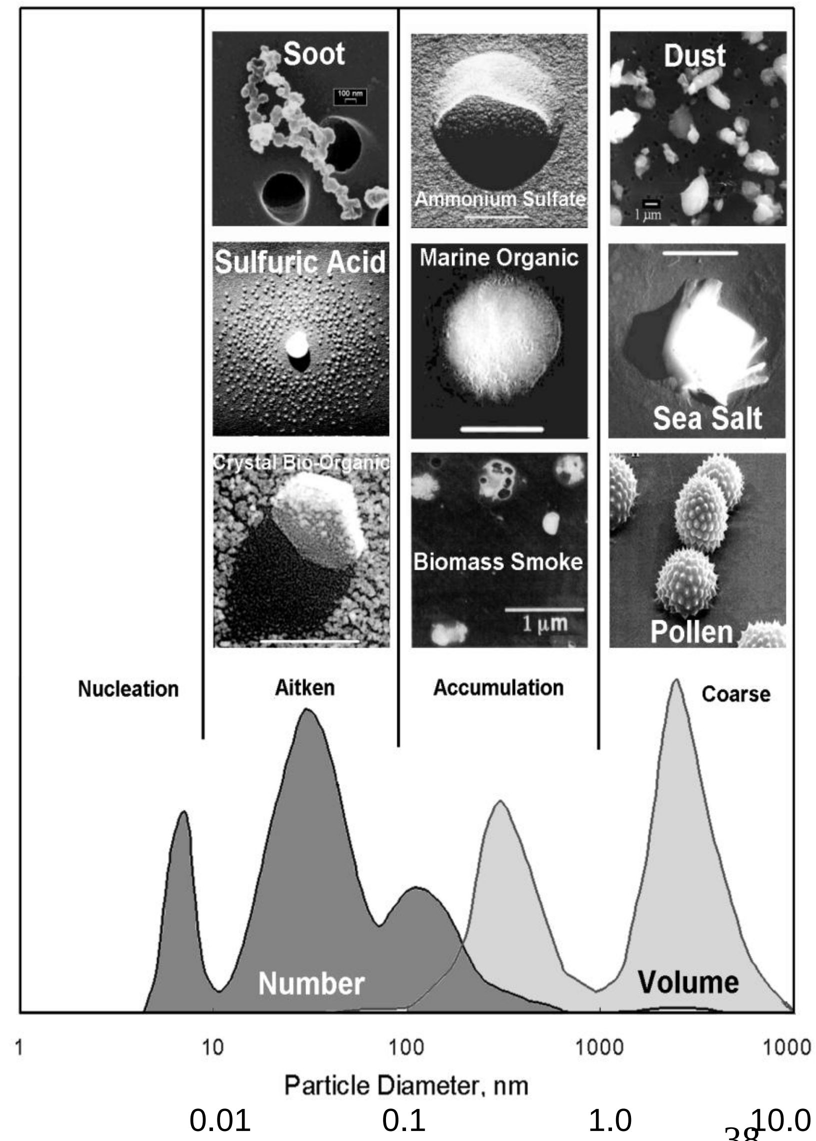
There are 3 modes :

- « **nucleation** »: radius is between 0.002 and 0.05 µm. They result from combustion processes, photo-chemical reactions, etc.

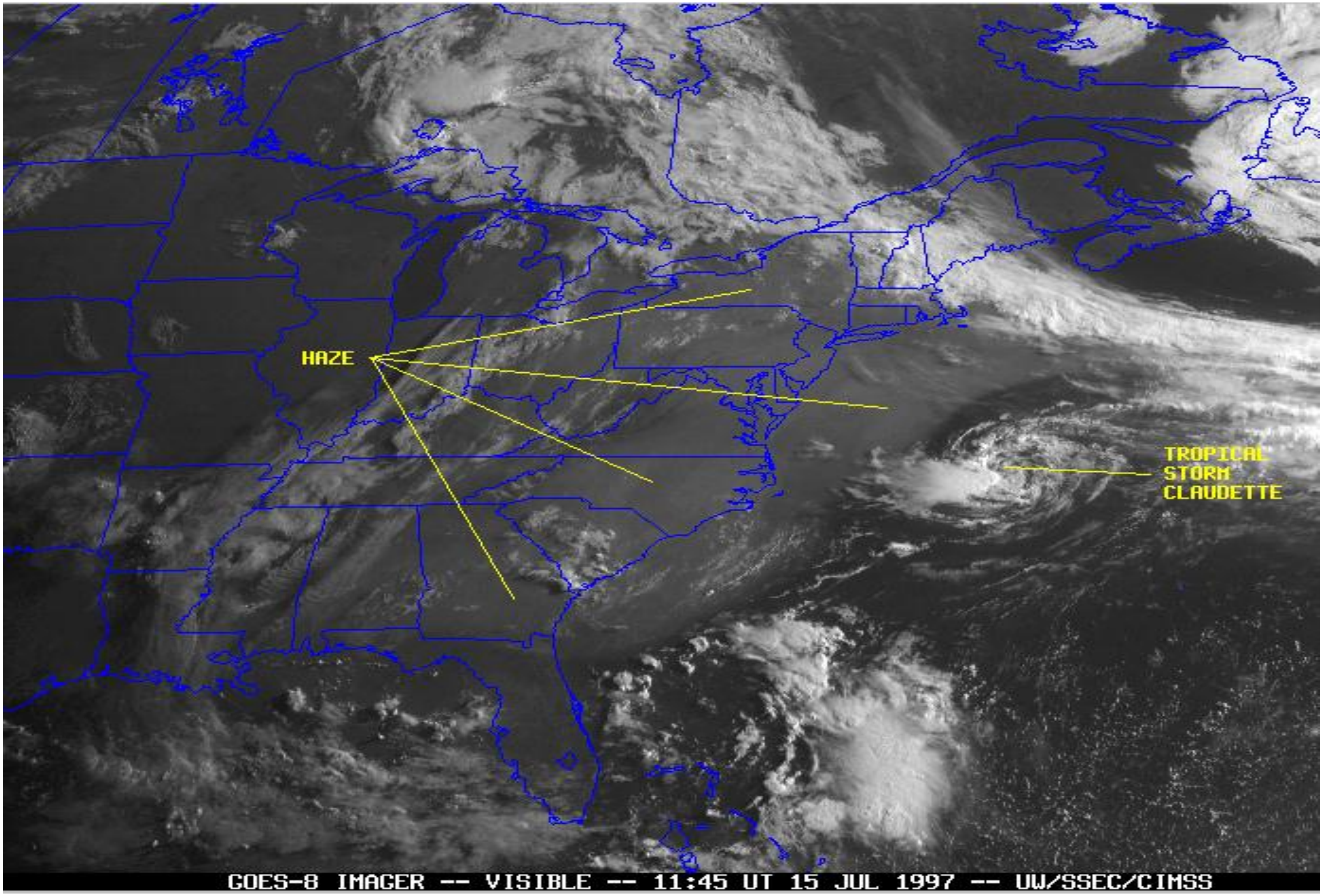
- « **accumulation »:** radius is between 0.05 µm and 0.5 µm. Coagulation processes.

- « **coarse** »: larger than 1 µm. From mechanical processes like aeolian erosion.

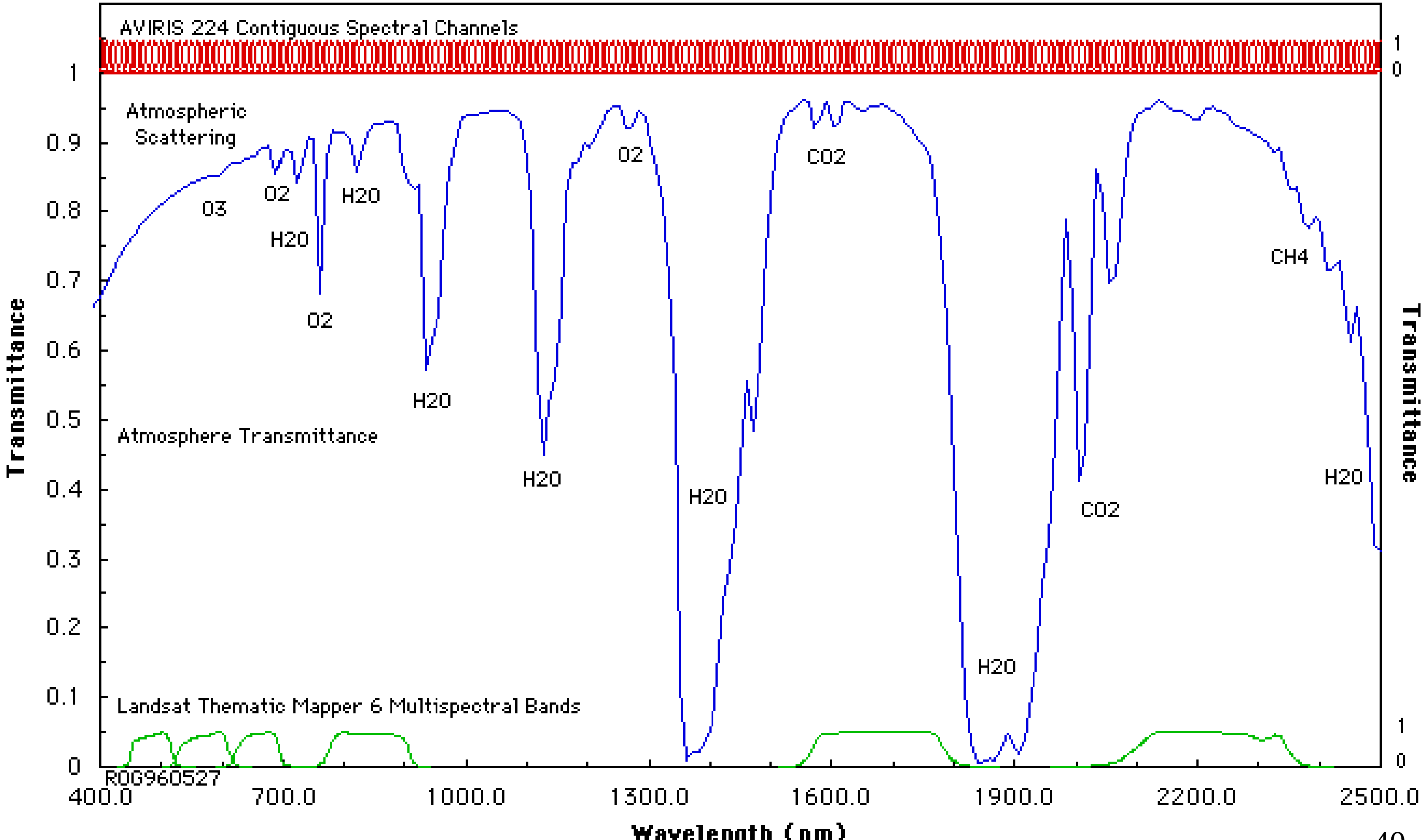
« fine » particles (nucleation and accumulation) result from anthropogenic activities, coarse particles come from natural processes.



Scattering of early morning sun light from haze

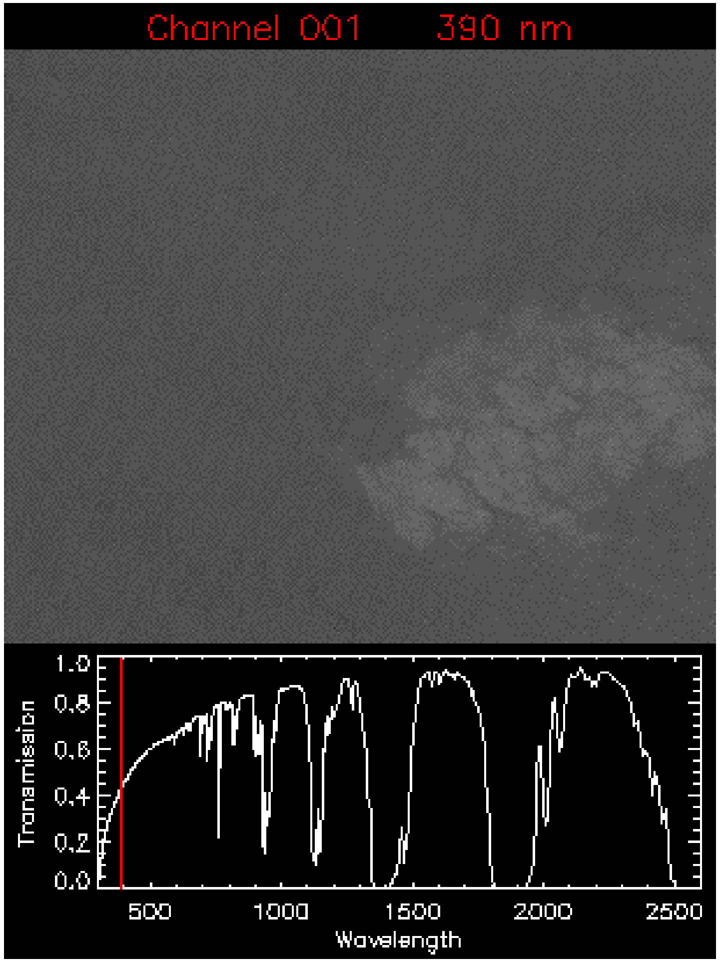


Measurements in the Solar Reflected Spectrum across the region covered by AVIRIS



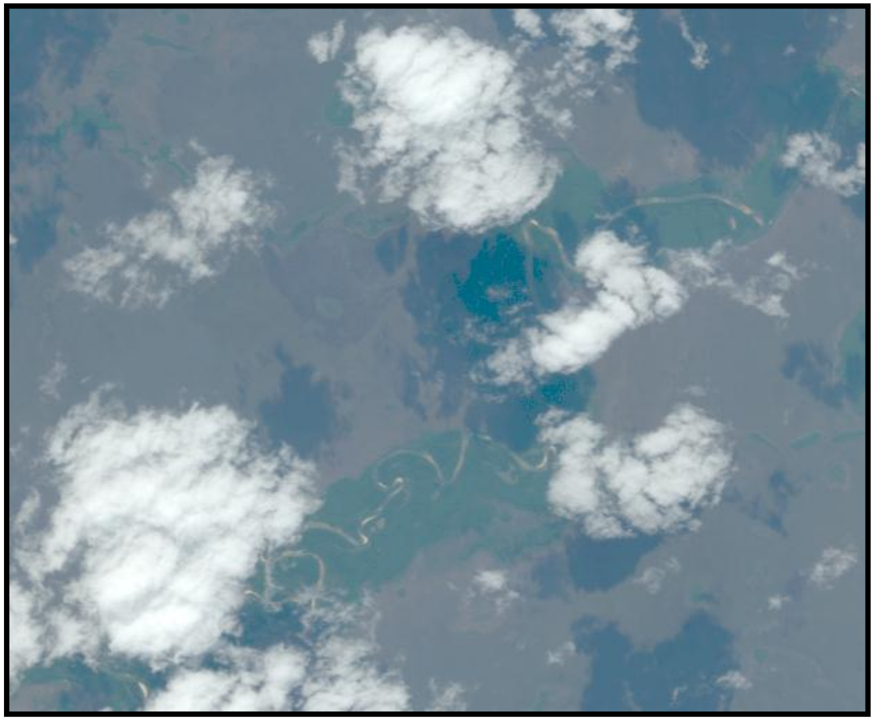
AVIRIS Movie #1

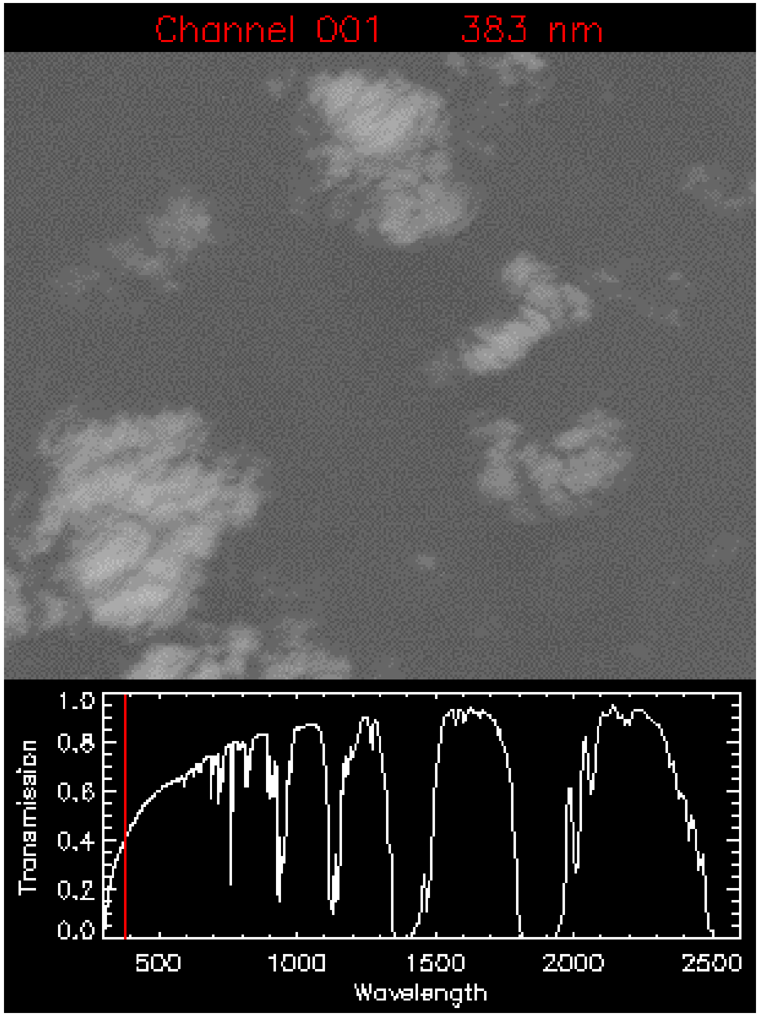
AVIRIS Image - Linden CA 20-Aug-1992
224 Spectral Bands: 0.4 - 2.5 μm
Pixel: 20m x 20m Scene: 10km x 10km



AVIRIS Movie #2

AVIRIS Image - Porto Nacional, Brazil
20-Aug-1995
224 Spectral Bands: 0.4 - 2.5 μm
Pixel: 20m x 20m Scene: 10km x 10km





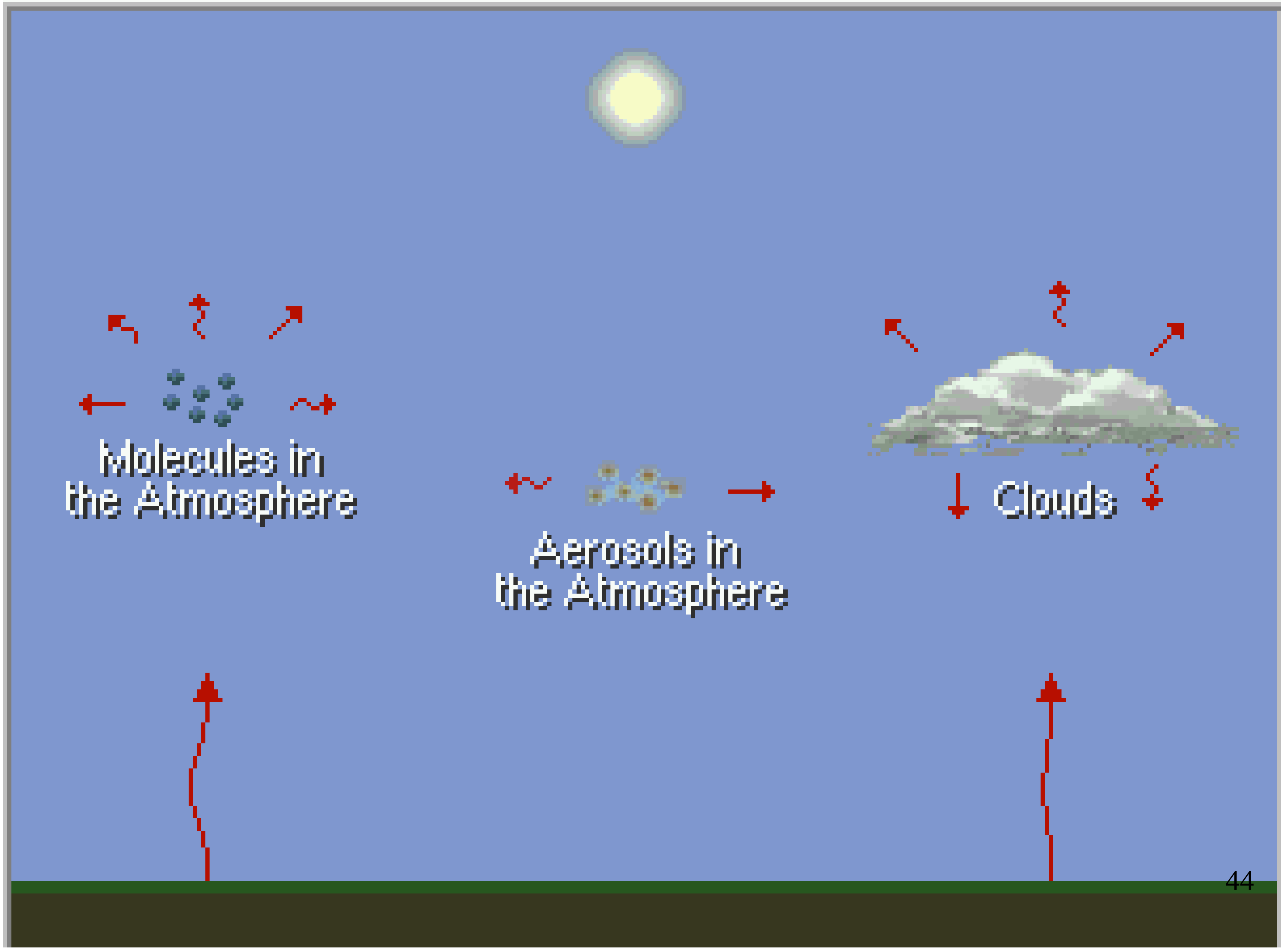
Relevant Material in Applications of Meteorological Satellites

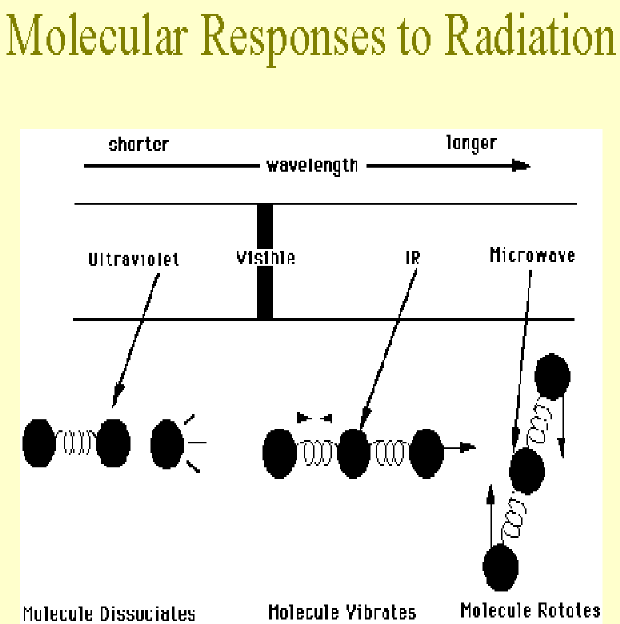
CHAPTER 2 - NATURE OF RADIATION

CHAPTER 3 - ABSORPTION, EMISSION, REFLECTION, AND SCATTERING

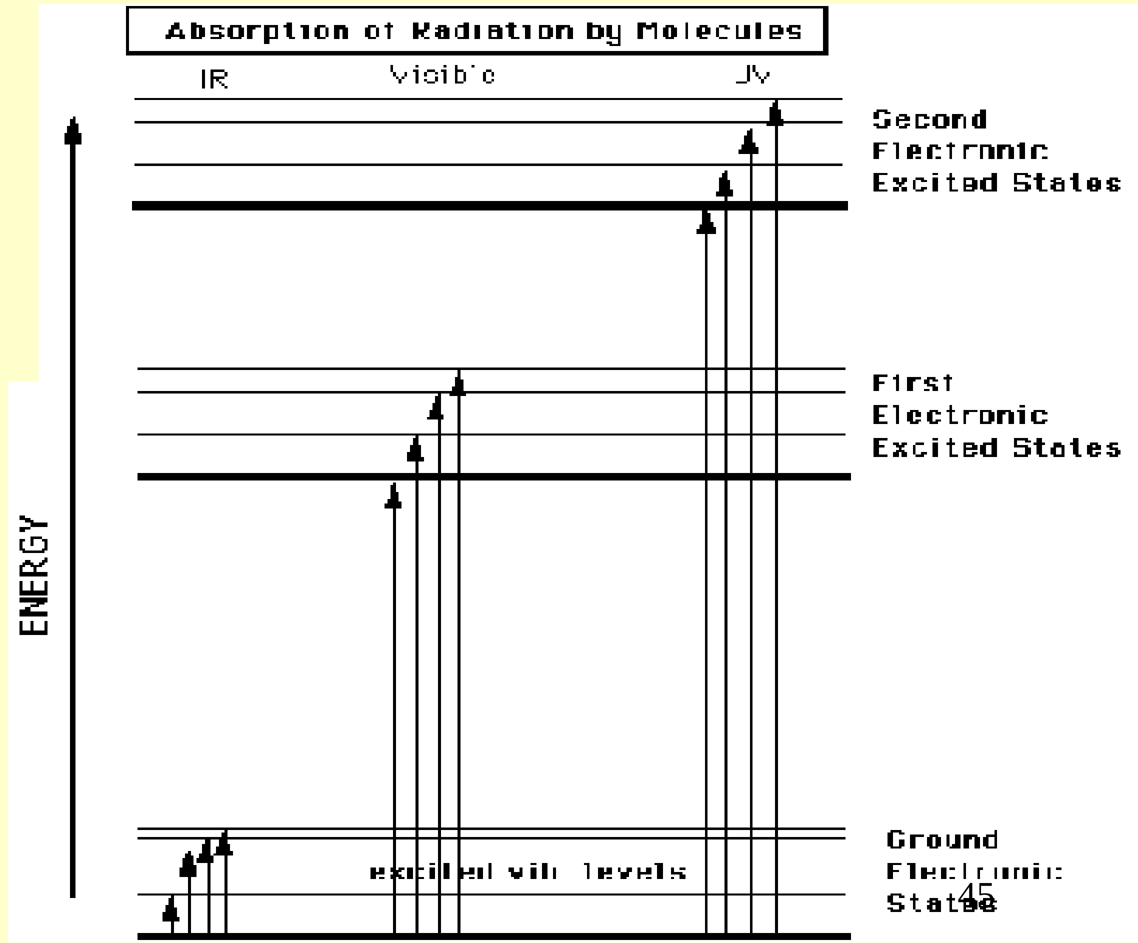
→ CHAPTER 5 - THE RADIATIVE TRANSFER EQUATION (RTE)

Re-emission of Infrared Radiation





Molecular absorption of IR by vibrational and rotational excitation



CO_2, H_2O, and O_3

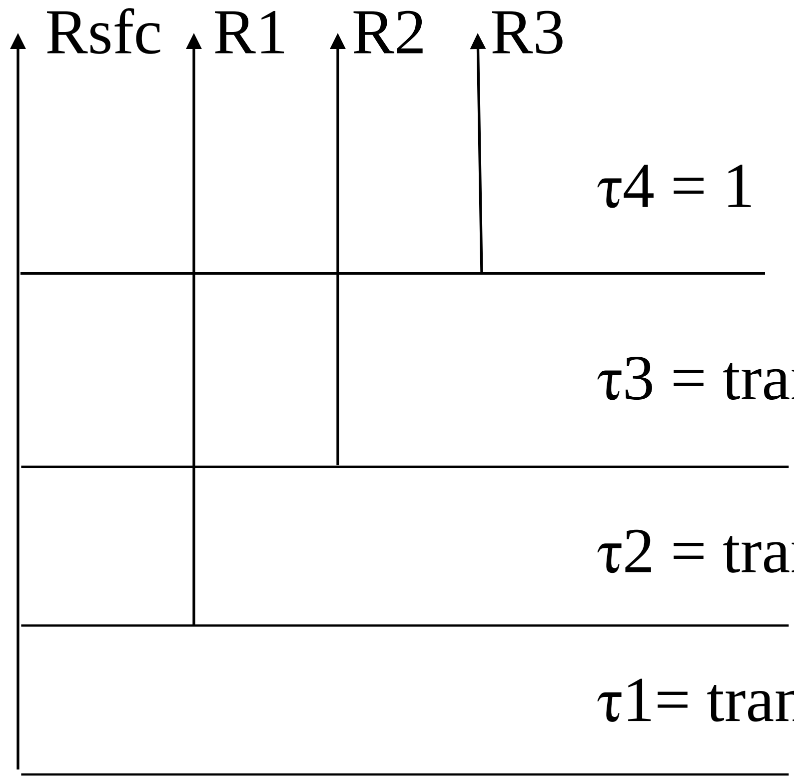
Radiative Transfer Equation

The radiance leaving the earth-atmosphere system sensed by a satellite borne radiometer is the sum of radiation emissions from the earth-surface and each atmospheric level that are transmitted to the top of the atmosphere. Considering the earth's surface to be a blackbody emitter (emissivity equal to unity), the upwelling radiance intensity, I_λ, for a cloudless atmosphere is given by the expression

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T_{sfc})\, \tau_\lambda(sfc - top) + \sum_{layers} \varepsilon_\lambda^{layer} B_\lambda(T_{layer})\, \tau_\lambda(layer - top)$$

where the first term is the surface contribution and the second term is the atmospheric contribution to the radiance to space.

Satellite observation comes from the sfc and the layers in the atm



$\tau 4 = 1$

$\tau 3$ = transmittance of upper layer of atm

$\tau 2$ = transmittance of middle layer of atm

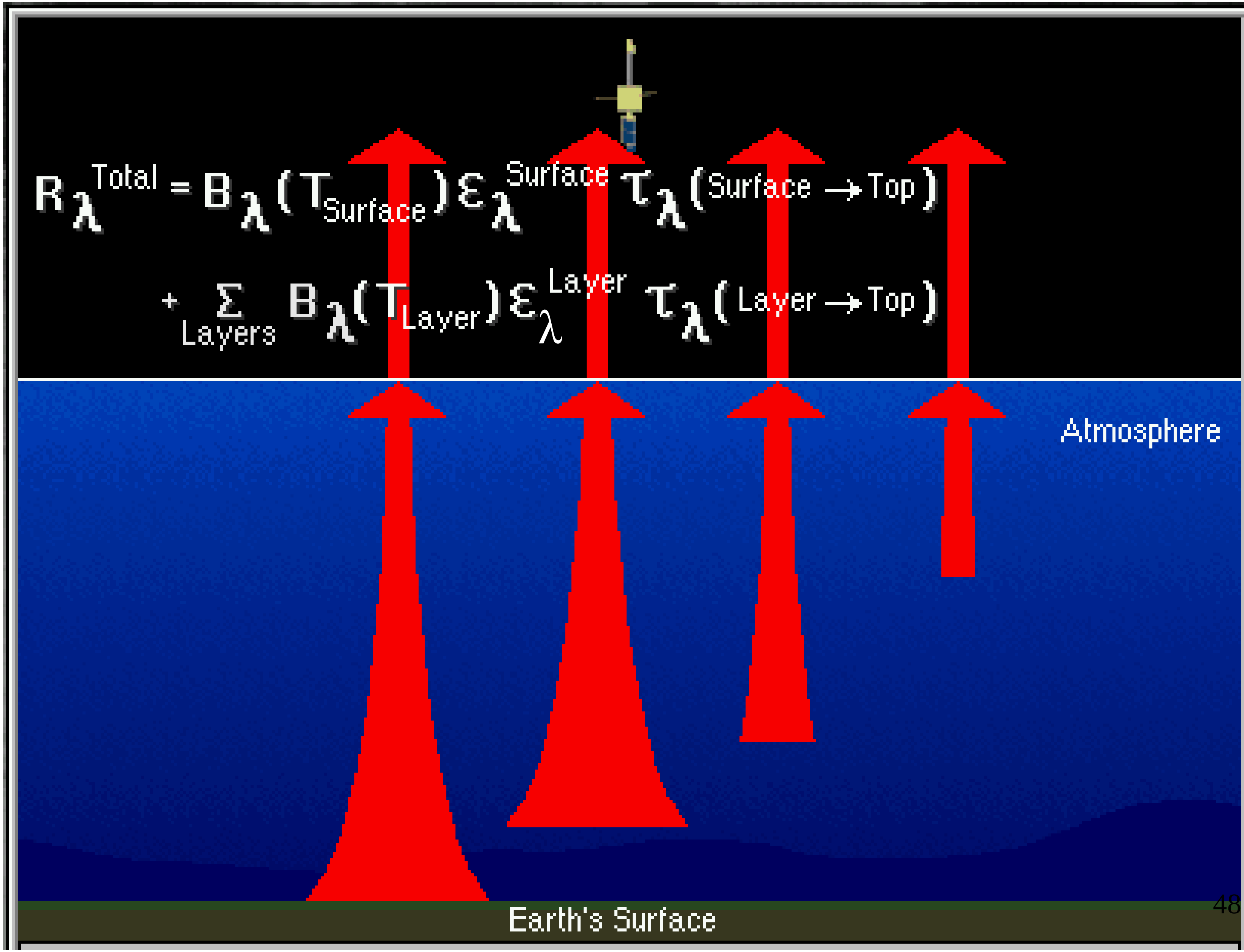
$\tau 1$= transmittance of lower layer of atm

εsfc for earth surface

recalling that $\varepsilon i = 1 - \tau i$ for each layer, then

$$Robs = \varepsilon sfc \; Bsfc \; \tau 1 \; \tau 2 \; \tau 3 + (1-\tau 1) \; B1 \; \tau 2 \; \tau 3 + (1- \tau 2) \; B2 \; \tau 3 + (1- \tau 3) \; B3$$

Radiative Transfer through the Atmosphere



The emissivity of an infinitesimal layer of the atmosphere at pressure p is equal to the absorptance (one minus the transmittance of the layer). Consequently,

$$\varepsilon_\lambda(\text{layer})\ \tau_\lambda(\text{layer to top}) = [1 - \tau_\lambda(\text{layer})]\ \tau_\lambda(\text{layer to top})$$

Since transmittance is multiplicative

$$\tau_\lambda(\text{layer to top}) - \tau_\lambda(\text{layer})\ \tau_\lambda(\text{layer to top}) = -\Delta\tau_\lambda(\text{layer to top})$$

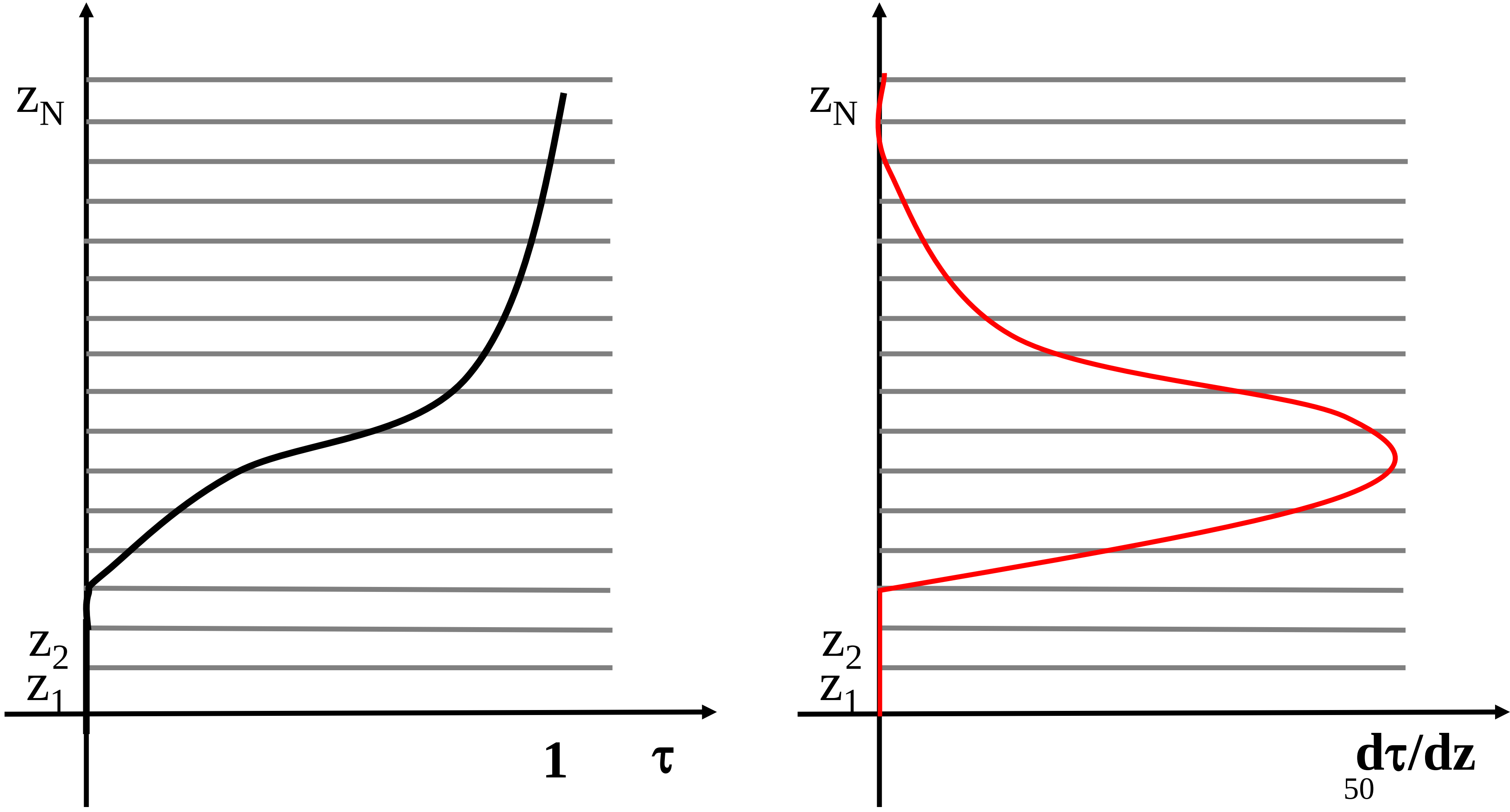
So we can write

$$I_\lambda = \varepsilon_\lambda^{sfc}\ B_\lambda(T(p_s))\ \tau_\lambda(p_s) - \sum_p B_\lambda(T(p))\ \Delta\tau_\lambda(p)\ .$$

which when written in integral form reads

$$I_\lambda = \varepsilon_\lambda^{sfc}\ B_\lambda(T(p_s))\ \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p))\ [\ d\tau_\lambda(p) / dp\ ]\ dp\ .$$

Weighting Functions



In standard notation,

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) + \sum_p \varepsilon_\lambda(\Delta p) B_\lambda(T(p)) \tau_\lambda(p)$$

The emissivity of an infinitesimal layer of the atmosphere at pressure p is equal to the absorptance (one minus the transmittance of the layer). Consequently,

$$\varepsilon_\lambda(\Delta p) \tau_\lambda(p) = [1 - \tau_\lambda(\Delta p)] \tau_\lambda(p)$$

Since transmittance is an exponential function of depth of absorbing constituent,

$$\tau_\lambda(\Delta p) \tau_\lambda(p) = \exp [- \int_p^{p+\Delta p} k_\lambda q g^{-1} dp] * \exp [- \int_o^p k_\lambda q g^{-1} dp] = \tau_\lambda(p + \Delta p)$$

Therefore

$$\varepsilon_\lambda(\Delta p) \tau_\lambda(p) = \tau_\lambda(p) - \tau_\lambda(p + \Delta p) = - \Delta\tau_\lambda(p) .$$

So we can write

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) - \sum_p B_\lambda(T(p)) \Delta\tau_\lambda(p) .$$

which when written in integral form reads

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p)) [d\tau_\lambda(p) / dp] dp .$$

To investigate the RTE further consider the atmospheric contribution to the radiance to space of an infinitesimal layer of the atmosphere at height z, $dI_\lambda(z) = B_\lambda(T(z))\, d\tau_\lambda(z)$.

Assume a well-mixed isothermal atmosphere where the density drops off exponentially with height $\rho = \rho_o \exp(-\gamma z)$, and assume k_λ is independent of height, so that the optical depth can be written for normal incidence

$$\sigma_\lambda = \int_z^\infty k_\lambda \rho \, dz = \gamma^{-1} k_\lambda \rho_o \exp(-\gamma z)$$

and the derivative with respect to height

$$\frac{d\sigma_\lambda}{dz} = -k_\lambda \rho_o \exp(-\gamma z) = -\gamma \sigma_\lambda \, .$$

Therefore, we may obtain an expression for the detected radiance per unit thickness of the layer as a function of optical depth,

$$\frac{dI_\lambda(z)}{dz} = B_\lambda(T_{const}) \frac{d\tau_\lambda(z)}{dz} = B_\lambda(T_{const})\, \gamma\, \sigma_\lambda \exp(-\sigma_\lambda) \; .$$

The level which is emitting the most detected radiance is given by

$$\frac{d}{dz}\left\{\frac{dI_\lambda(z)}{dz}\right\} = 0 \text{ , or where } \sigma_\lambda = 1.$$

Most of monochromatic radiance detected is emitted by layers near level of unit optical depth.

When reflection from the earth surface is also considered, the Radiative Transfer Equation for infrared radiation can be written

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T_s)\, \tau_\lambda(p_s) + \int_{p_s}^{o} B_\lambda(T(p))\, F_\lambda(p)\, [d\tau_\lambda(p)/\, dp]\, dp$$

where

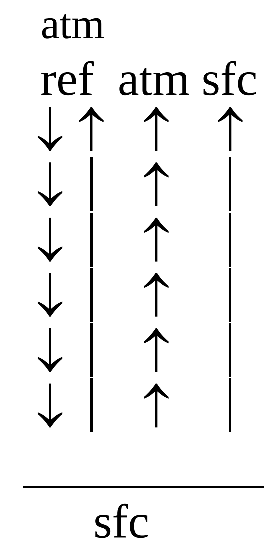
$$F_\lambda(p) = \{ 1 + (1 - \varepsilon_\lambda)\, [\tau_\lambda(p_s) / \tau_\lambda(p)]^2 \}$$

The first term is the spectral radiance emitted by the surface and attenuated by the atmosphere, often called the boundary term and the second term is the spectral radiance emitted to space by the atmosphere directly or by reflection from the earth surface.

The atmospheric contribution is the weighted sum of the Planck radiance contribution from each layer, where the weighting function is [$d\tau_\lambda(p)$ / dp]. This weighting function is an indication of where in the atmosphere the majority of the radiation for a given spectral band comes from.

Including surface emissivity

$$I^{sfc}_{\lambda} = \varepsilon_\lambda B_\lambda(T_s)\, \tau_\lambda(p_s) + (1-\varepsilon_\lambda)\, \tau_\lambda(p_s) \int_{o}^{p_s} B_\lambda(T(p))\, \frac{\partial \tau'_\lambda(p)}{\partial \ln p}\, d \ln p$$



$$I_\lambda = \varepsilon_\lambda B_\lambda(T_s)\, \tau_\lambda(p_s) + (1-\varepsilon_\lambda)\, \tau_\lambda(p_s) \int_{o}^{p_s} B_\lambda(T(p))\, \frac{\partial \tau'_\lambda(p)}{\partial \ln p}\, d \ln p$$

$$+ \int_{p_s}^{o} B_\lambda(T(p))\, \frac{\partial \tau_\lambda(p)}{\partial \ln p}\, d \ln p$$

using $\tau'_\lambda(p) = \tau_\lambda(p_s) / \tau_\lambda(p)$ then.

$$\frac{\partial \tau'_\lambda(p)}{\partial \ln p} = - \frac{\tau_\lambda(p_s)}{(\tau_\lambda(p))^2}\, \frac{\partial \tau_\lambda(p)}{\partial \ln p}$$

Thus

$$I_\lambda = \varepsilon_\lambda B_\lambda(T(p_s))\, \tau_\lambda(p_s) + \int_{p_s}^{o} B(T(p))\, F_\lambda(p)\, \frac{\partial \tau_\lambda(p)}{\partial \ln p}\, d \ln p$$

where

$$F_\lambda(p) = \left\{ 1 + (1 - \varepsilon_\lambda) \left[\frac{\tau_\lambda(p_s)}{\tau_\lambda(p)}\right]^2 \right\} .$$

The transmittance to the surface can be expressed in terms of transmittance to the top of the atmosphere by remembering

$$\tau'_\lambda(p) = \exp\left[-\frac{1}{g} \int_p^{p_s} k_\lambda(p)\, g(p)\, dp \right]$$

$$= \exp\left[-\int_o^{p_s} + \int_o^{p} \right]$$

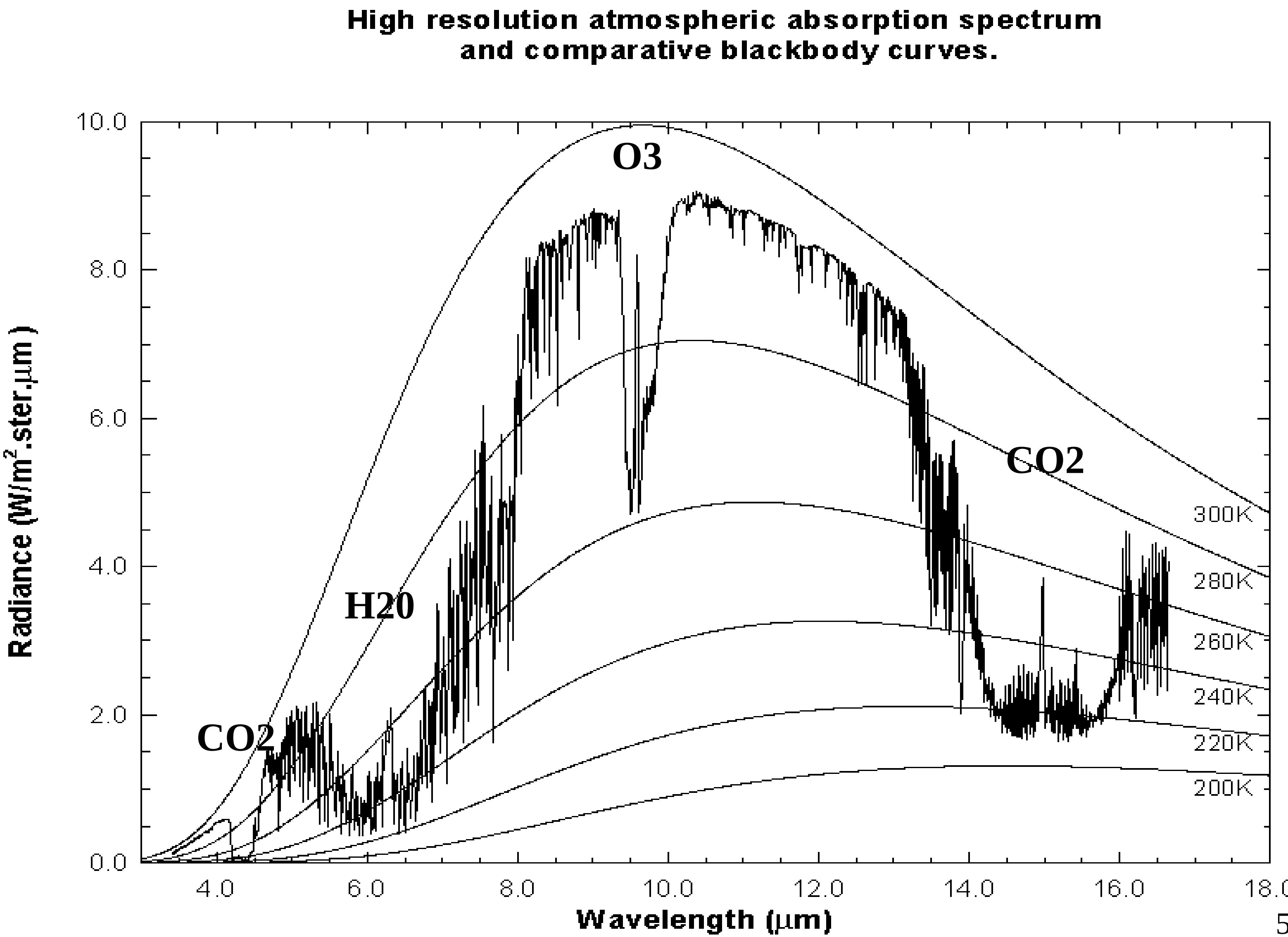
$$= \tau_\lambda(p_s) / \tau_\lambda(p) \; .$$

So

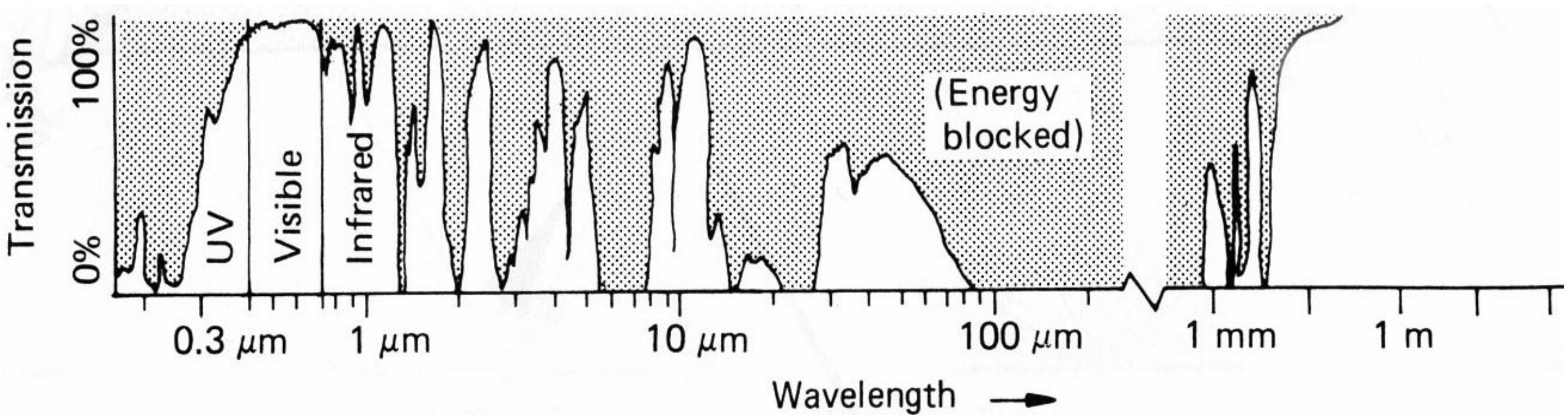
$$\frac{\partial \tau'_\lambda(p)}{\partial \ln p} = - \frac{\tau_\lambda(p_s)}{(\tau_\lambda(p))^2} \frac{\partial \tau_\lambda(p)}{\partial \ln p} \; .$$

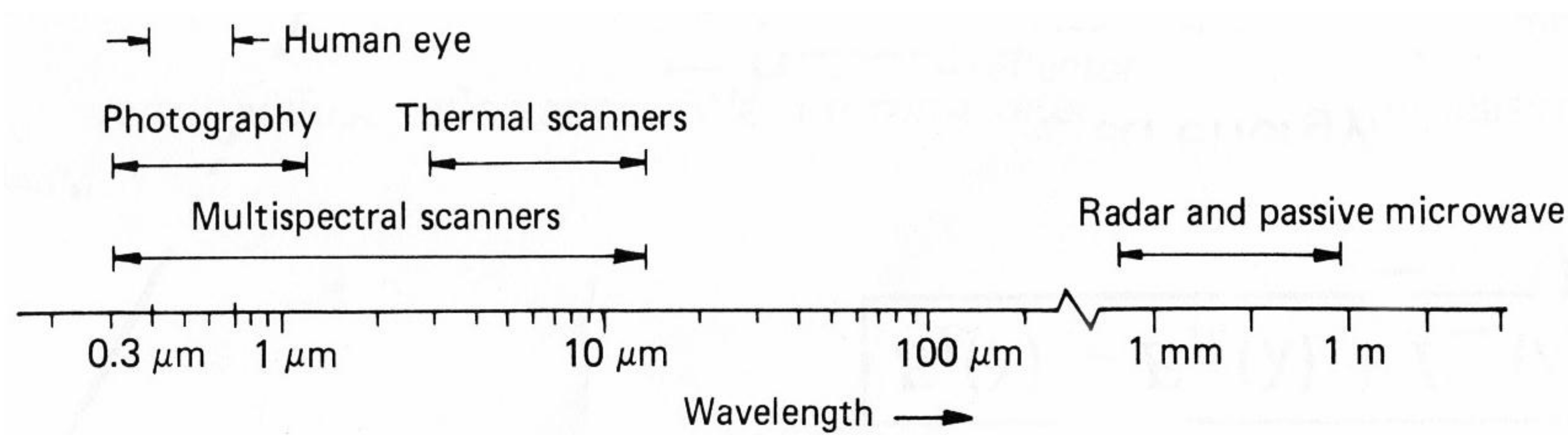
[remember that $\tau_\lambda(p_s, p)\, \tau_\lambda(p, 0) = \tau_\lambda(p_s, 0)$ and $\tau_\lambda(p_s, p) = \tau_\lambda(p, p_s)$]

Earth emitted spectra overlaid on Planck function envelopes

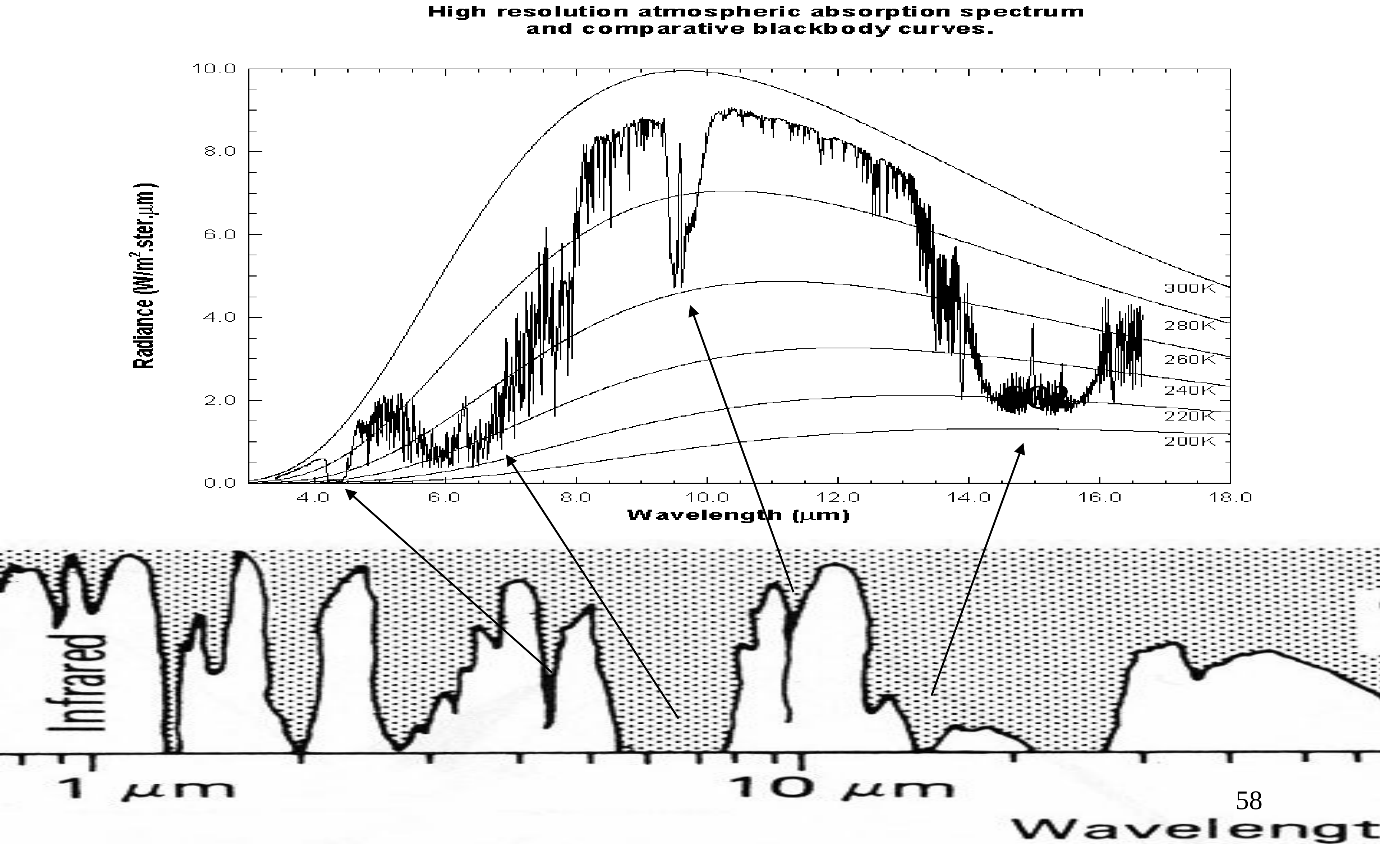


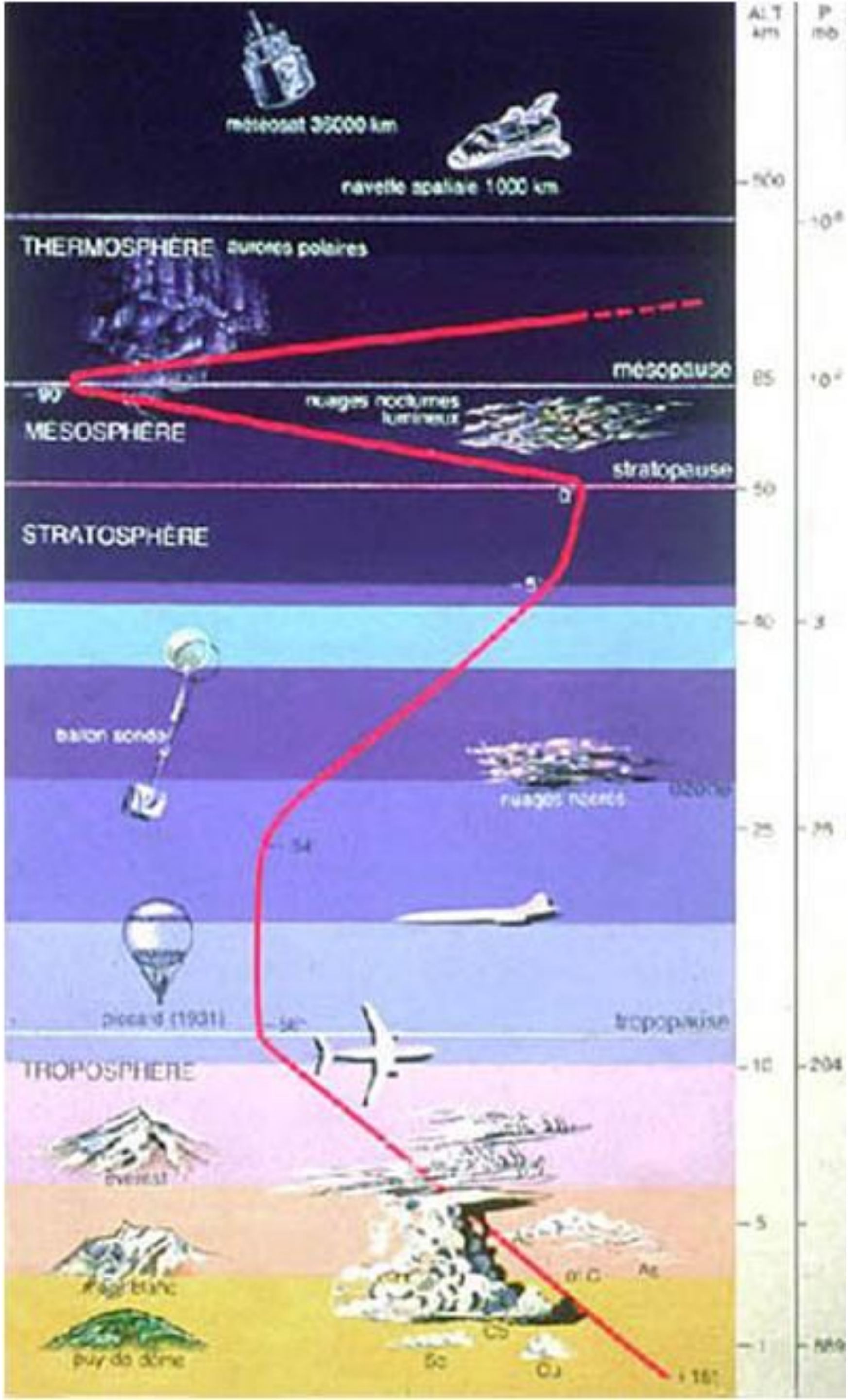
Spectral Characteristics of Atmospheric Transmission and Sensing Systems

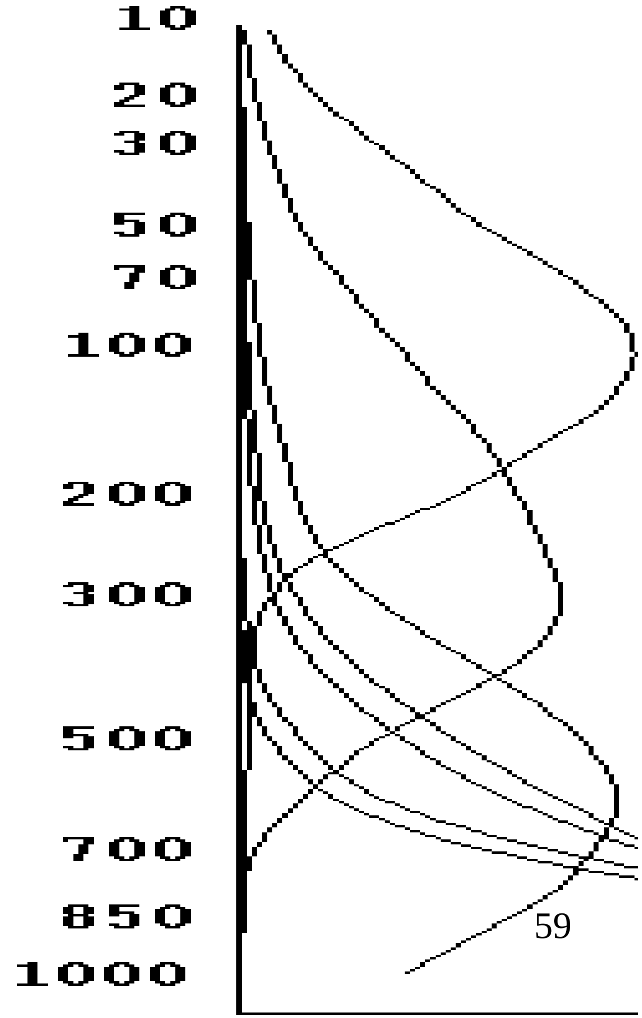




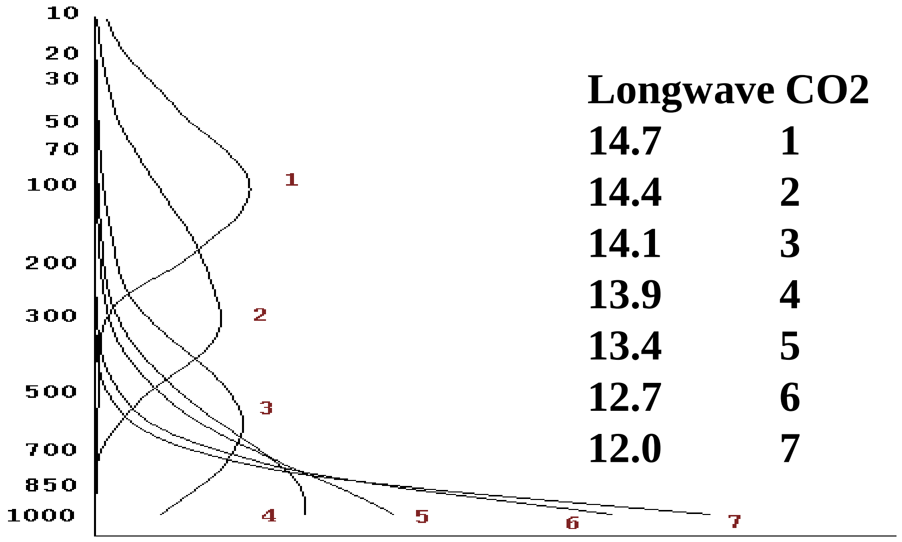
Earth emitted spectra overlaid on Planck function envelopes



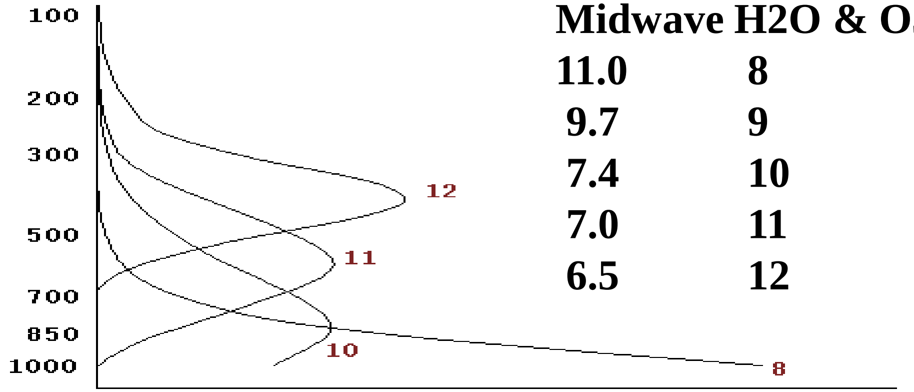

ALT km
P mb
THERMOSPHÈRE
aurores polaires
mésopause
MÉSOSPHÈRE
stratopause
STRATOSPHÈRE
ballon sonde
tropopause
TROPOSPHÈRE
Everest
puy de dôme
500
85
50
40
25
10
5
1


10
20
30
50
70
100
200
300
500
700
850
1000

Weighting Functions

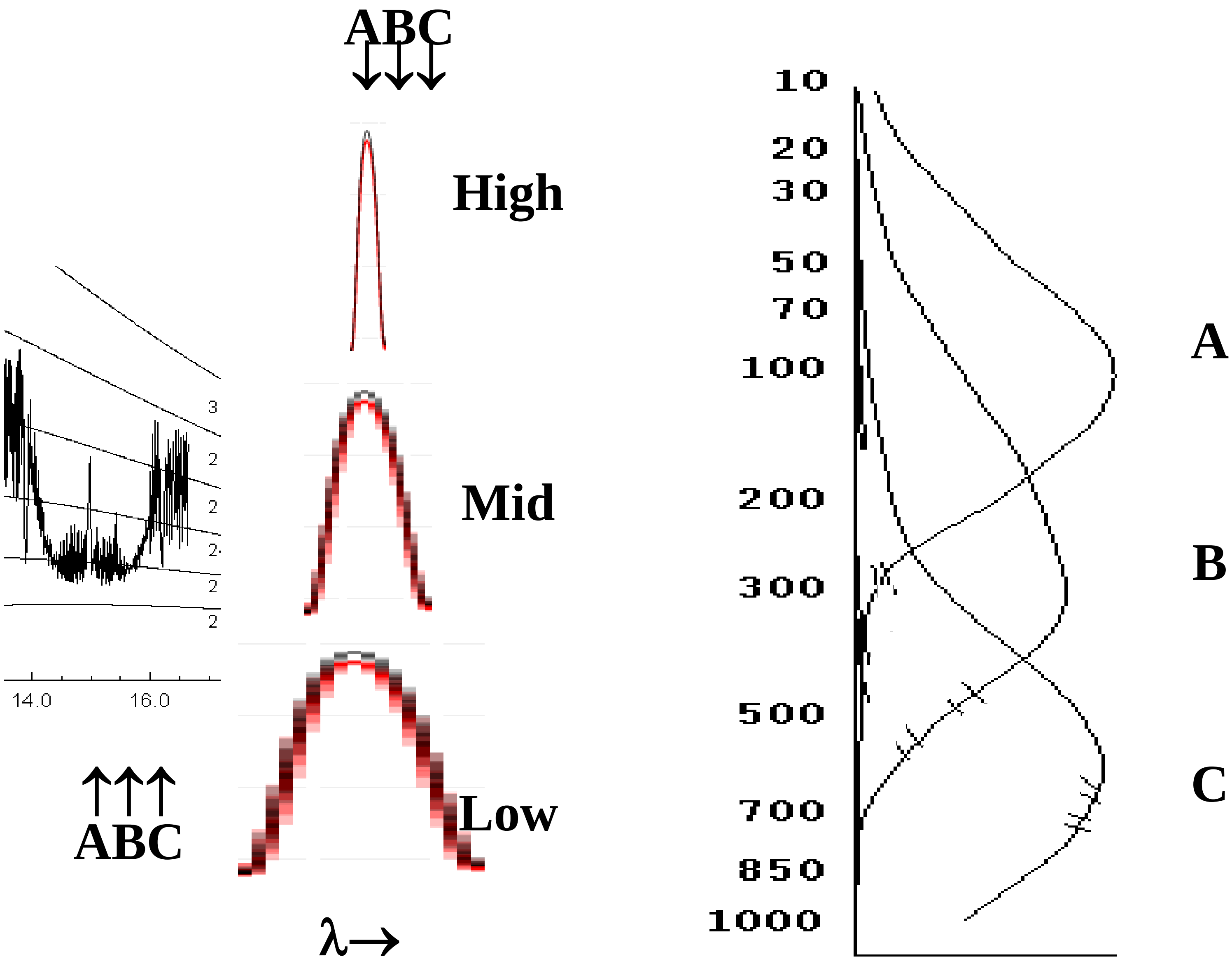


Longwave CO_2			
14.7	1	680	CO_2, strat temp
14.4	2	696	CO_2, strat temp
14.1	3	711	CO_2, upper trop temp
13.9	4	733	CO_2, mid trop temp
13.4	5	748	CO_2, lower trop temp
12.7	6	790	H_2O, lower trop moisture
12.0	7	832	H_2O, dirty window

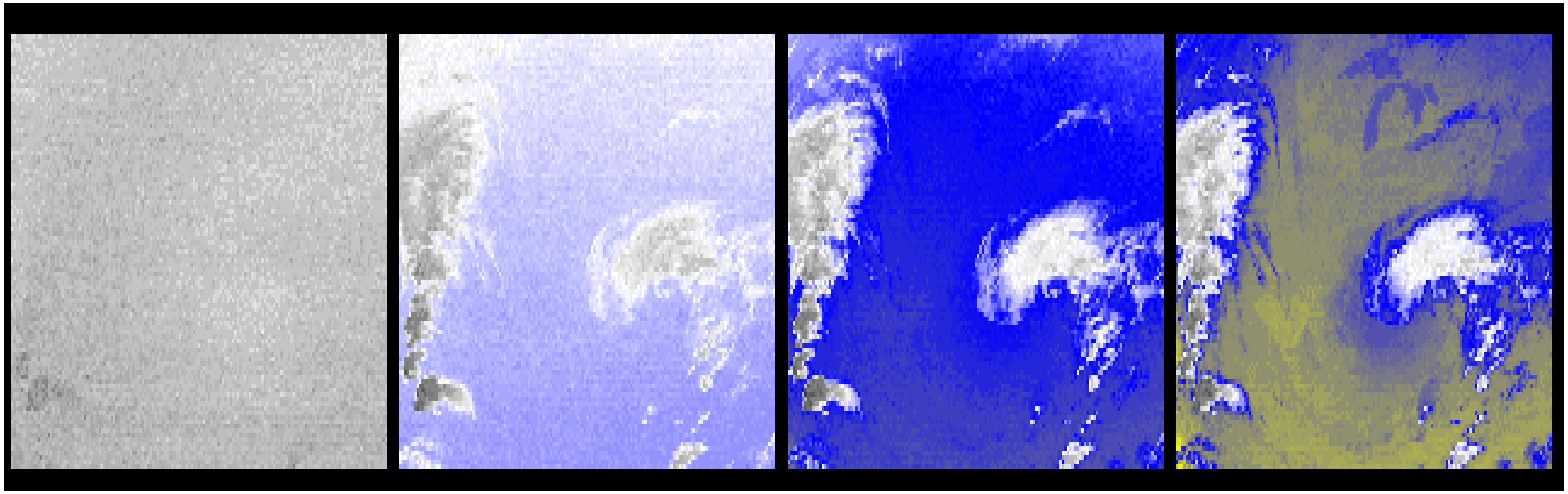


Midwave H_2O & O_3			
11.0	8	907	window
9.7	9	1030	O_3, strat ozone
7.4	10	1345	H_2O, lower mid trop moisture
7.0	11	1425	H_2O, mid trop moisture
6.5	12	1535	H_2O, upper trop moisture

line broadening with pressure helps to explain weighting functions



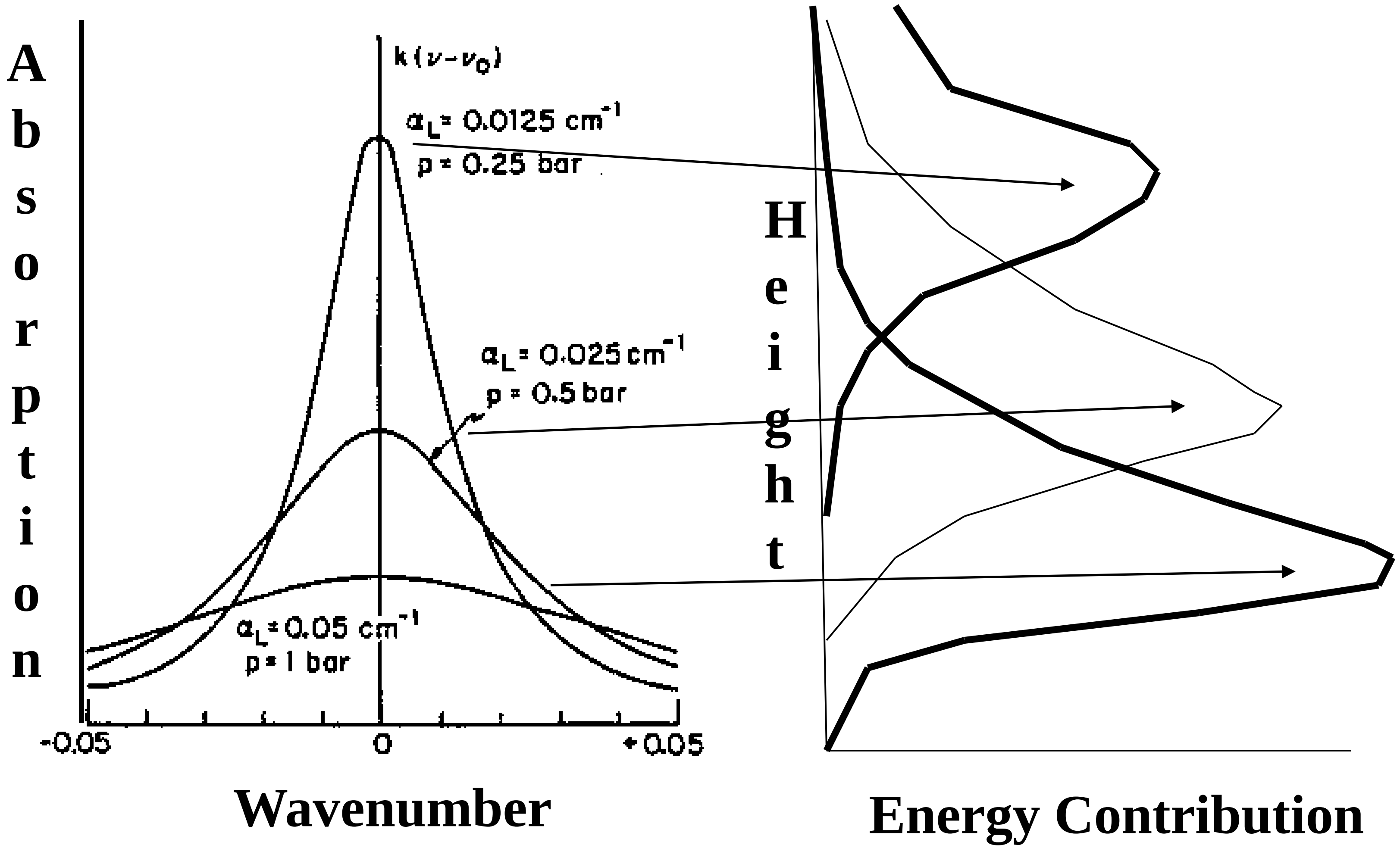
CO2 channels see to different levels in the atmosphere



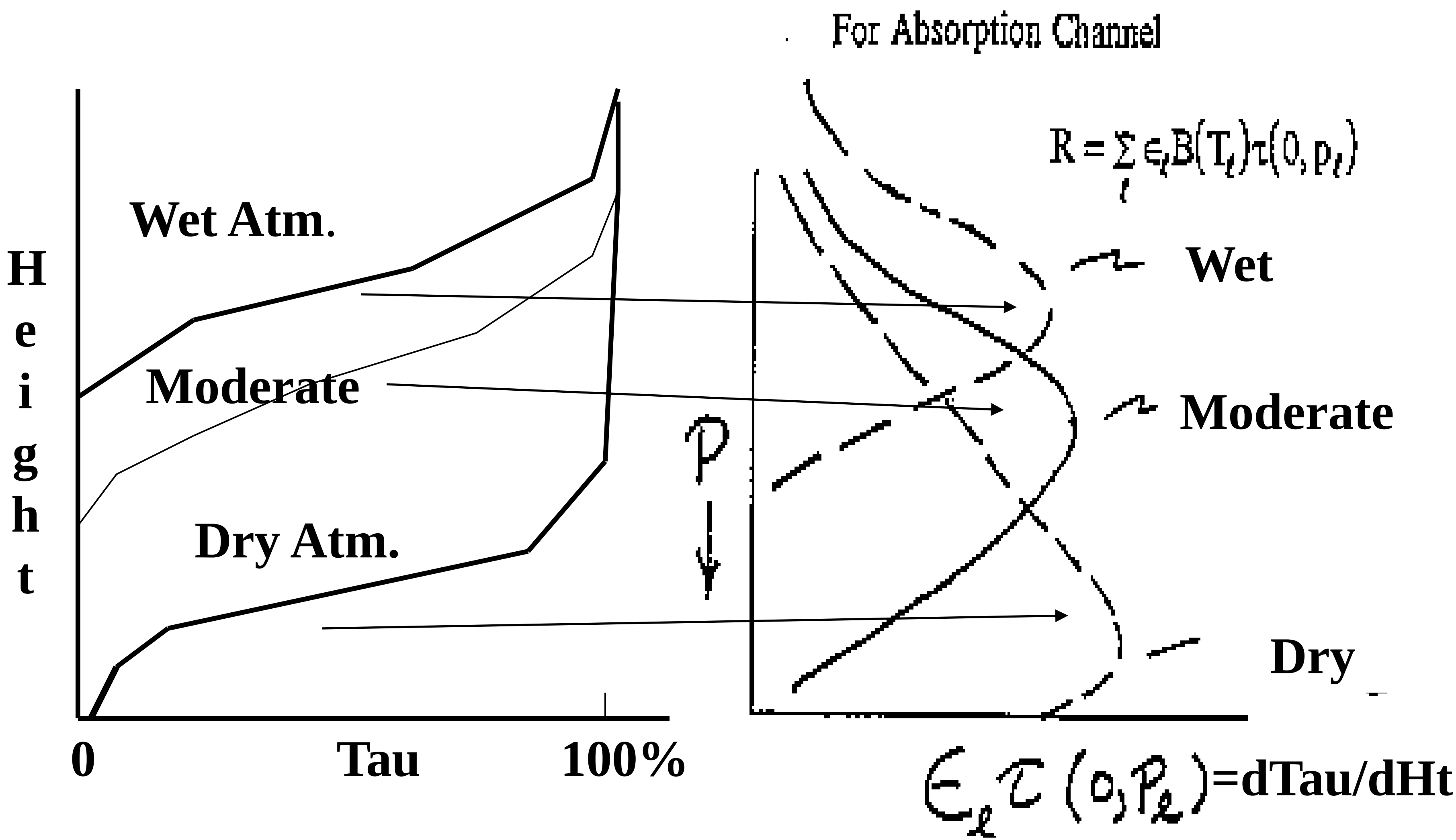
14.2 um 13.9 um 13.6 um 13.3 um

line broadening with pressure helps to explain weighting functions

$$\tau_\nu (z \rightarrow \infty) = e^{-k_\nu u(z)}$$

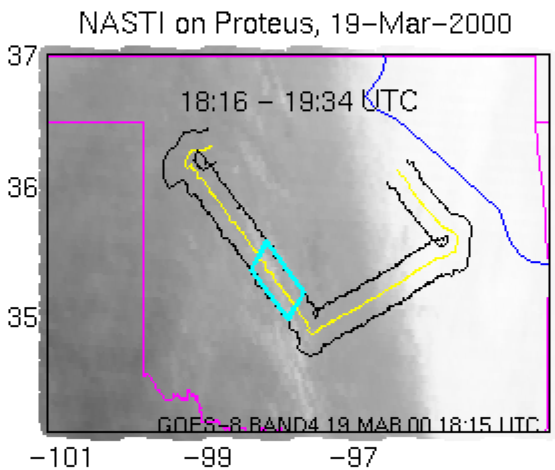


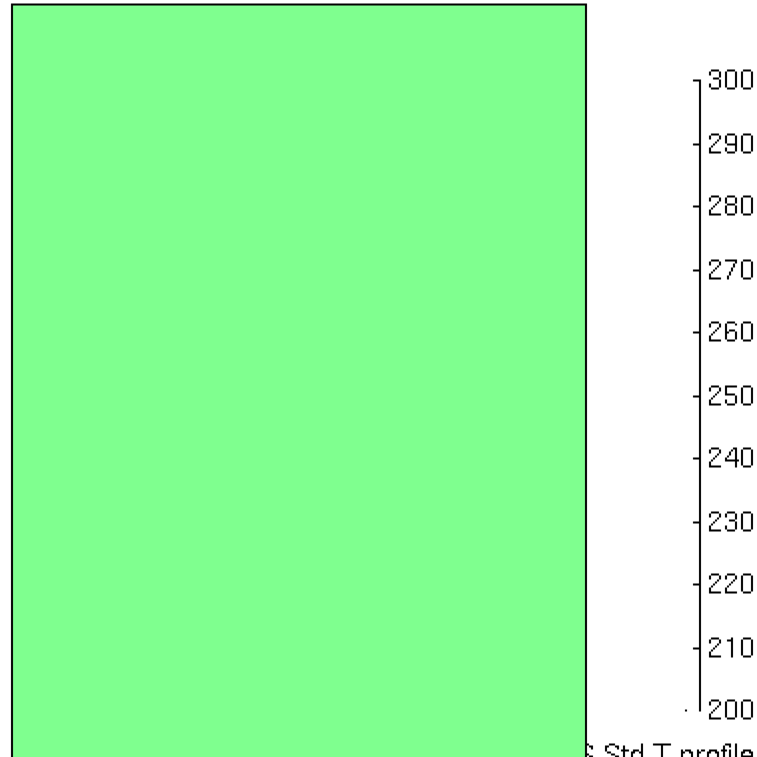
For a given water vapor spectral channel the weighting function depends on the amount of water vapor in the atmospheric column

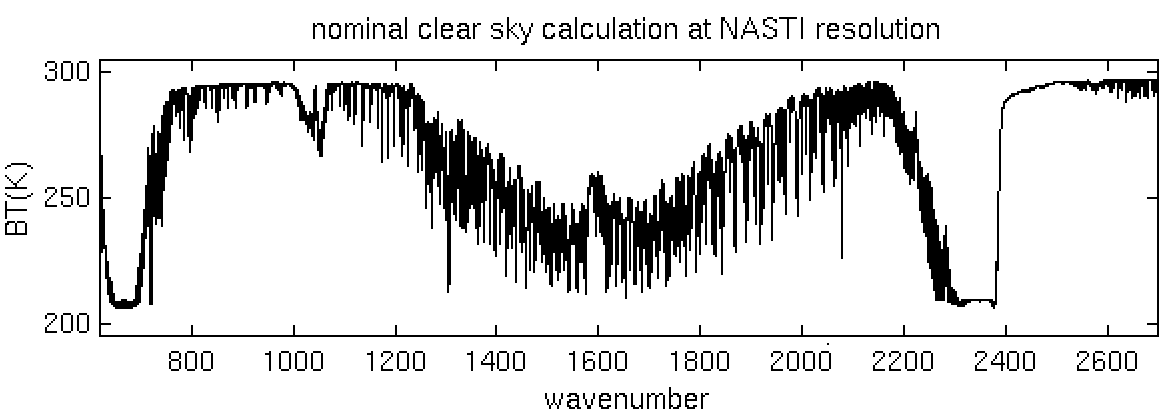


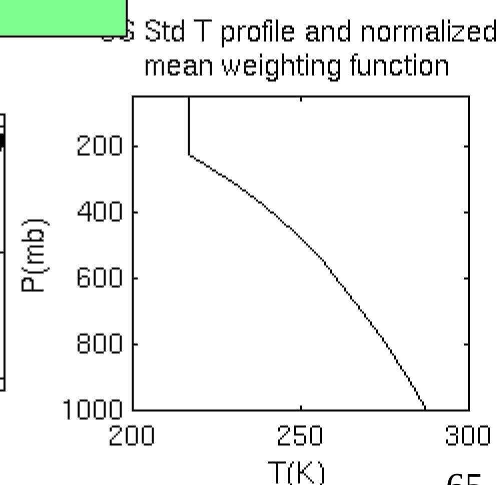
CO2 is about the same everywhere, the weighting function for a given CO2 spectral channel is the same everywhere

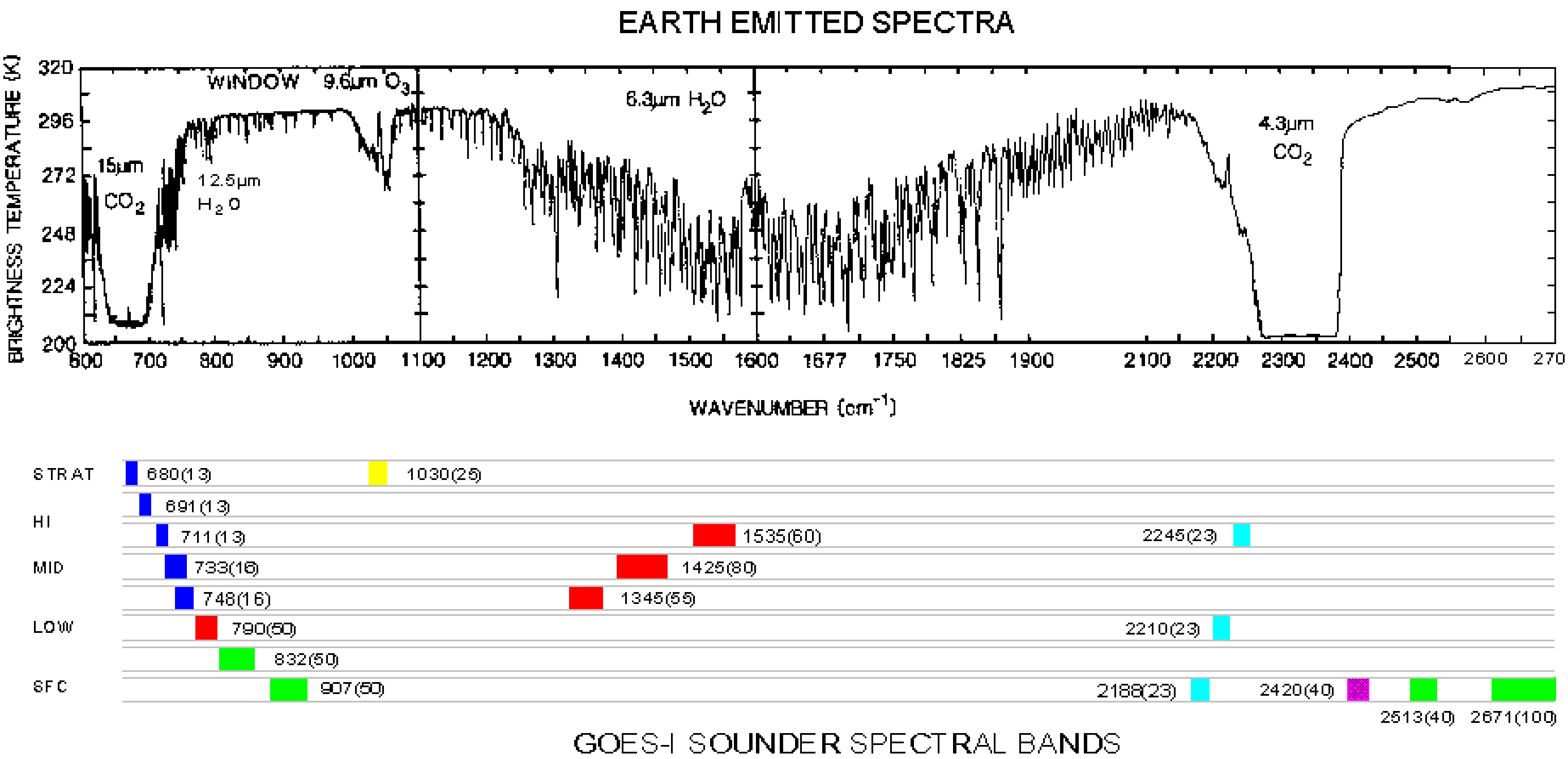
Improvements with Hyperspectral IR Data





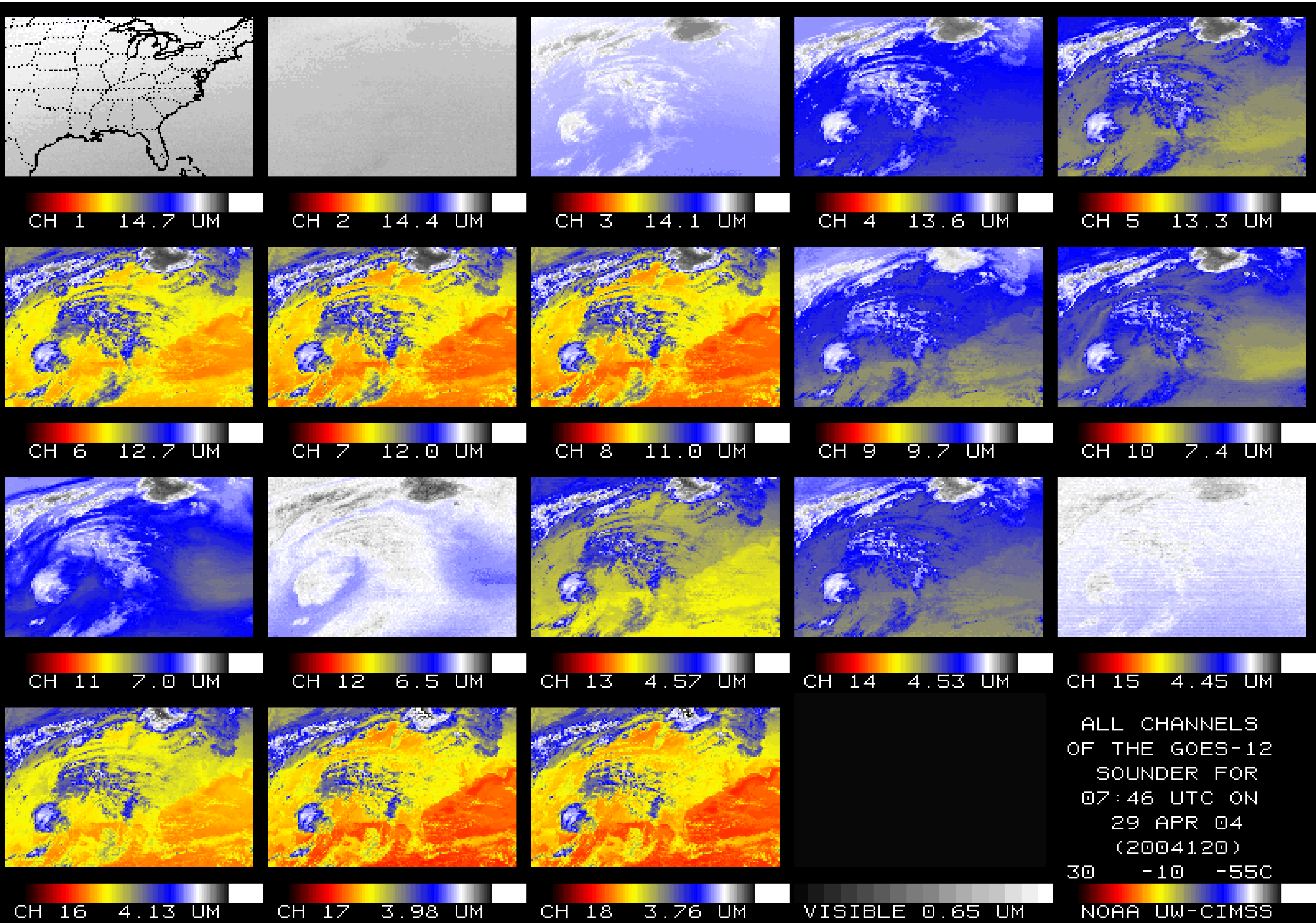






COOPERATIVE INSTITUTE FOR METEOROLOGICAL SATELLITE STUDIES

GOES-12 Sounder – Brightness Temperature (Radiances) – 12 bands



Characteristics of RTE

* Radiance arises from deep and overlapping layers

* The radiance observations are not independent

* There is no unique relation between the spectrum of the outgoing radiance and T(p) or Q(p)

* T(p) is buried in an exponent in the denominator in the integral

* Q(p) is implicit in the transmittance

* Boundary conditions are necessary for a solution; the better the first guess the better the final solution

Profile Retrieval from Sounder Radiances

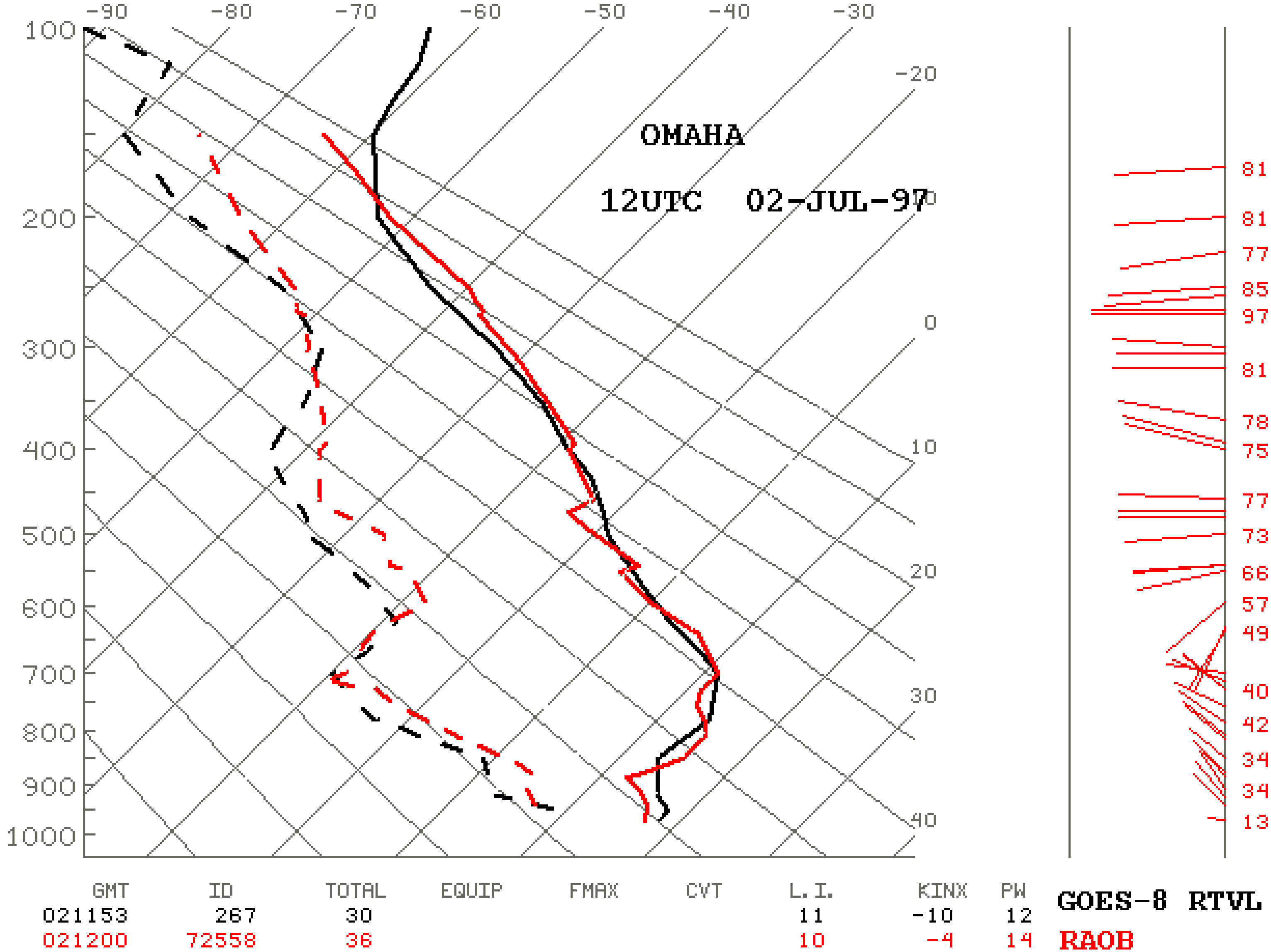
$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p)) F_\lambda(p) [d\tau_\lambda(p) / dp] dp .$$

I1, I2, I3, ... , In are measured with the sounder
P(sfc) and T(sfc) come from ground based conventional observations
$\tau_\lambda(p)$ are calculated with physics models (using for CO2 and O3)
$\varepsilon_\lambda^{sfc}$ is estimated from a priori information (or regression guess)

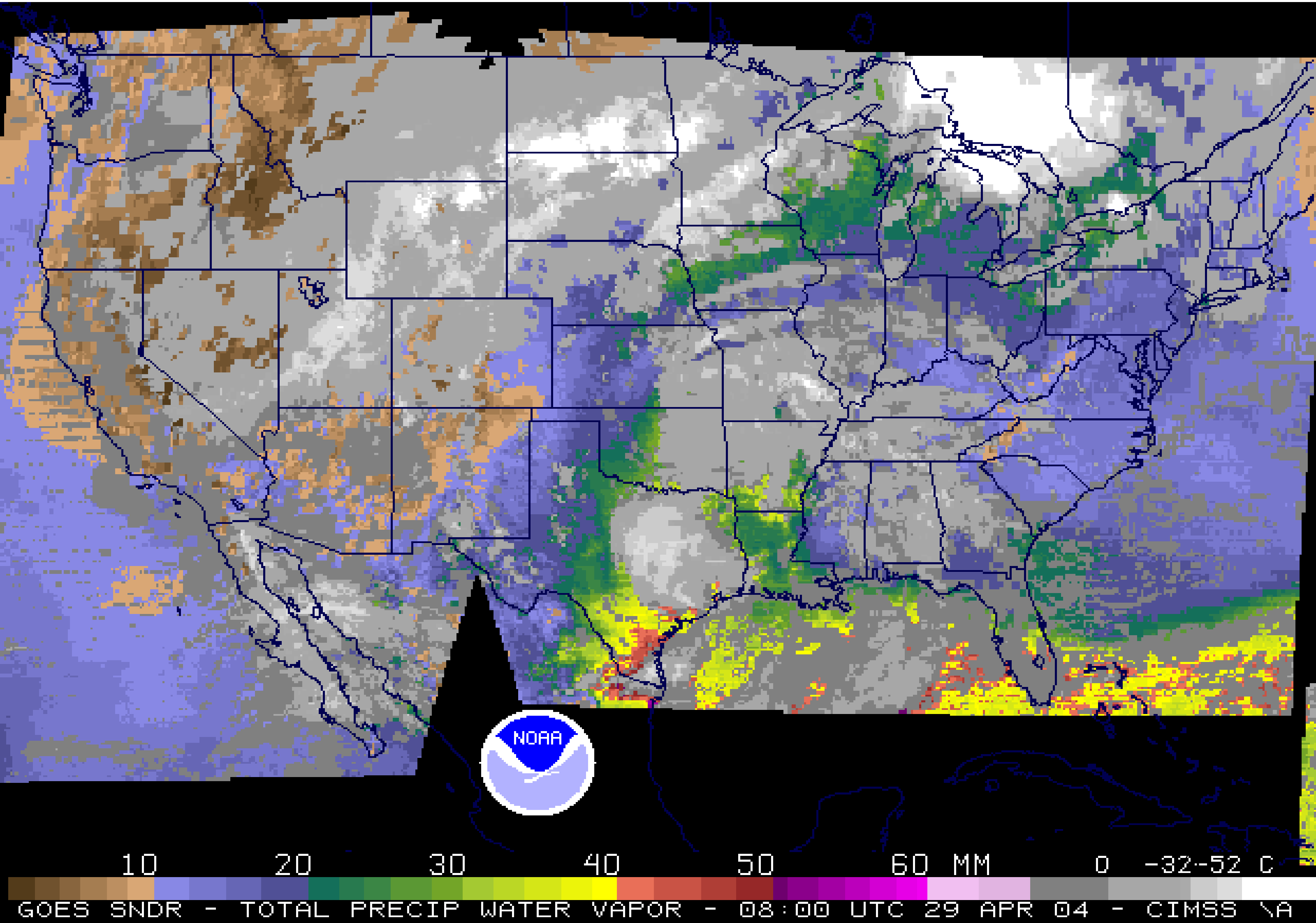
First guess solution is inferred from (1) in situ radiosonde reports,
(2) model prediction, or (3) blending of (1) and (2)

Profile retrieval from perturbing guess to match measured sounder radiances

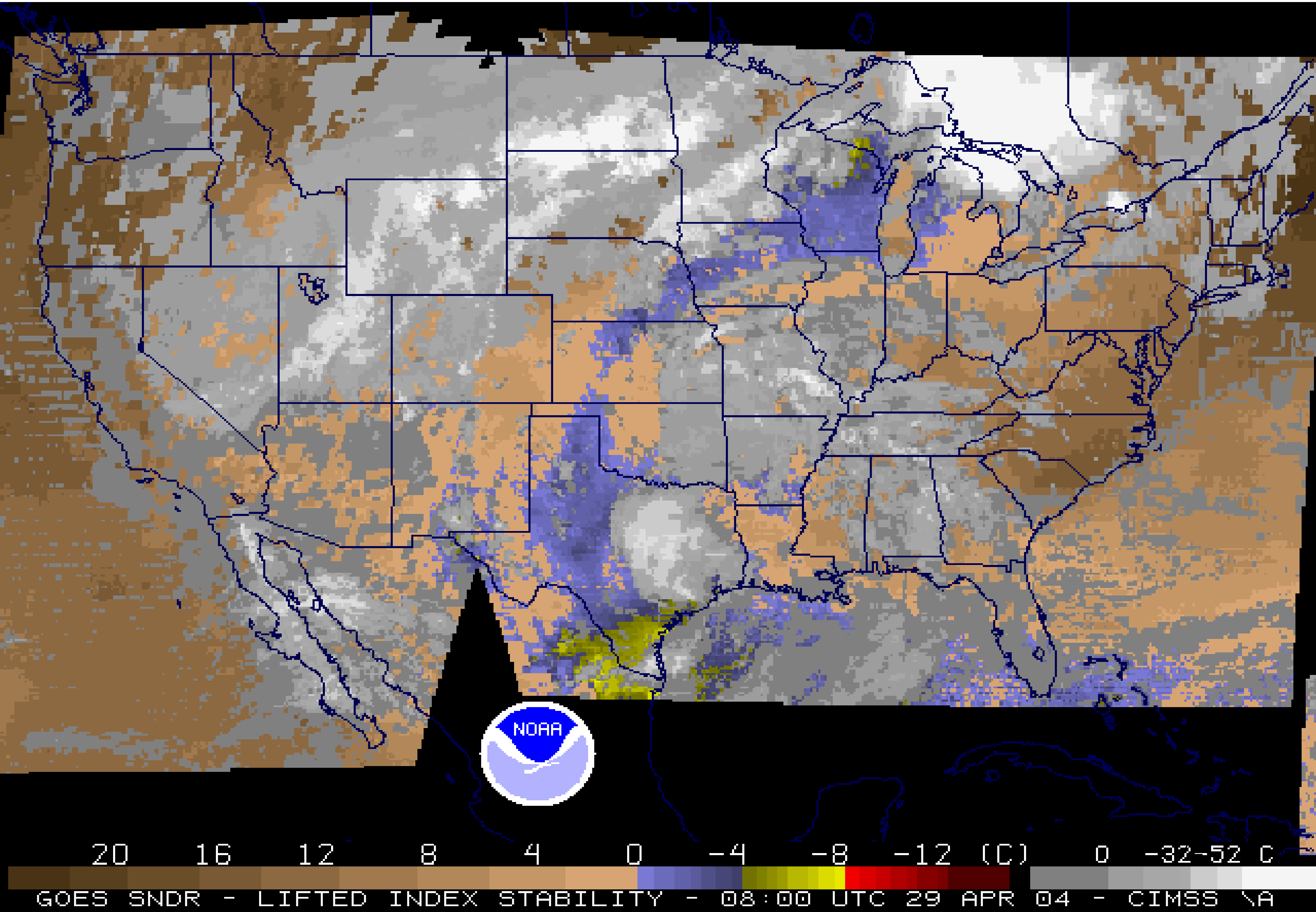
Example GOES Sounding

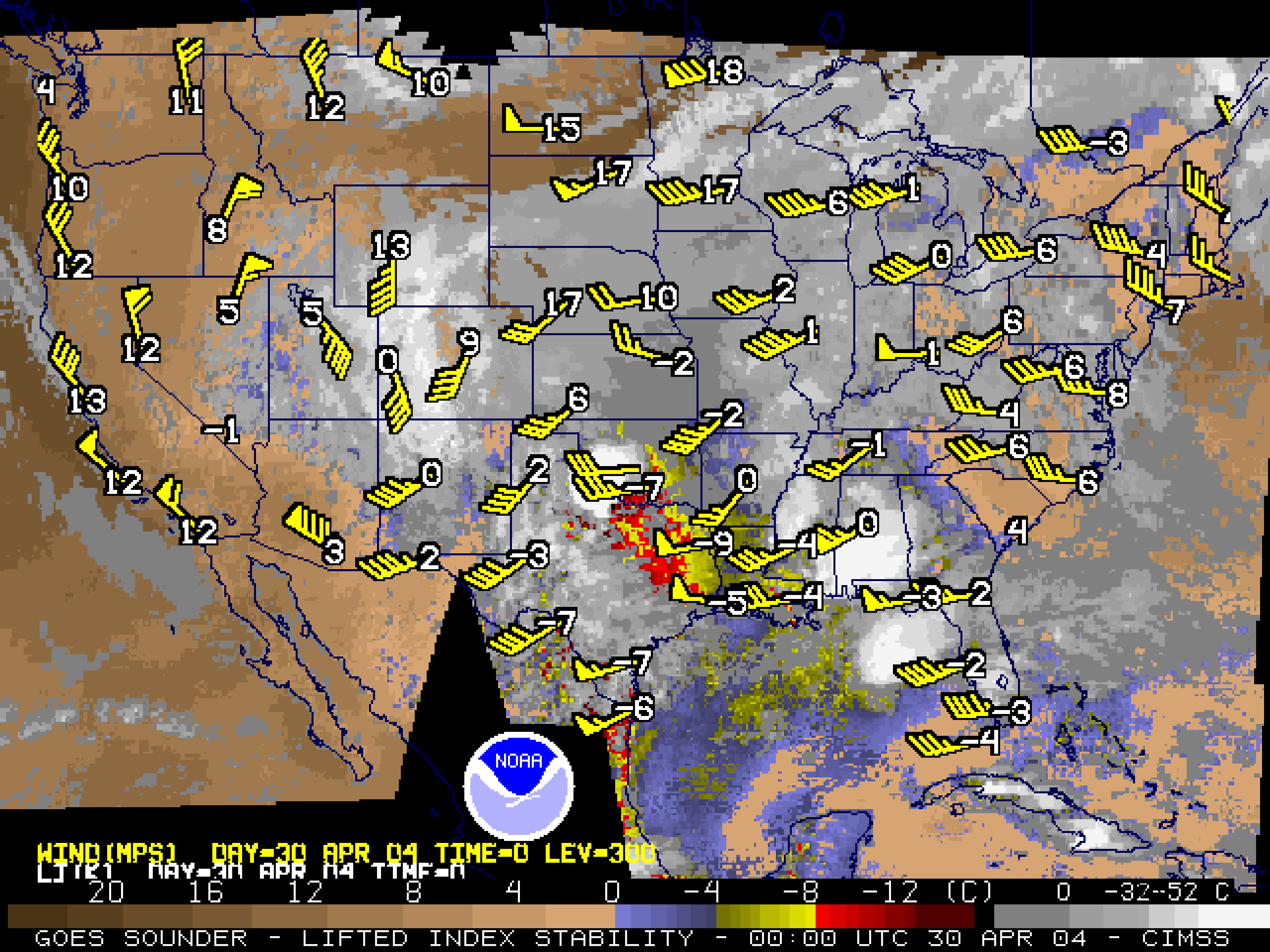


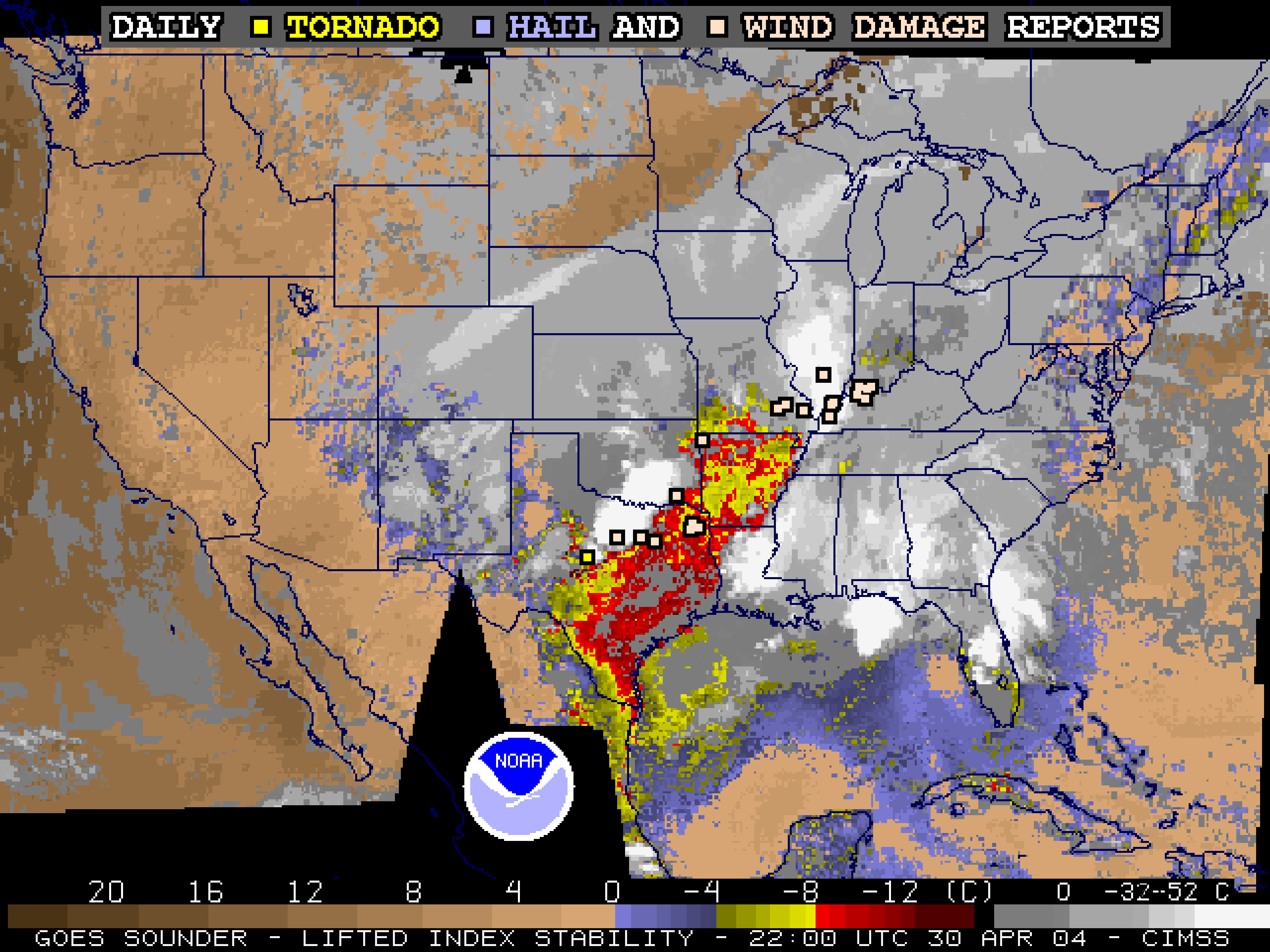
GOES Sounders –Total Precipitable Water



GOES Sounders –Lifted Index Stability




WIND(MPS) DAY=30 APR 04 TIME=0 LEV=300
20 16 12 8 4 0 -4 -8 -12 (C) 0 -32--52 C
GOES SOUNDER - LIFTED INDEX STABILITY - 00:00 UTC 30 APR 04 - CIMSS


DAILY ■ TORNADO ■ HAIL AND ■ WIND DAMAGE REPORTS
NOAA
20 16 12 8 4 0 -4 -8 -12 (C) 0 -32-52 C
GOES SOUNDER - LIFTED INDEX STABILITY - 22:00 UTC 30 APR 04 - CIMSS

Sounder Retrieval Products

$$I_\lambda = \varepsilon_\lambda(sfc)\, B_\lambda(T(ps))\, \tau_\lambda(ps) - \int_o^{ps} B_\lambda(T(p))\, F_\lambda(p)\, [\, d\tau_\lambda(p) / dp \,]\, dp\ .$$

Direct

brightness temperatures

Derived in Clear Sky

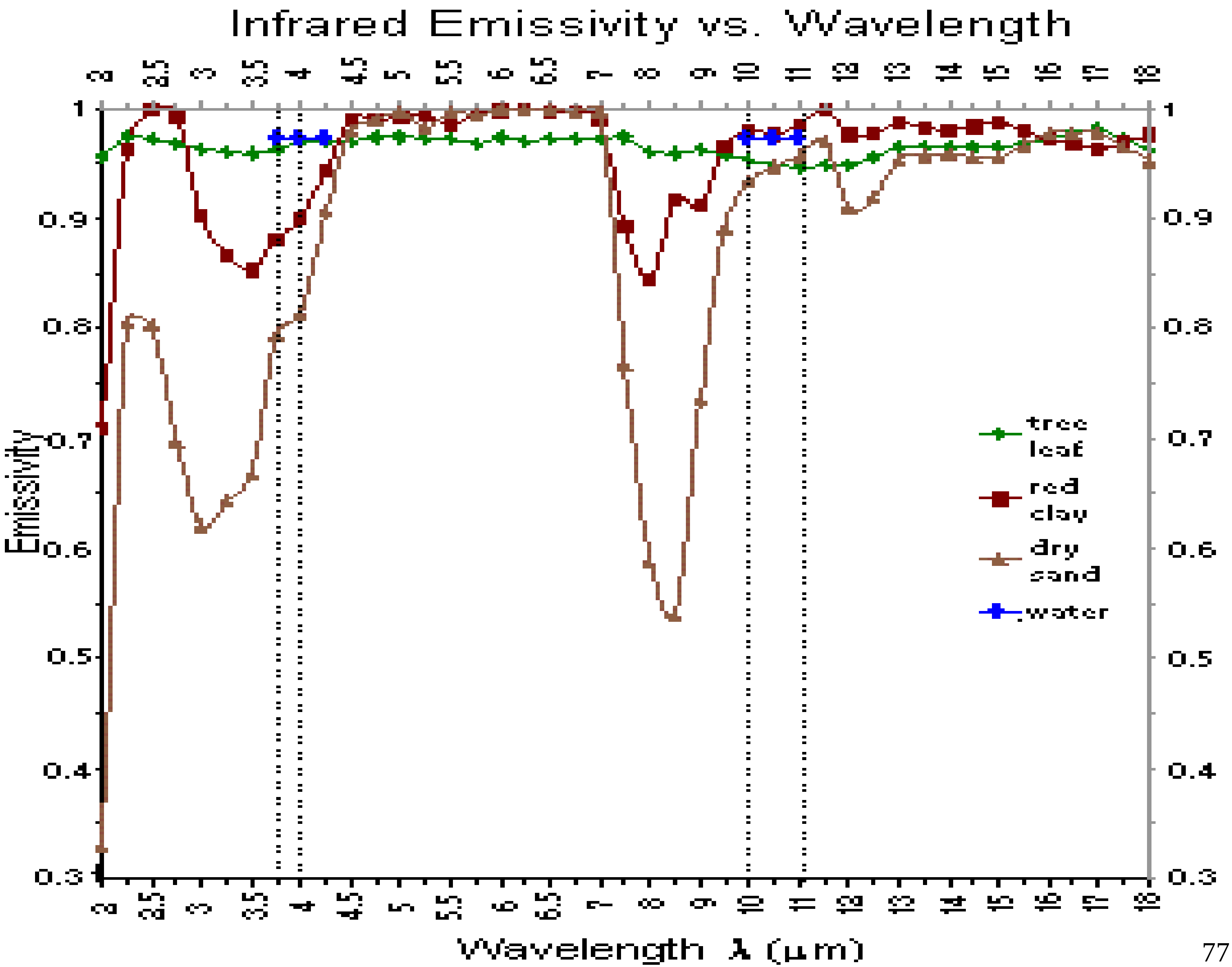
20 retrieved temperatures (at mandatory levels)
20 geo-potential heights (at mandatory levels)
11 dewpoint temperatures (at 300 hPa and below)
3 thermal gradient winds (at 700, 500, 400 hPa)
1 total precipitable water vapor
1 surface skin temperature
2 stability index (lifted index, CAPE)

Derived in Cloudy conditions

3 cloud parameters (amount, cloud top pressure, and cloud top temperature)

Mandatory Levels (in hPa)

sfc	780	300	70
1000	700	250	50
950	670	200	30
920	500	150	20
850	400	100	10



Infrared Emissivity vs. Wavelength
Emissivity
Wavelength λ (µm)
tree leaf
red clay
dry sand
water

PND/COMET

Microwave RTE

Lectures in Brienza

19 Sep 2011

Paul Menzel
UW/CIMSS/AOS

Radiation is governed by Planck's Law

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

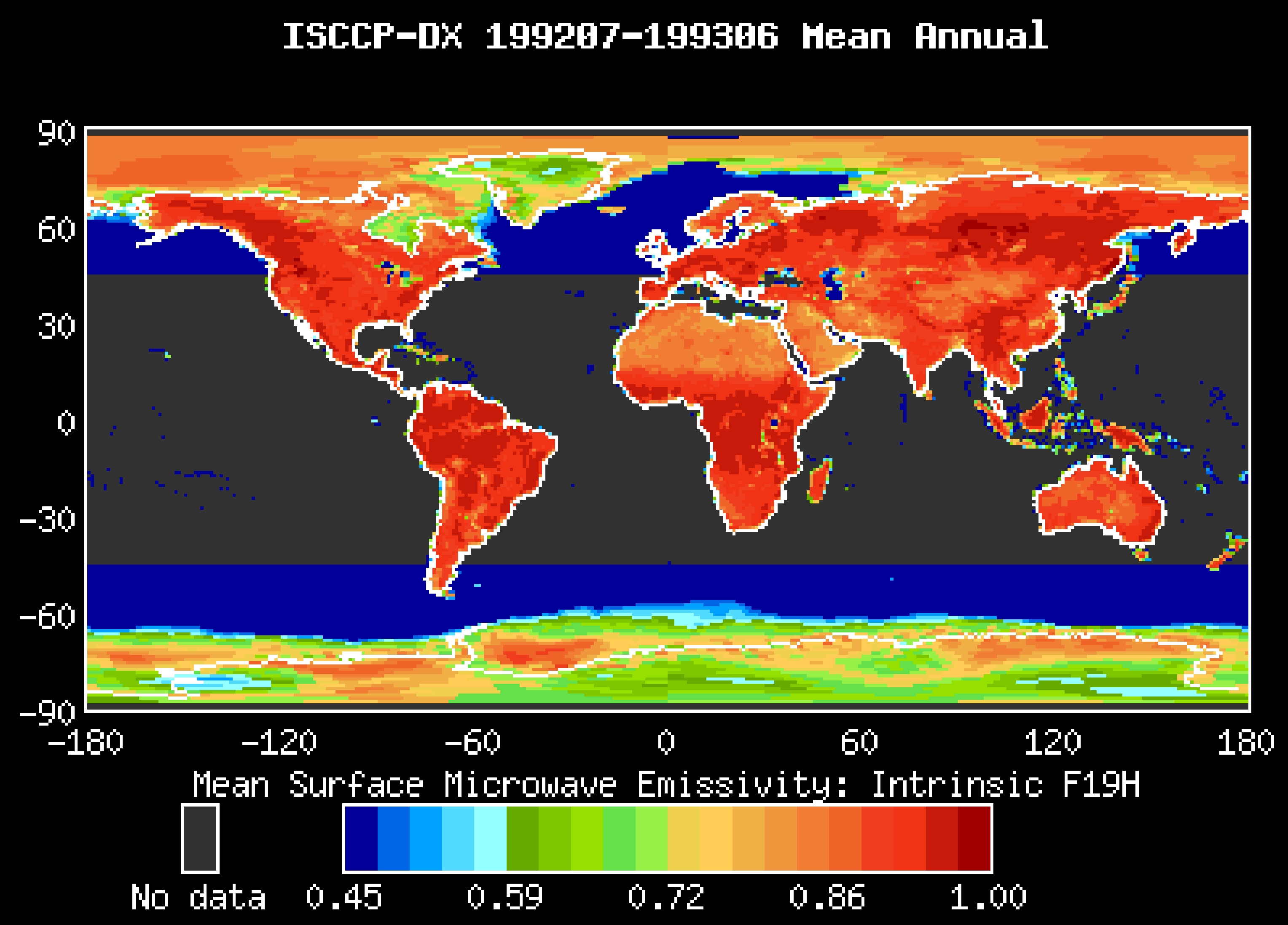
In microwave region $c_2/\lambda T << 1$ so that

$$e^{c_2/\lambda T} = 1 + c_2/\lambda T + \text{second order}$$

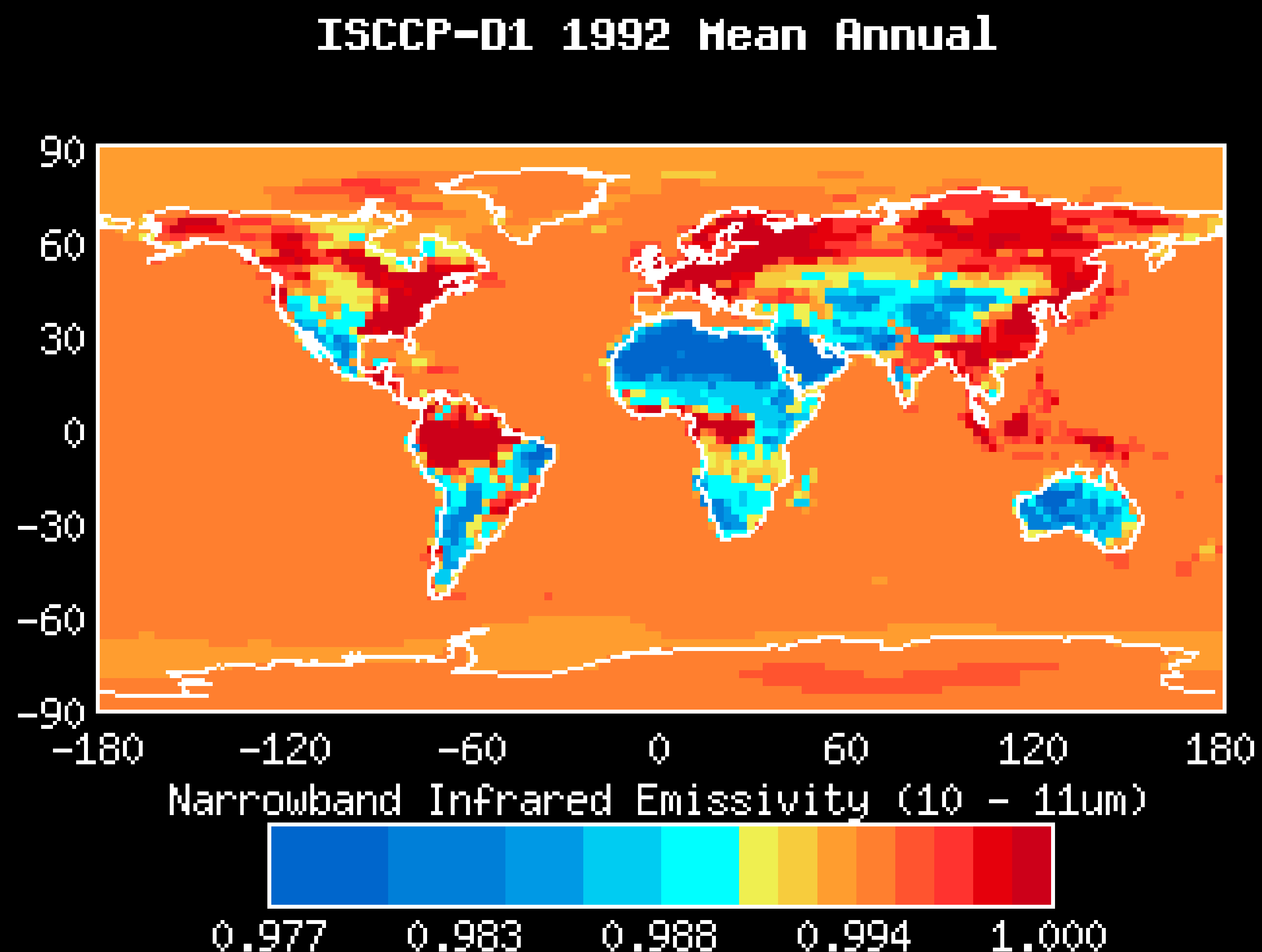
And classical Rayleigh Jeans radiation equation emerges

$$B_\lambda(T) \approx [c_1 / c_2] [T / \lambda^4]$$

Radiance is linear function of brightness temperature.


ISCCP-DX 199207-199306 Mean Annual
90
60
30
0
-30
-60
-90
-180
-120
-60
0
60
120
180
Mean Surface Microwave Emissivity: Intrinsic F19H
No data
0.45
0.59
0.72
0.86
1.00

19H Ghz


ISCCP-D1 1992 Mean Annual
90
60
30
0
-30
-60
-90
-180
-120
-60
0
60
120
180
Narrowband Infrared Emissivity (10 - 11um)
0.977
0.983
0.988
0.994
1.000

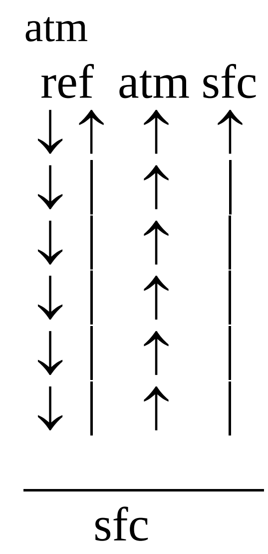
10 – 11 um

Microwave Form of RTE

$$I^{sfc}_\lambda = \varepsilon_\lambda B_\lambda(T_s)\,\tau_\lambda(p_s) + (1-\varepsilon_\lambda)\,\tau_\lambda(p_s) \int_o^{p_s} B_\lambda(T(p))\,\frac{\partial \tau'_\lambda(p)}{\partial \ln p}\, d\ln p$$

$$I_\lambda = \varepsilon_\lambda B_\lambda(T_s)\,\tau_\lambda(p_s) + (1-\varepsilon_\lambda)\,\tau_\lambda(p_s) \int_o^{p_s} B_\lambda(T(p))\,\frac{\partial \tau'_\lambda(p)}{\partial \ln p}\, d\ln p$$

$$+ \int_{p_s}^{o} B_\lambda(T(p))\,\frac{\partial \tau_\lambda(p)}{\partial \ln p}\, d\ln p$$



In the microwave region $c_2/\lambda T << 1$, so the Planck radiance is linearly proportional to the brightness temperature

$$B_\lambda(T) \approx [c_1 / c_2]\,[T / \lambda^4]$$

So

$$T_{b\lambda} = \varepsilon_\lambda T_s(p_s)\,\tau_\lambda(p_s) + \int_{p_s}^{o} T(p)\,F_\lambda(p)\,\frac{\partial \tau_\lambda(p)}{\partial \ln p}\, d\ln p$$

where

$$F_\lambda(p) = \left\{ 1 + (1-\varepsilon_\lambda)\left[\frac{\tau_\lambda(p_s)}{\tau_\lambda(p)}\right]^2 \right\} .$$

Transmittance

$\tau(a,b) = \tau(b,a)$
$\tau(a,c) = \tau(a,b) * \tau(b,c)$

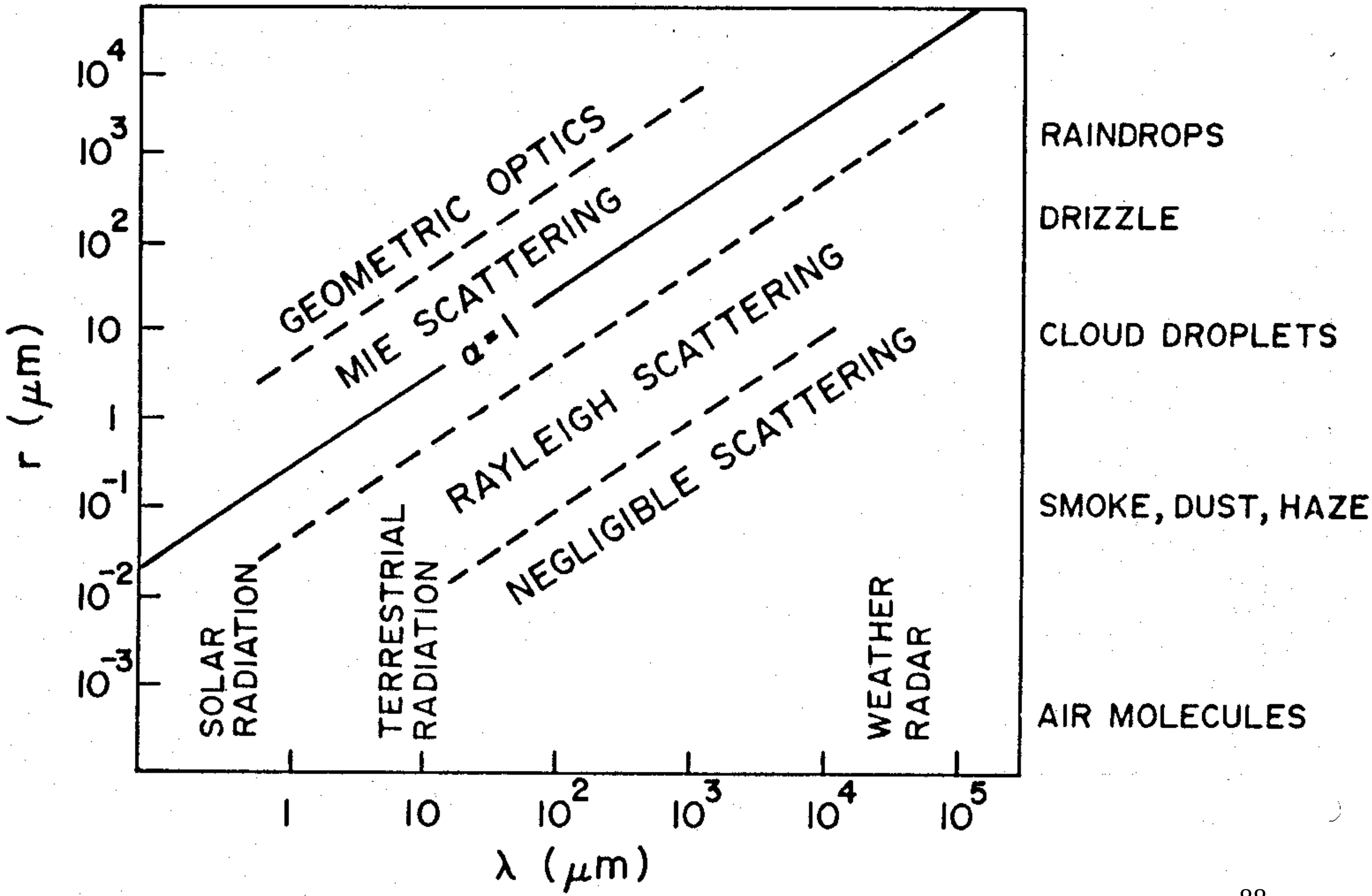
Thus downwelling in terms of upwelling can be written

$\tau'(p,ps) = \tau(ps,p) = \tau(ps,0) / \tau(p,0)$

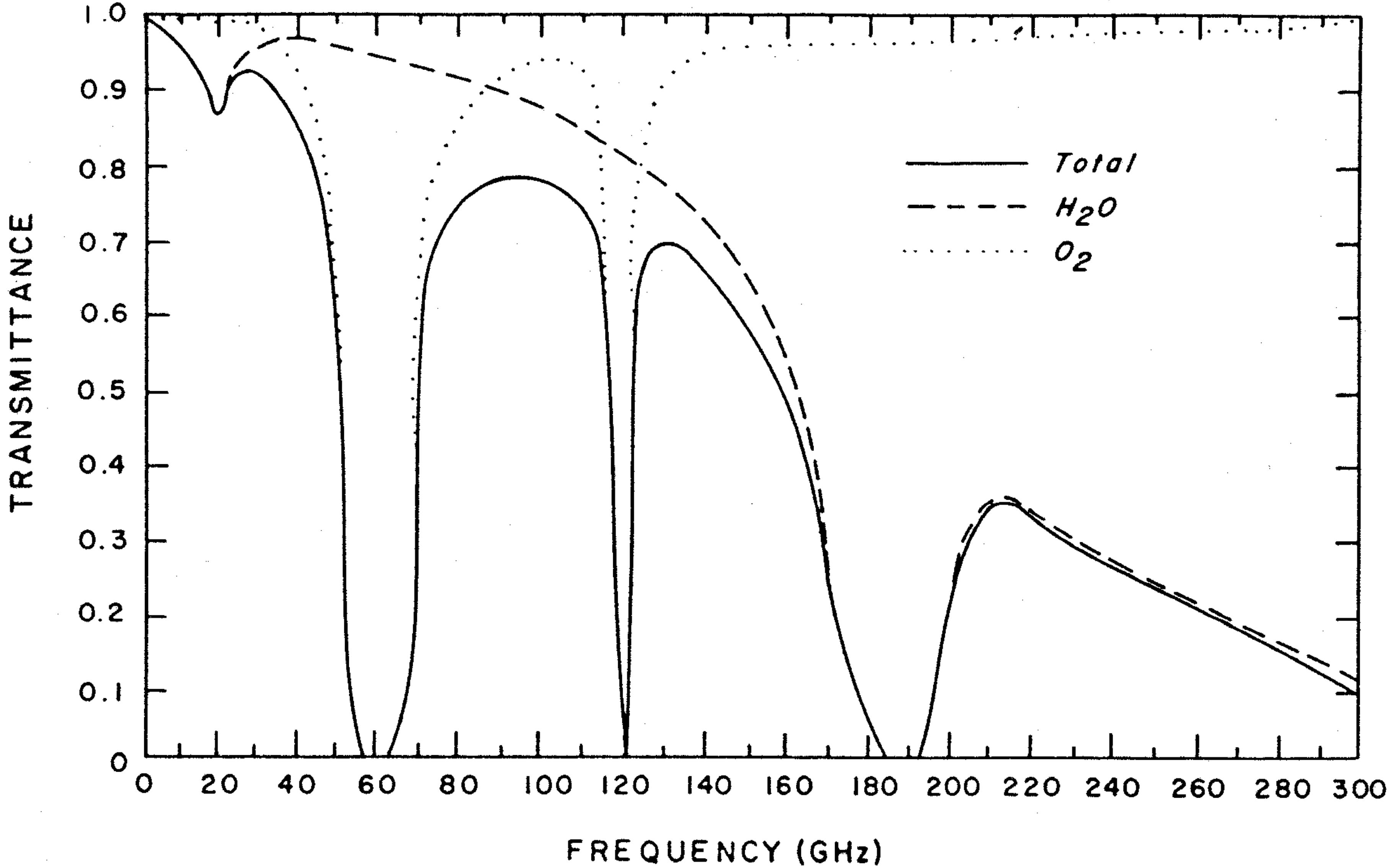
and

$d\tau'(p,ps) = - d\tau(p,0) * \tau(ps,0) / [\tau(p,0)]^2$

WAVELENGTH			FREQUENCY		WAVENUMBER
cm	μm	Å	Hz	GHz	cm^{-1}
10^{-5} Near Ultraviolet (UV)	0.1	1,000	$3x10^{15}$		
$4x10^{-5}$ Visible	0.4	4,000	$7.5x10^{14}$		
$7.5x10^{-5}$ Near Infrared (IR)	0.75	7,500	$4x10^{14}$		13,333
$2x10^{-3}$ Far Infrared (IR)	20	$2x10^{5}$	$1.5x10^{13}$		500
0.1 Microwave (MW)	10^{3}		$3x10^{11}$	300	10

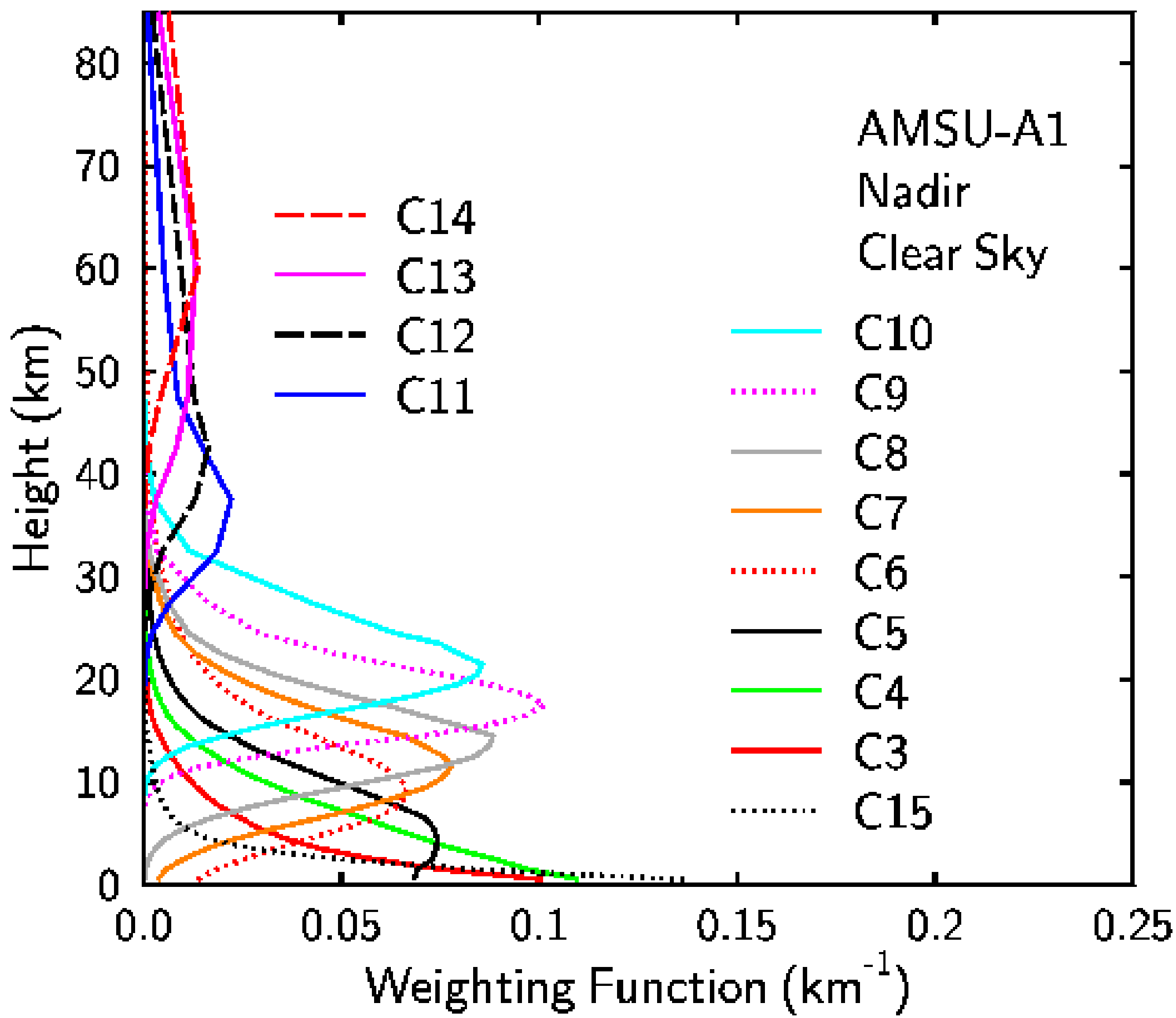


r (μm)
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
GEOMETRIC OPTICS
MIE SCATTERING
α=1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
1
10
10^2
10^3
10^4
10^5
λ (μm)
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
TRANSMITTANCE
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300
FREQUENCY (GHz)
Total
H_2O
O_2

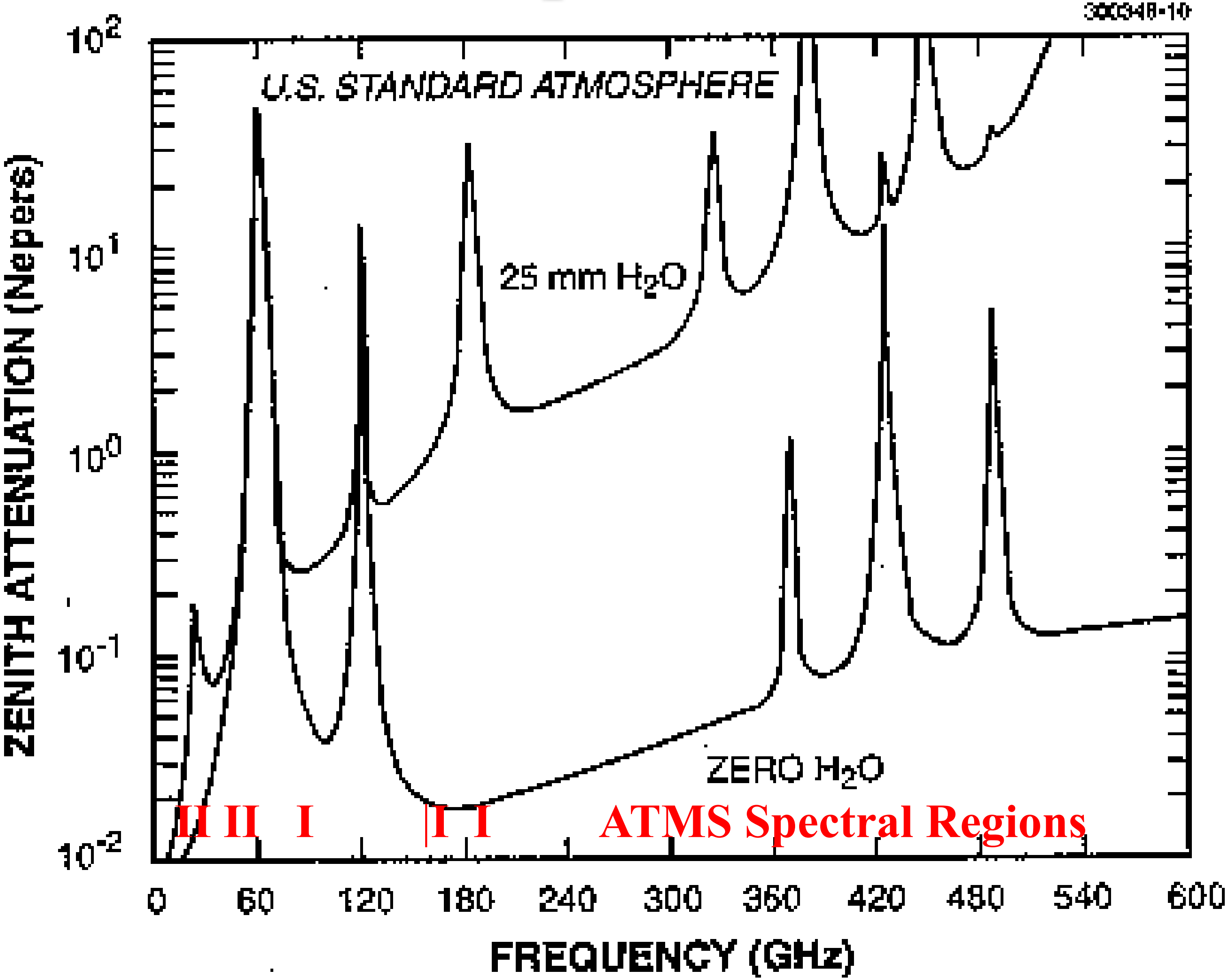
Microwave spectral bands

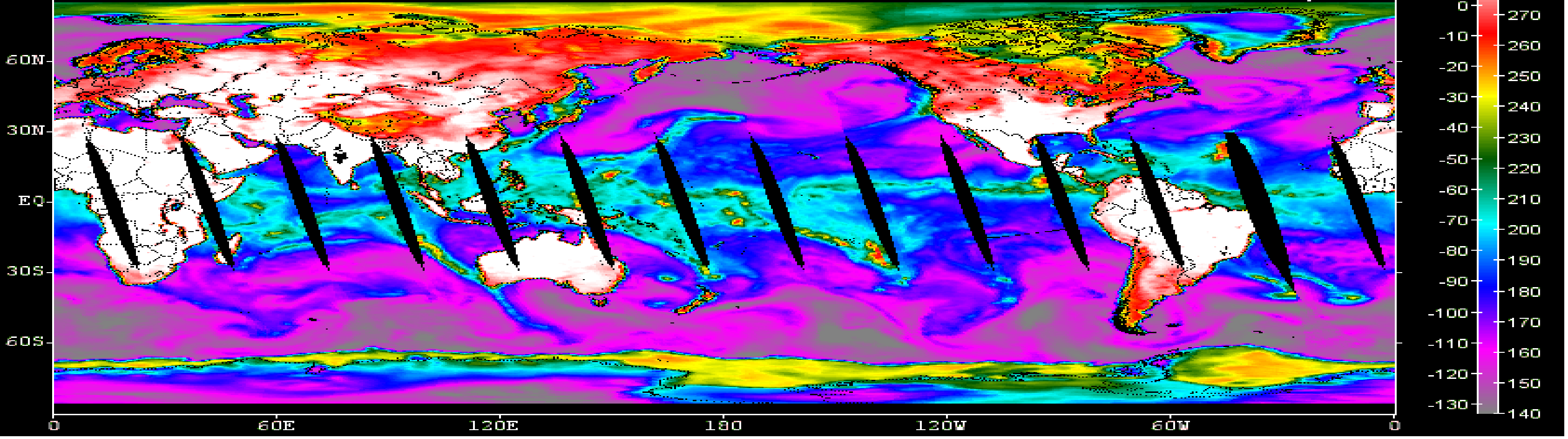
23.8 GHz	dirty window H_2O absorption
31.4 GHz	window
60 GHz	O_2 sounding
120 GHz	O_2 sounding
183 GHz	H_2O sounding



23.8, 31.4, 50.3, 52.8, 53.6, 54.4, 54.9, 55.5, 57.3 (6 chs), 89.0 GHz

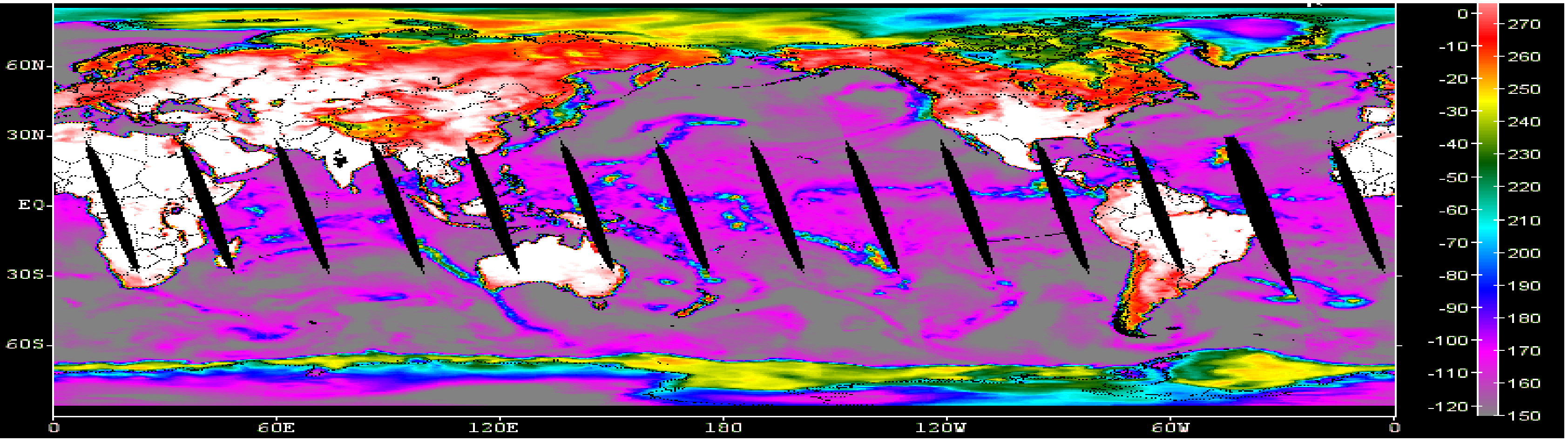
Microwave Spectral Features



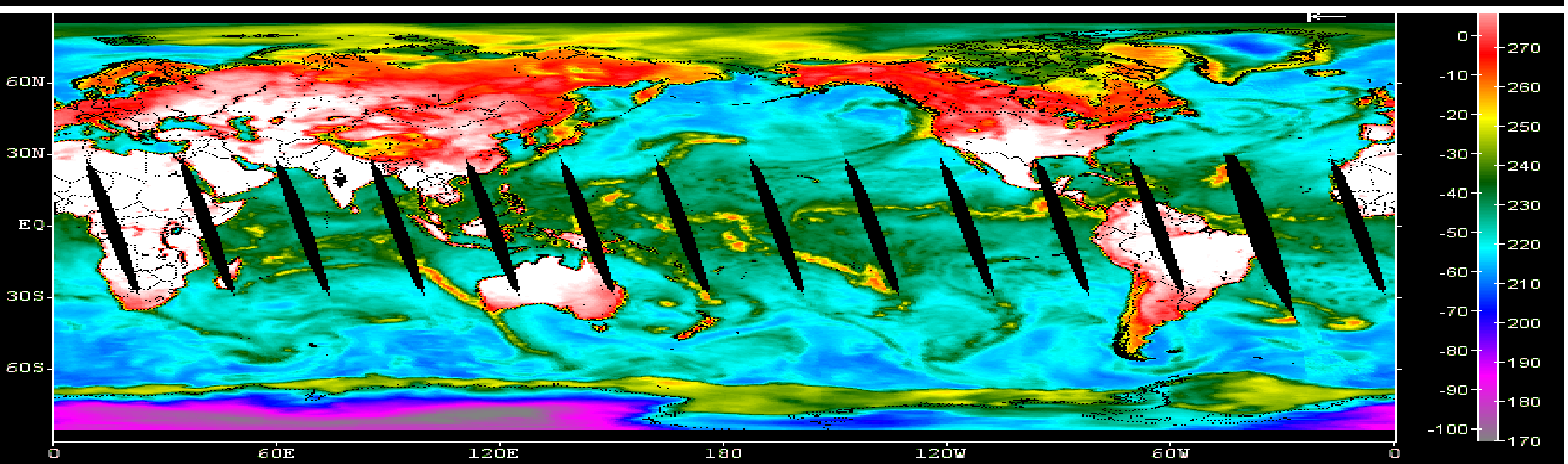


AMSU

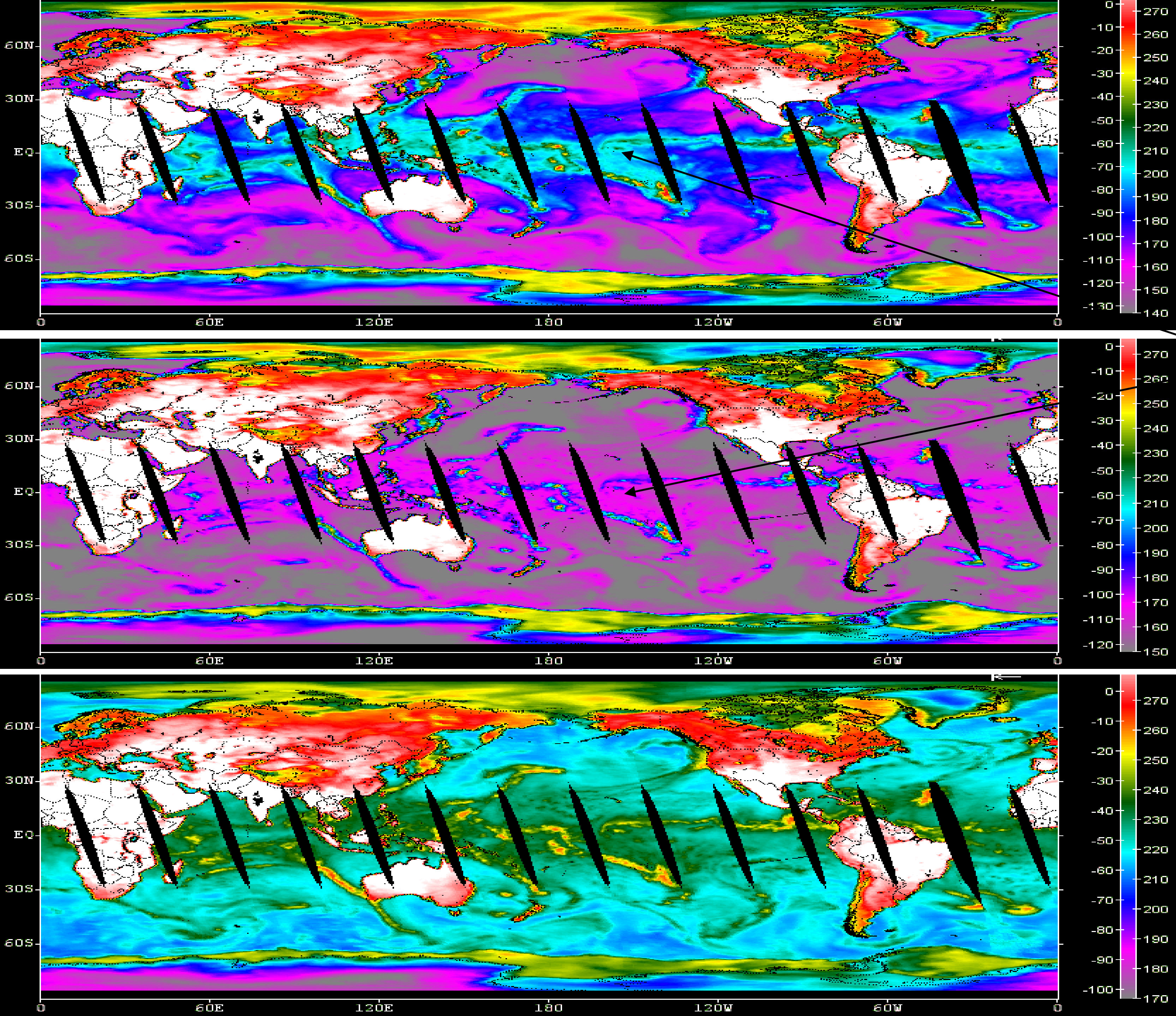
23.8



31.4



50.3 GHz



AMSU

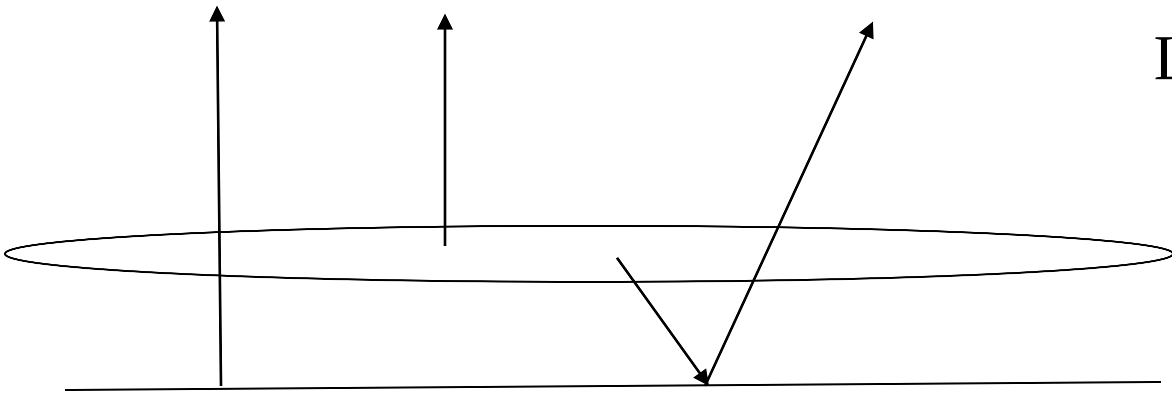
23.8 dirty window

atm Q warms BT

31.4 window

50.3 GHz

Low mist over ocean



τ_m , ε_m

r_s , ε_s

$$T_b = \varepsilon_s T_s \tau_m + \varepsilon_m T_m + \varepsilon_m r_s \tau_m T_m$$

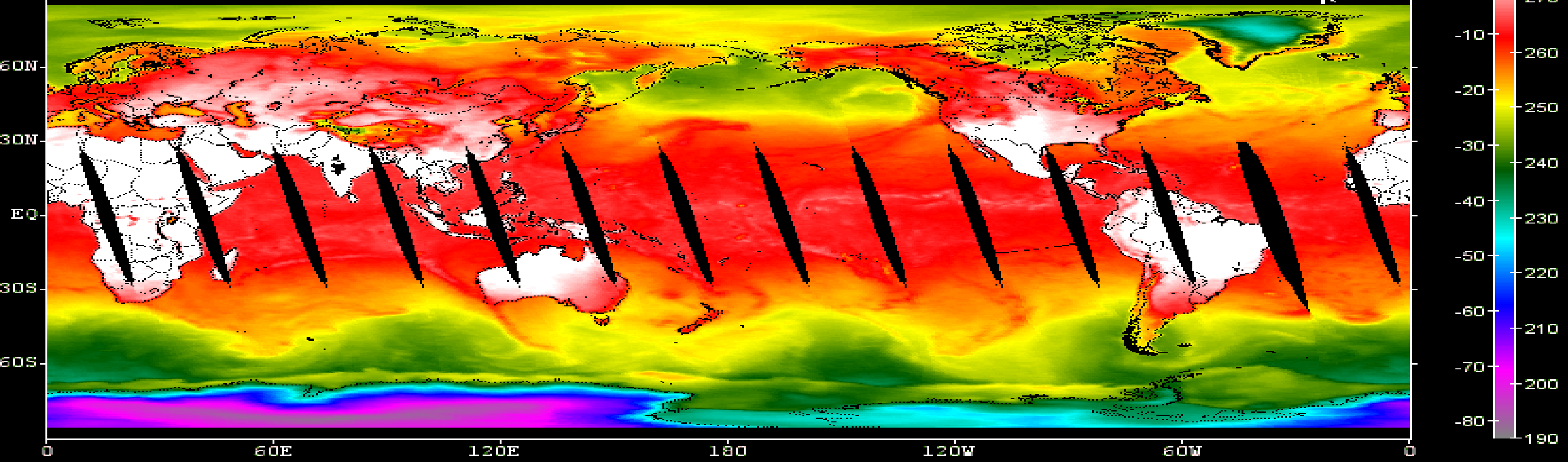
$$T_b = \varepsilon_s T_s (1-\sigma_m) + \sigma_m T_m + \sigma_m (1-\varepsilon_s) (1-\sigma_m) T_m$$

So temperature difference of low moist over ocean from clear sky over ocean is given by

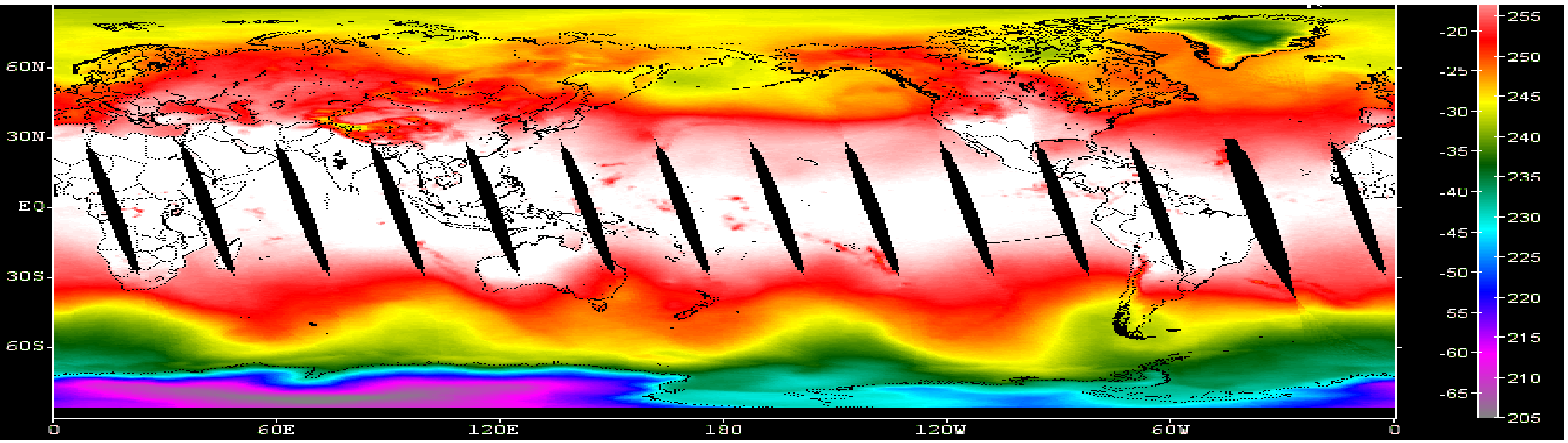
$$\Delta T_b = - \varepsilon_s \sigma_m T_s + \sigma_m T_m + \sigma_m (1-\varepsilon_s) (1-\sigma_m) T_m$$

For $\varepsilon_s \sim 0.5$ and $T_s \sim T_m$ this is always positive for $0 < \sigma_m < 1$

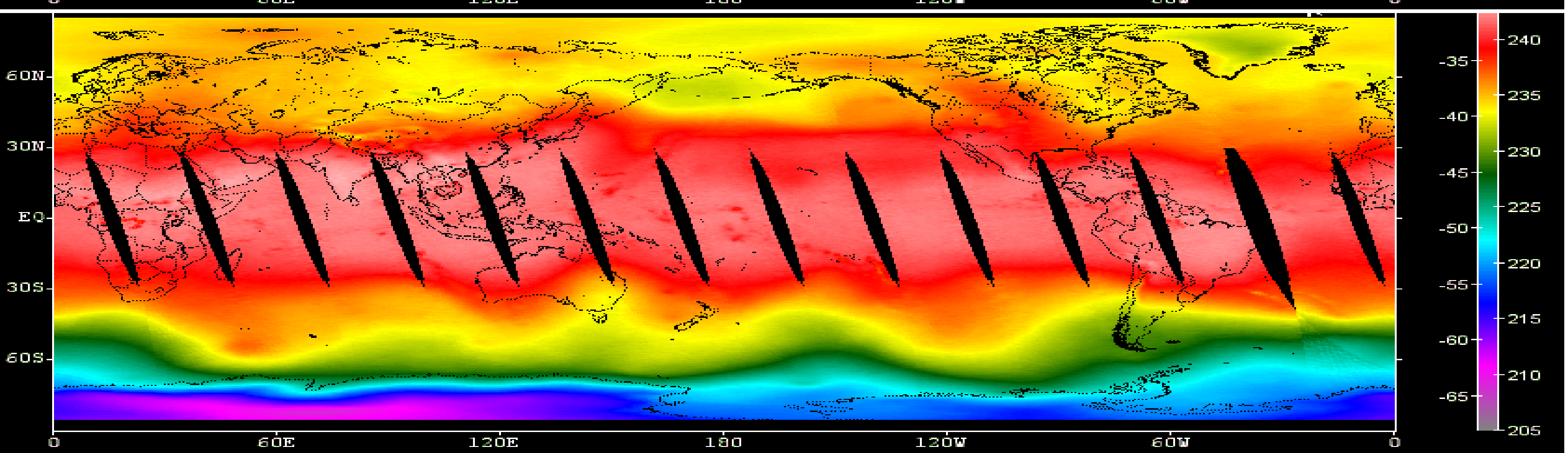
AMSU



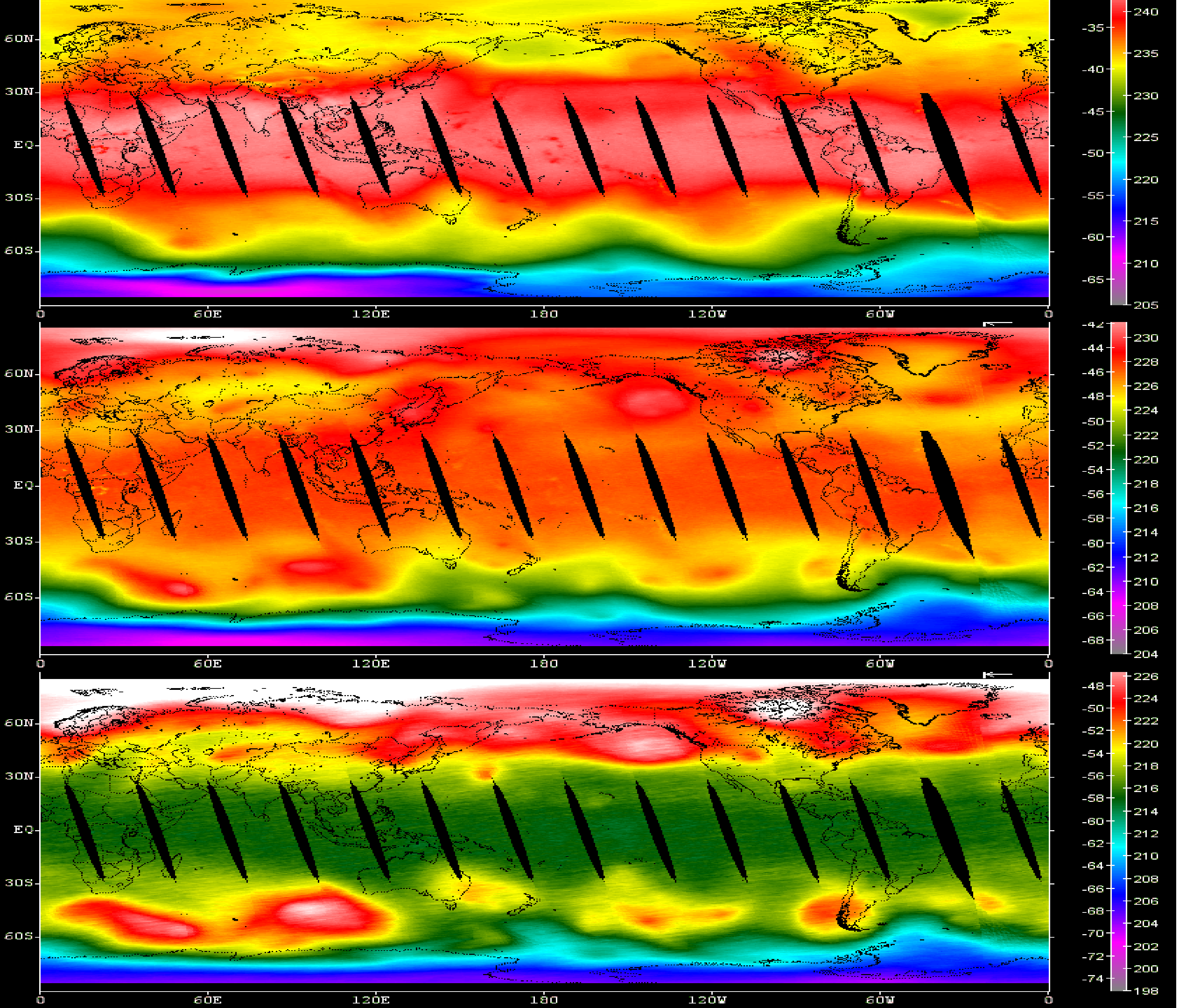
52.8



53.6



54.4 GHz

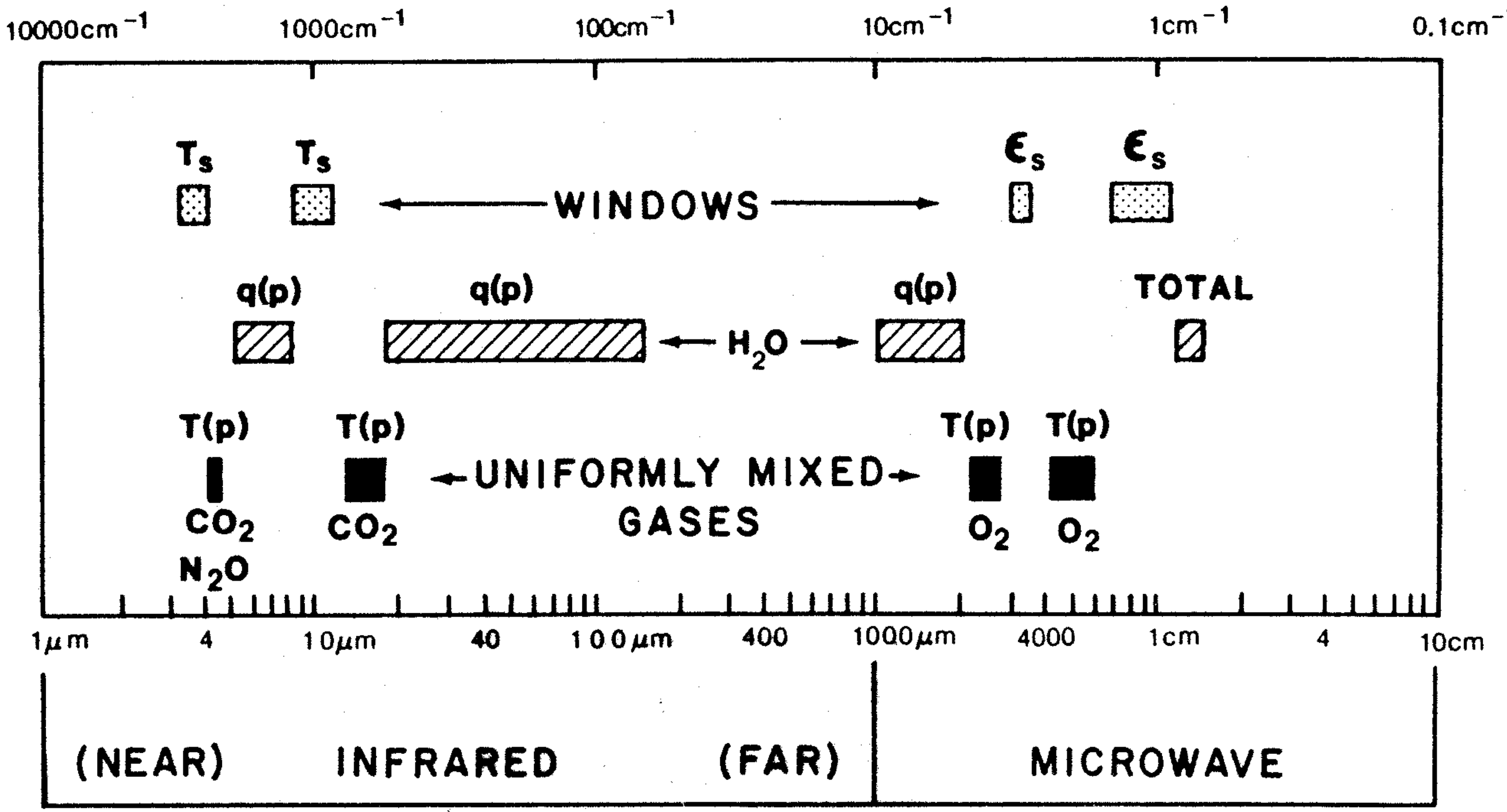


AMSU

54.4

54.9

55.5 GHz



Spectral regions used for remote sensing of the earth atmosphere and surface from satellites. ε indicates emissivity, q denotes water vapour, and T represents temperature.