Radiation and the Radiative Transfer Equation

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Relevant Material in Applications of Meteorological Satellites

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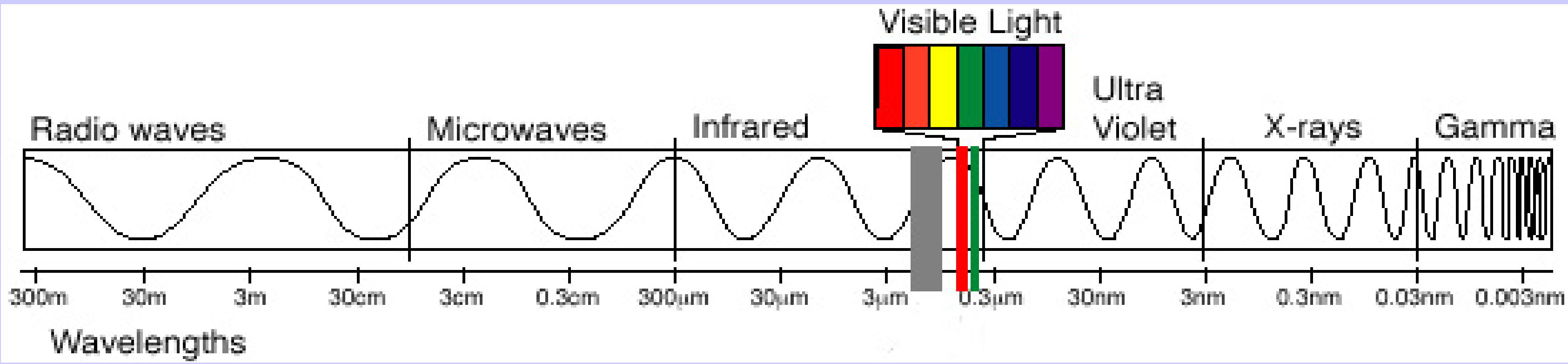
All satellite remote sensing systems involve the measurement of electromagnetic radiation.

Electromagnetic radiation has the properties of both waves and discrete particles, although the two are never manifest simultaneously.

Electromagnetic radiation is usually quantified according to its wave-like properties; for many applications it considered to be a continuous train of sinusoidal shapes.

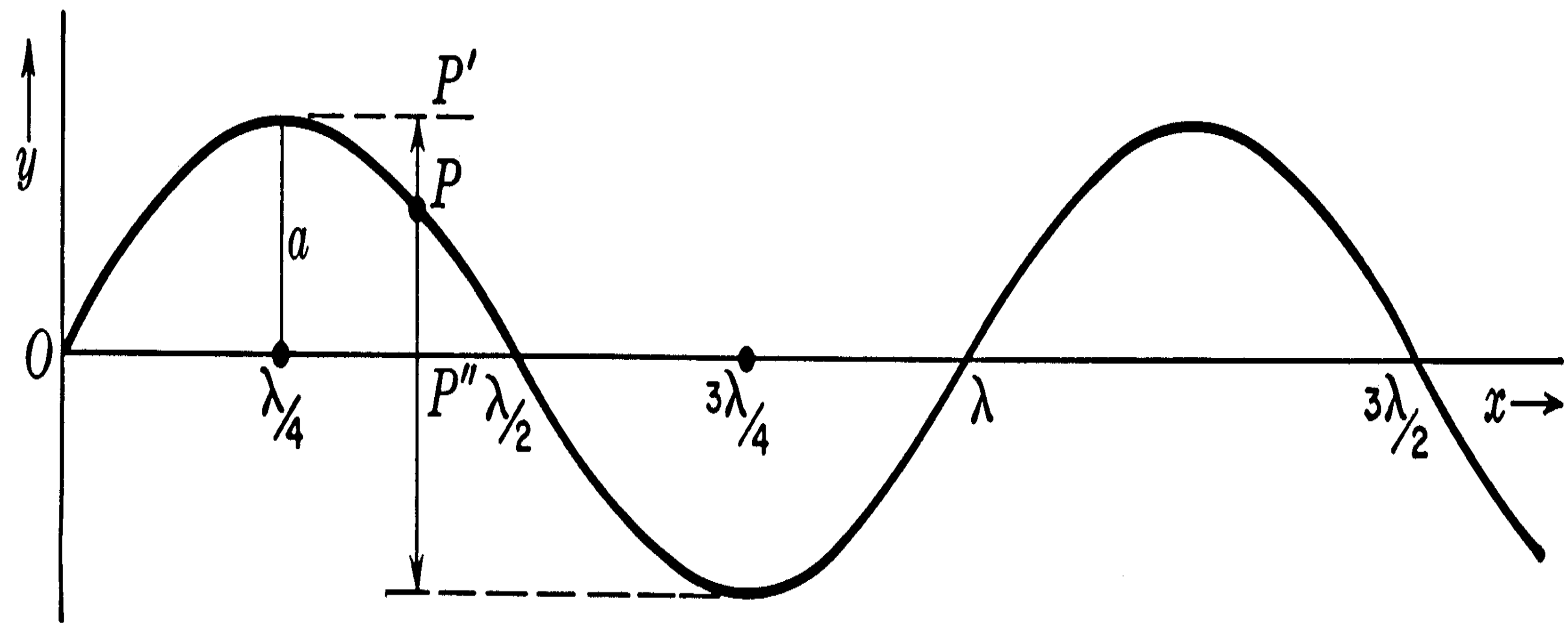
The Electromagnetic Spectrum



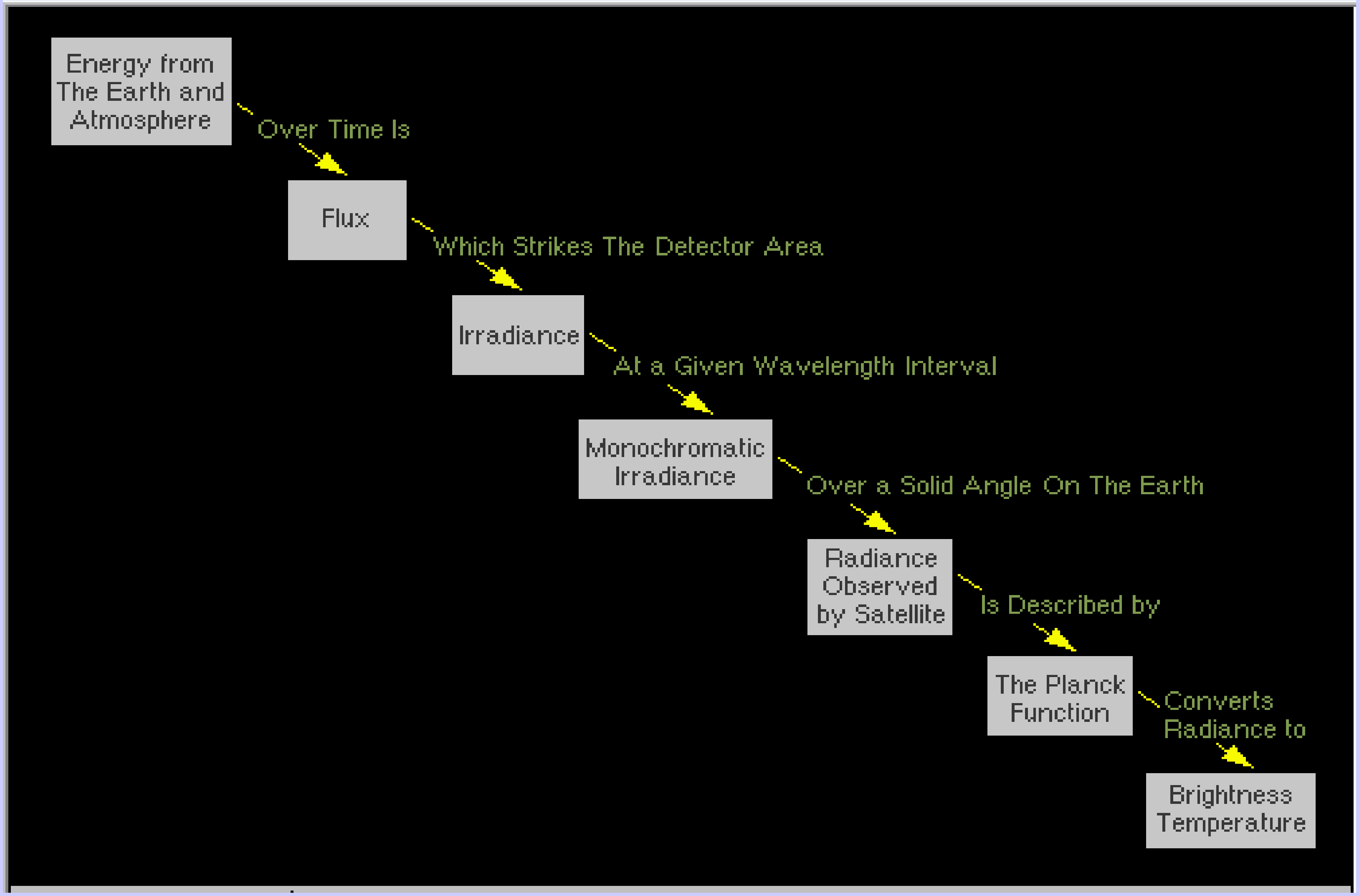
Remote sensing uses radiant energy that is reflected and emitted from Earth at various "wavelengths" of the electromagnetic spectrum

Our eyes are sensitive to the visible portion of the EM spectrum

Radiation is characterized by wavelength λ and amplitude a



Terminology of radiant energy



Definitions of Radiation

QUANTITY	SYMBOL	UNITS
Energy	dQ	Joules
Flux	dQ/dt	Joules/sec = Watts
Irradiance	dQ/dt/dA	Watts/meter^2
Monochromatic Irradiance	dQ/dt/dA/dλ	W/m^2/micron
	or	
	dQ/dt/dA/dν	W/m^2/cm^{-1}
Radiance	dQ/dt/dA/dλ/dΩ	W/m^2/micron/ster
	or	
	dQ/dt/dA/dν/dΩ	W/m^2/cm^{-1}/ster

Radiation from the Sun

The rate of energy transfer by electromagnetic radiation is called the radiant flux, which has units of energy per unit time. It is denoted by

$$F = dQ / dt$$

and is measured in joules per second or watts. For example, the radiant flux from the sun is about 3.90 x 10**26 W.

The radiant flux per unit area is called the irradiance (or radiant flux density in some texts). It is denoted by

$$E = dQ / dt / dA$$

and is measured in watts per square metre. The irradiance of electromagnetic radiation passing through the outermost limits of the visible disk of the sun (which has an approximate radius of 7 x 10**8 m) is given by

$$E\ (\text{sun sfc}) = \frac{3.90 \times 10^{26}}{4\pi\,(7 \times 10^{8})^{2}} = 6.34 \times 10^{7}\ \text{W m}^{-2}\ .$$

The solar irradiance arriving at the earth can be calculated by realizing that the flux is a constant, therefore

$$E\ (\text{earth sfc}) \times 4\pi R_{es}^2 = E\ (\text{sun sfc}) \times 4\pi R_s^2,$$

where R_{es} is the mean earth to sun distance (roughly 1.5×10^{11} m) and R_s is the solar radius. This yields

$$E\ (\text{earth sfc}) = 6.34 \times 10^7\ (7 \times 10^8 / 1.5 \times 10^{11})^2 = 1380\ \text{W m}^{-2}.$$

The irradiance per unit wavelength interval at wavelength λ is called the monochromatic irradiance,

$$E_\lambda = dQ / dt / dA / d\lambda\ ,$$

and has the units of watts per square metre per micrometer. With this definition, the irradiance is readily seen to be

$$E = \int_o^\infty E_\lambda\, d\lambda\ .$$

In general, the irradiance upon an element of surface area may consist of contributions which come from an infinity of different directions. It is sometimes necessary to identify the part of the irradiance that is coming from directions within some specified infinitesimal arc of solid angle $d\Omega$. The irradiance per unit solid angle is called the radiance,

$$I = dQ / dt / dA / d\lambda / d\Omega,$$

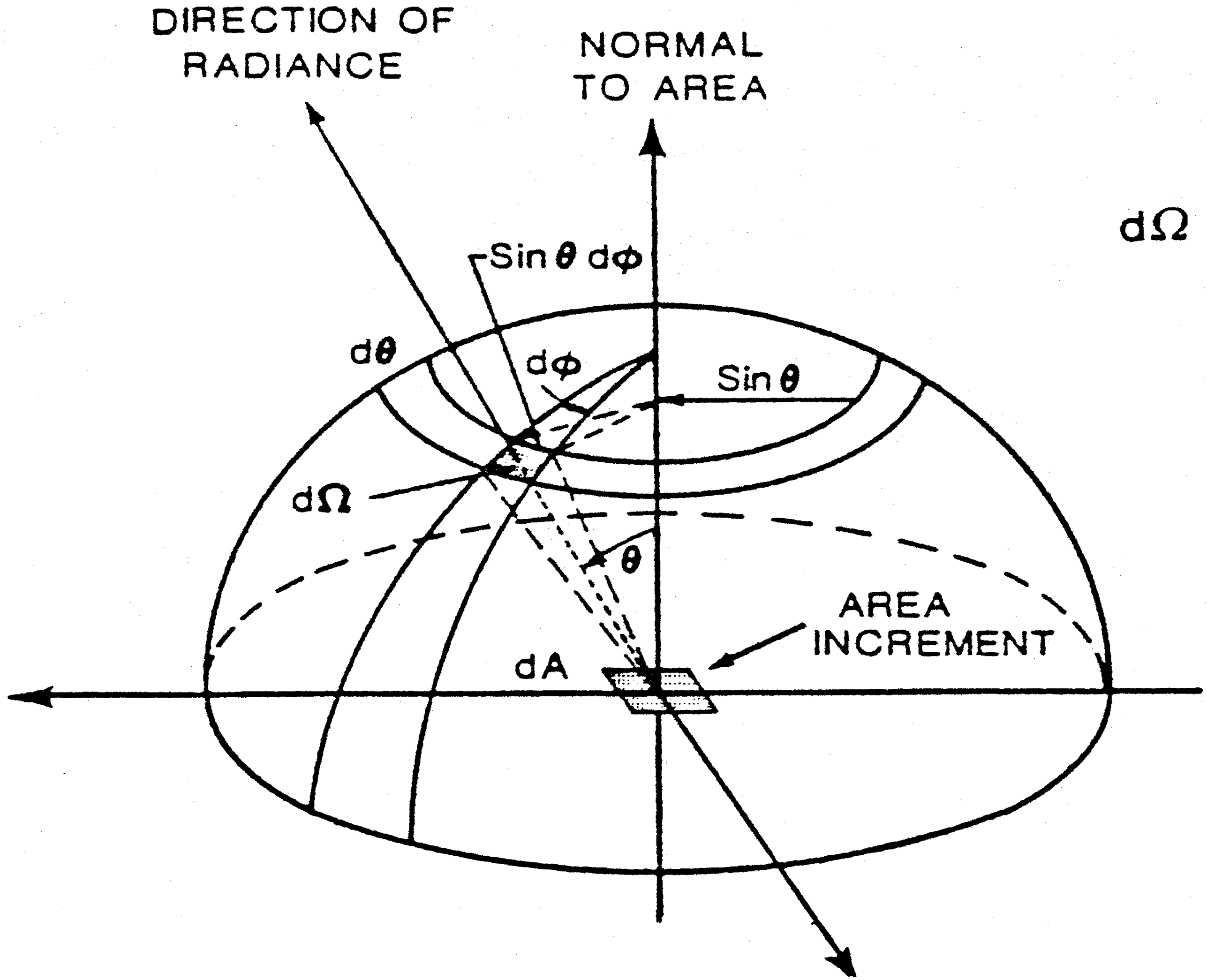
and is expressed in watts per square metre per micrometer per steradian. This quantity is often also referred to as intensity and denoted by the letter B (when referring to the Planck function).

If the zenith angle, θ, is the angle between the direction of the radiation and the normal to the surface, then the component of the radiance normal to the surface is then given by $I \cos \theta$. The irradiance represents the combined effects of the normal component of the radiation coming from the whole hemisphere; that is,

$$E = \int_{\Omega} I \cos \theta \, d\Omega \qquad \text{where in spherical coordinates } d\Omega = \sin \theta \, d\theta \, d\varphi \, .$$

Radiation whose radiance is independent of direction is called isotropic radiation. In this case, the integration over $d\Omega$ can be readily shown to be equal to π so that

$$E = \pi I \, .$$



$$d\Omega = \sin\theta \, d\theta \, d\varphi$$

spherical coordinates and solid angle considerations

Radiation is governed by Planck's Law

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

Summing the Planck function at one temperature over all wavelengths yields the energy of the radiating source

$$E = \sum_{\lambda} B(\lambda, T) = \sigma T^4$$

Brightness temperature is uniquely related to radiance for a given wavelength by the Planck function.

Using wavenumbers

Planck's Law $B(\nu,T) = c_1\nu^3 / [e^{c_2\nu/T} - 1]$ $(mW/m^2/ster/cm^{-1})$

where ν = # wavelengths in one centimeter (cm-1)

T = temperature of emitting surface (deg K)

c_1 = 1.191044 x 10-5 $(mW/m^2/ster/cm^{-4})$

c_2 = 1.438769 (cm deg K)

Wien's Law $dB(\nu_{max},T) / dT = 0$ where $\nu(max) = 1.95T$

indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\nu_{max},T) \sim T^{**}3$.

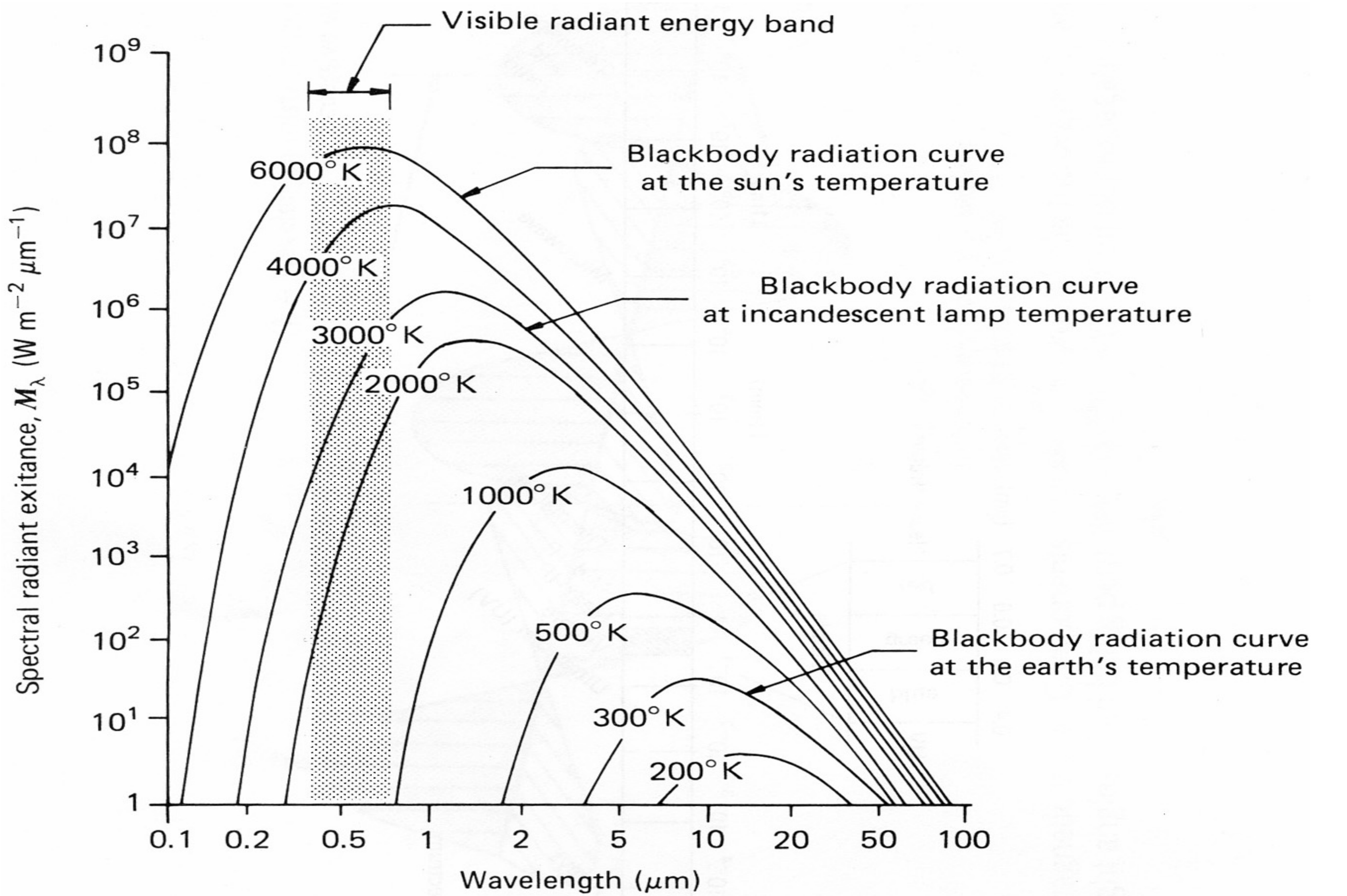
Stefan-Boltzmann Law $E = \pi \int_o^\infty B(\nu,T)\, d\nu = \sigma T^4$, where σ = 5.67 x 10-8 W/m2/deg4.

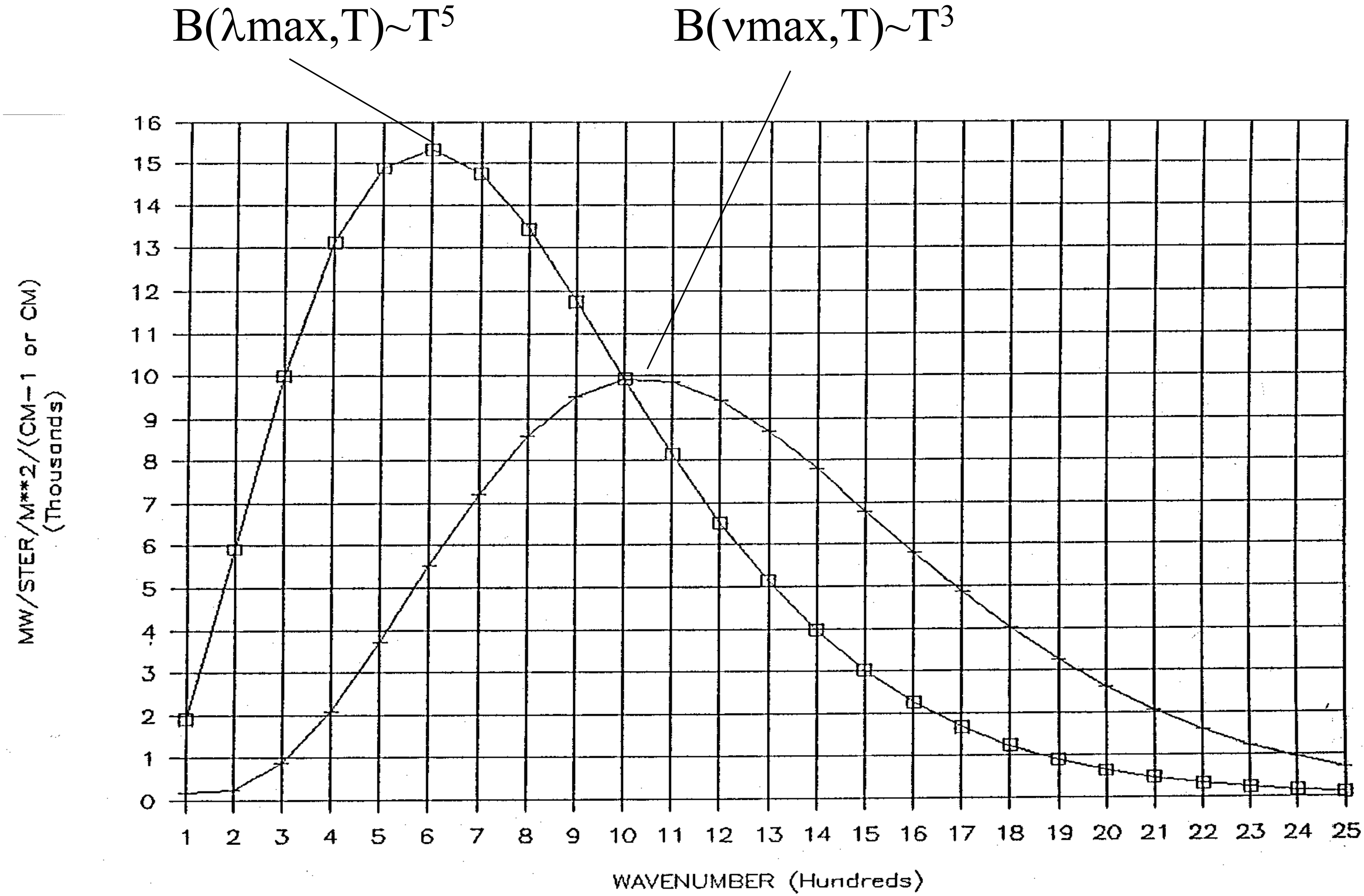
states that irradiance of a black body (area under Planck curve) is proportional to T^4 .

Brightness Temperature

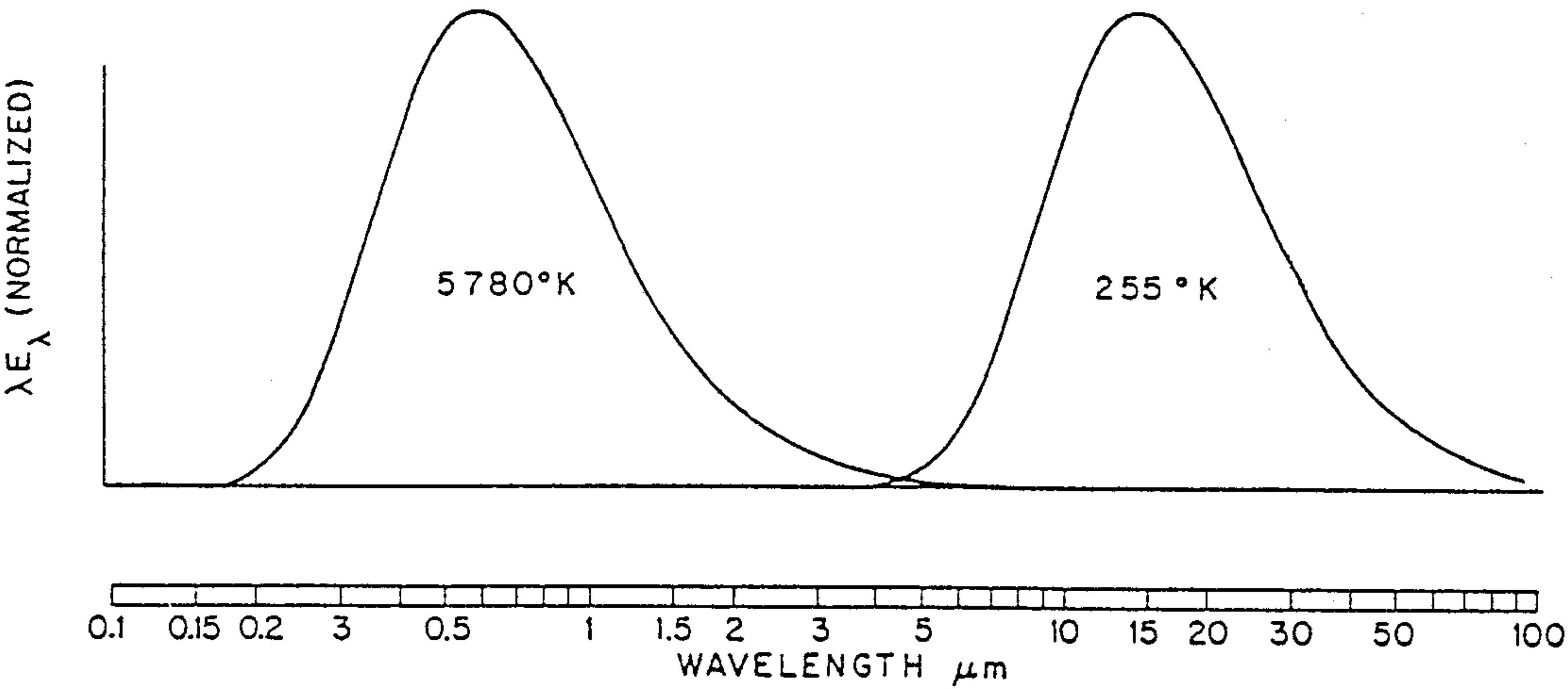
$$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$ is determined by inverting Planck function

Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures

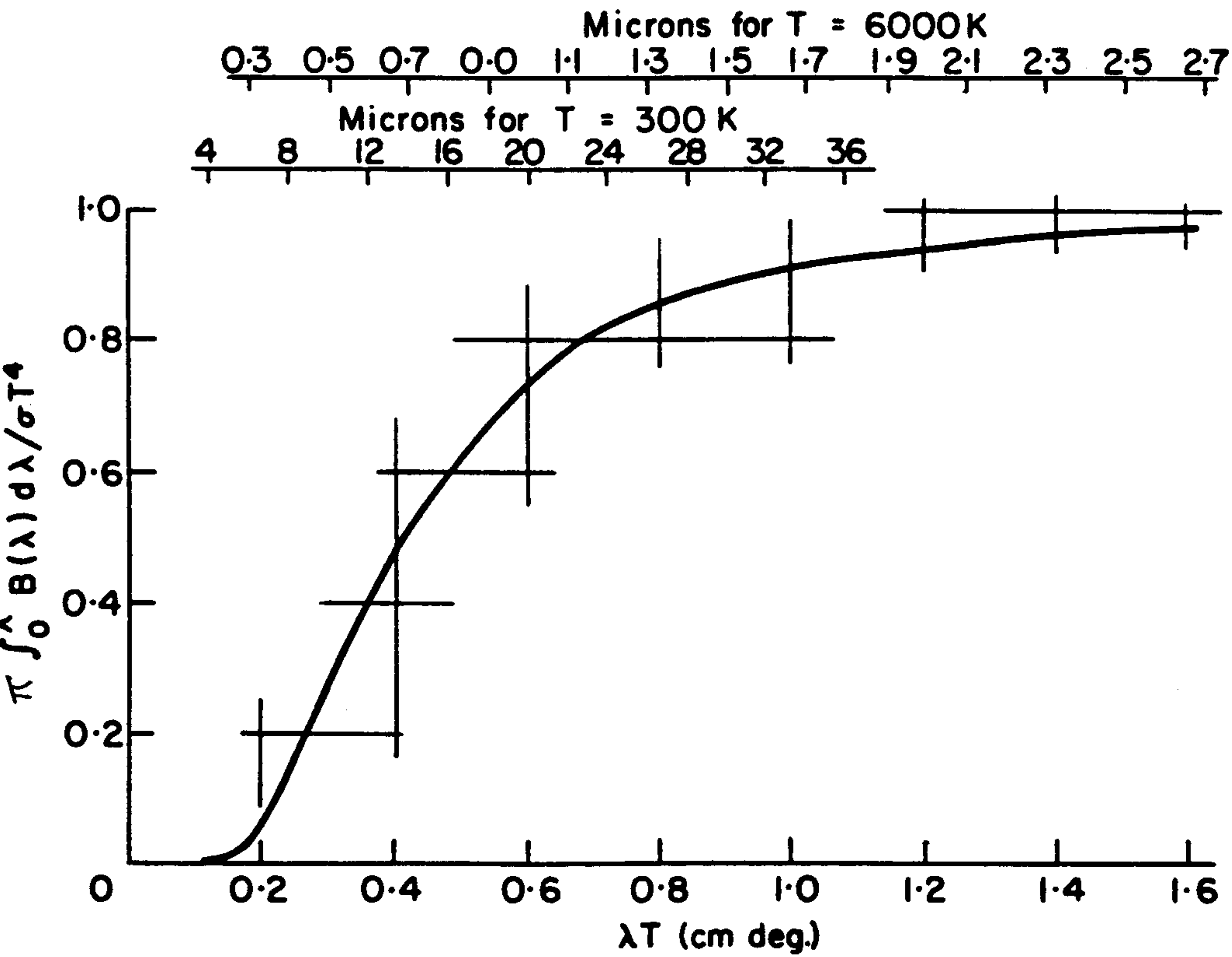




B(λ,T) versus B(ν,T)

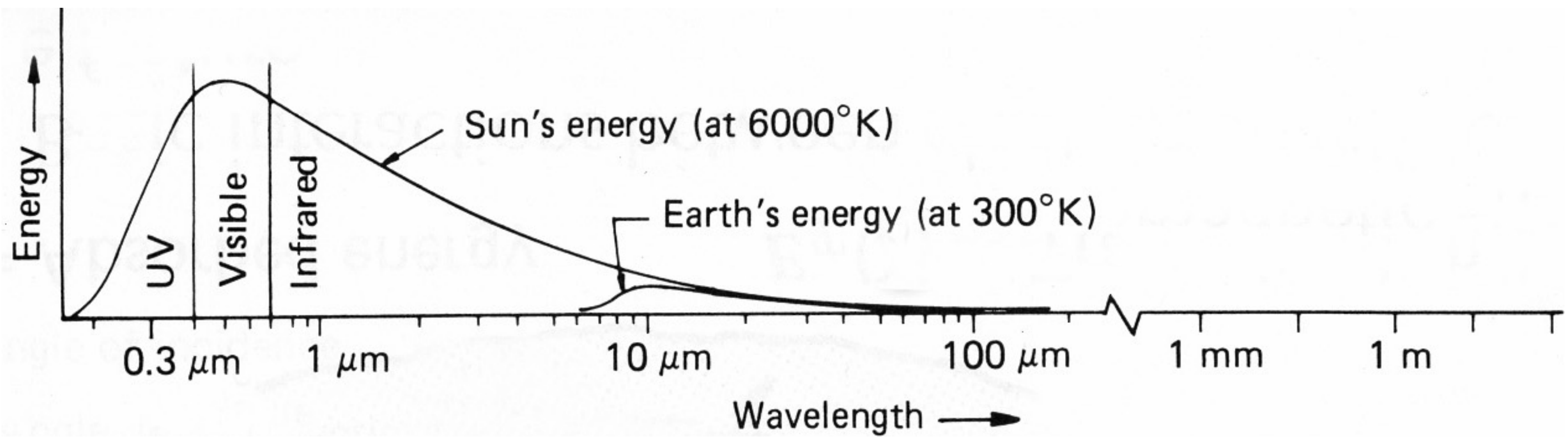


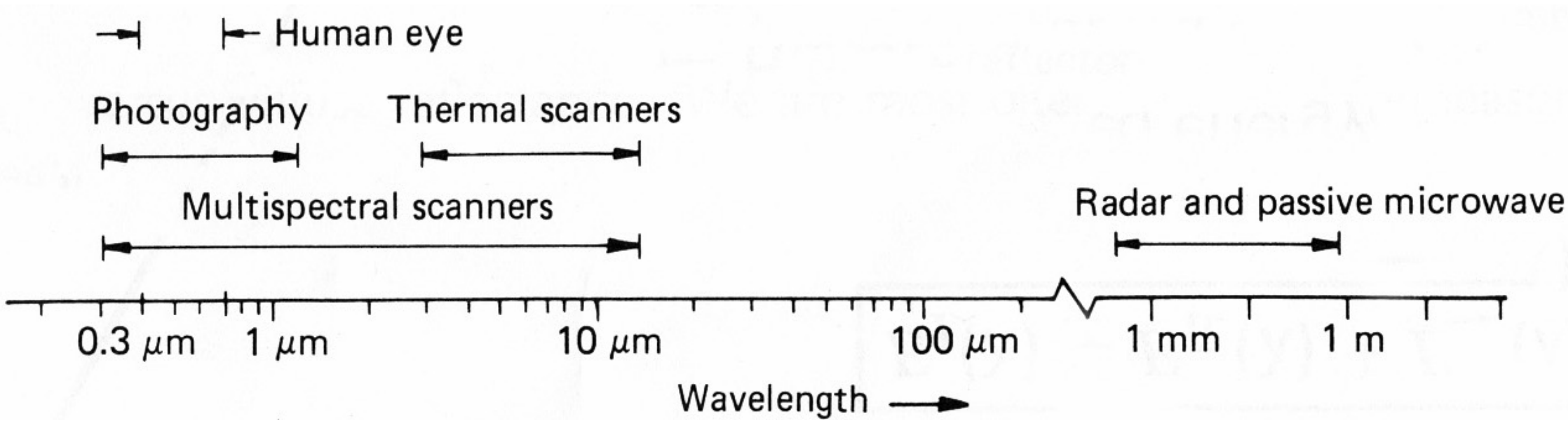
Normalized black body spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength so that the area under the curves is proportional to irradiance.



Microns for T = 6000 K
0·3 0·5 0·7 0·0 1·1 1·3 1·5 1·7 1·9 2·1 2·3 2·5 2·7
Microns for T = 300 K
4 8 12 16 20 24 28 32 36
1·0
0·8
0·6
0·4
0·2
0
0·2 0·4 0·6 0·8 1·0 1·2 1·4 1·6
λT (cm deg.)
π ∫₀^λ B(λ) dλ/σT⁴

Spectral Characteristics of Energy Sources and Sensing Systems





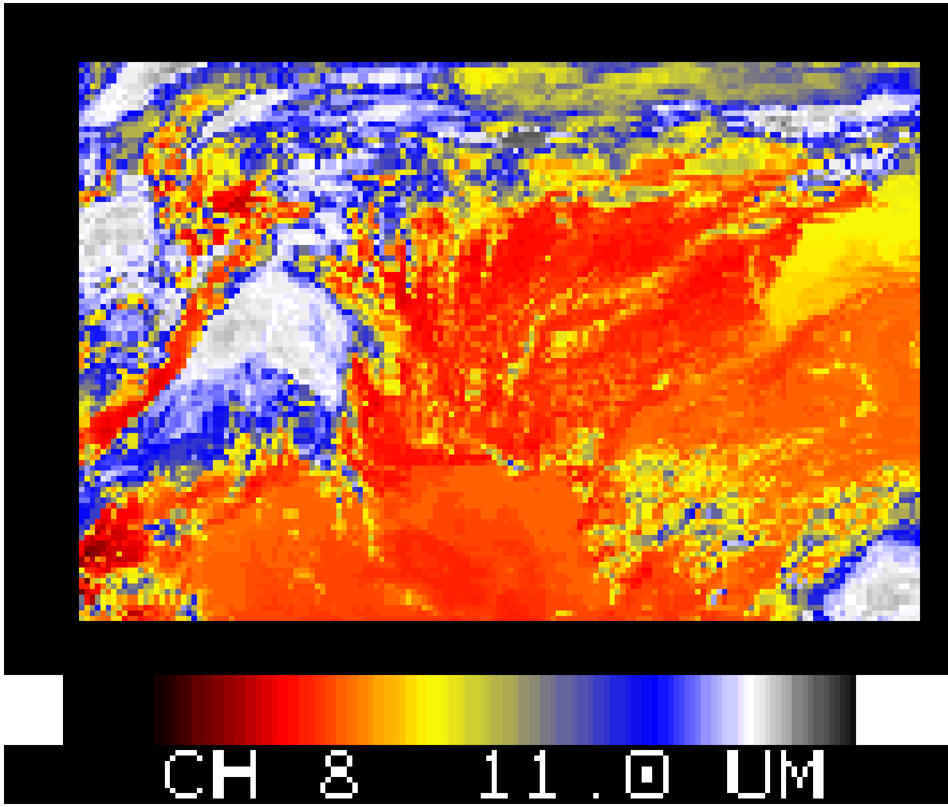
Temperature sensitivity, or the percentage change in radiance corresponding to a percentage change in temperature, α, is defined as

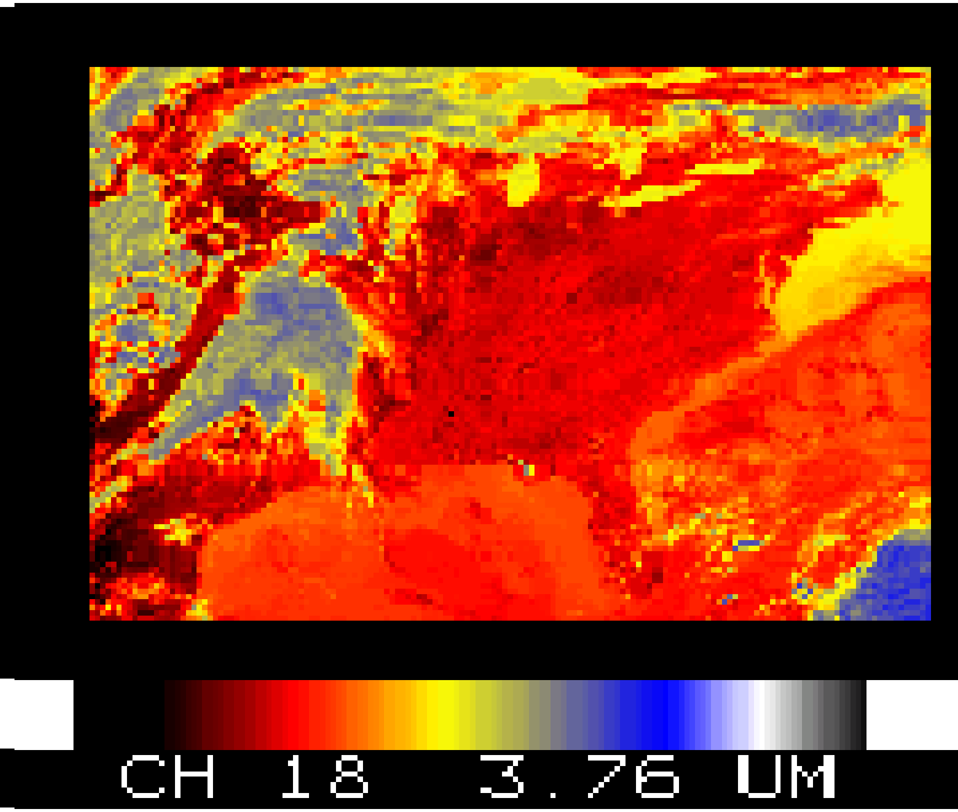
$$dB/B = \alpha \, dT/T.$$

The temperature sensivity indicates the power to which the Planck radiance depends on temperature, since B proportional to T^α satisfies the equation. For infrared wavelengths,

$$\alpha = c_2\nu/T = c_2/\lambda T.$$

Wavenumber	Typical Scene Temperature	Temperature Sensitivity
700	220	4.58
900	300	4.32
1200	300	5.76
1600	240	9.59
2300	220	15.04
2500	300	11.99

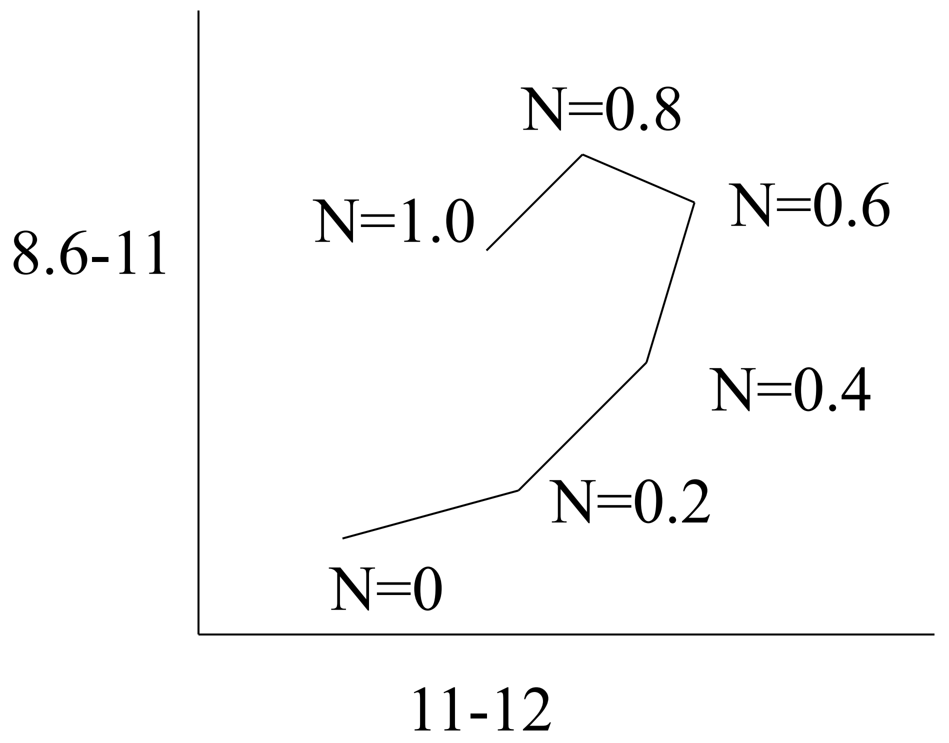


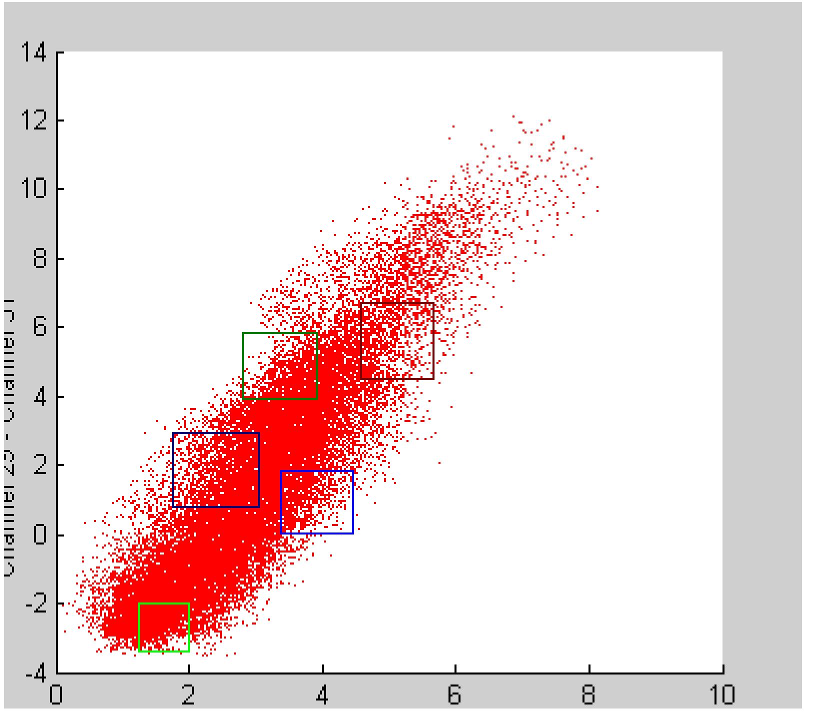


Cloud edges and broken clouds appear different in 11 and 4 um images.

T(11)**4=(1-N)*Tclr**4+N*Tcld**4~(1-N)*300**4+N*200**4
T(4)**12=(1-N)*Tclr**12+N*Tcld**12~(1-N)*300**12+N*200**12

Cold part of pixel has more influence for B(11) than B(4)





Broken clouds appear different in 8.6, 11 and 12 um images;
assume Tclr=300 and Tcld=230

$$T(11)-T(12)=[(1-N)*B11(Tclr)+N*B11(Tcld)]^{-1} - [(1-N)*B12(Tclr)+N*B12(Tcld)]^{-1}$$

$$T(8.6)-T(11)=[(1-N)*B8.6(Tclr)+N*B8.6(Tcld)]^{-1} - [(1-N)*B11(Tclr)+N*B11(Tcld)]^{-1}$$

Cold part of pixel has more influence at longer wavelengths

Emission, Absorption, Reflection, and Scattering

Blackbody radiation B_λ represents the upper limit to the amount of radiation that a real substance may emit at a given temperature for a given wavelength.

Emissivity ε_λ is defined as the fraction of emitted radiation R_λ to Blackbody radiation,

$$\varepsilon_\lambda = R_\lambda / B_\lambda \ .$$

In a medium at thermal equilibrium, what is absorbed is emitted (what goes in comes out) so

$$a_\lambda = \varepsilon_\lambda \ .$$

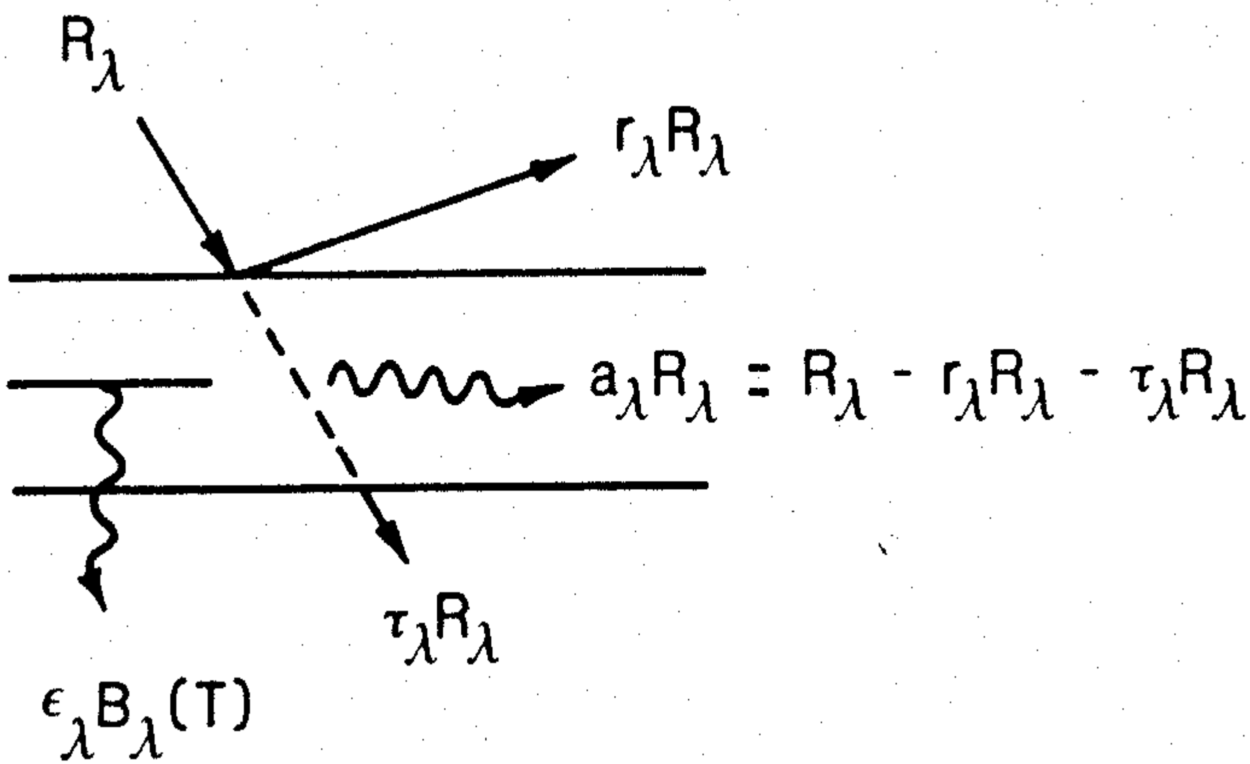
Thus, materials which are strong absorbers at a given wavelength are also strong emitters at that wavelength; similarly weak absorbers are weak emitters.

If a_λ, r_λ, and τ_λ represent the fractional absorption, reflectance, and transmittance, respectively, then conservation of energy says

$$a_\lambda + r_\lambda + \tau_\lambda = 1 \ .$$

For a blackbody $a_\lambda = 1$, it follows that $r_\lambda = 0$ and $\tau_\lambda = 0$ for blackbody radiation. Also, for a perfect window $\tau_\lambda = 1$, $a_\lambda = 0$ and $r_\lambda = 0$. For any opaque surface $\tau_\lambda = 0$, so radiation is either absorbed or reflected $a_\lambda + r_\lambda = 1$.

At any wavelength, strong reflectors are weak absorbers (i.e., snow at visible wavelengths), and weak reflectors are strong absorbers (i.e., asphalt at visible wavelengths).



"ENERGY CONSERVATION"

Planetary Albedo

Planetary albedo is defined as the fraction of the total incident solar irradiance, S, that is reflected back into space. Radiation balance then requires that the absorbed solar irradiance is given by

$$E = (1 - A) S/4.$$

The factor of one-fourth arises because the cross sectional area of the earth disc to solar radiation, πr^2, is one-fourth the earth radiating surface, $4\pi r^2$. Thus recalling that $S = 1380 \ Wm^{-2}$, if the earth albedo is 30 percent, then $E = 241 \ Wm^{-2}$.

Selective Absorption and Transmission

Assume that the earth behaves like a blackbody and that the atmosphere has an absorptivity a_S for incoming solar radiation and a_L for outgoing longwave radiation. Let Y_a be the irradiance emitted by the atmosphere (both upward and downward); Y_s the irradiance emitted from the earth's surface; and E the solar irradiance absorbed by the earth-atmosphere system. Then, radiative equilibrium requires

$$E - (1-a_L)\, Y_s - Y_a = 0 \text{ , at the top of the atmosphere,}$$
$$(1-a_S)\, E - Y_s + Y_a = 0 \text{ , at the surface.}$$

Solving yields

$$Y_s = \frac{(2-a_S)}{(2-a_L)} E \text{ , and}$$

$$Y_a = \frac{(2-a_L) - (1-a_L)(2-a_S)}{(2-a_L)} E \text{ .}$$

Since $a_L > a_S$, the irradiance and hence the radiative equilibrium temperature at the earth surface is increased by the presence of the atmosphere. With $a_L = .8$ and $a_S = .1$ and $E = 241$ Wm^{-2}, Stefans Law yields a blackbody temperature at the surface of 286 K, in contrast to the 255 K it would be if the atmospheric absorptance was independent of wavelength ($a_S = a_L$). The atmospheric gray body temperature in this example turns out to be 245 K.

Expanding on the previous example, let the atmosphere be represented by two layers and let us compute the vertical profile of radiative equilibrium temperature. For simplicity in our two layer atmosphere, let $a_S = 0$ and $a_L = a = .5$, u indicate upper layer, l indicate lower layer, and s denote the earth surface. Schematically we have:

$\downarrow E \quad \uparrow (1-a)^2 Y_s \quad \uparrow (1-a) Y_l \quad \uparrow Y_u$	top of the atmosphere
$\downarrow E \quad \uparrow (1-a) Y_s \quad \uparrow Y_l \quad \downarrow Y_u$	middle of the atmosphere
$\downarrow E \quad \uparrow Y_s \quad \downarrow Y_l \quad \downarrow (1-a) Y_u$	earth surface.

Radiative equilibrium at each surface requires

$$E = .25\,Y_s + .5\,Y_l + Y_u\,,$$
$$E = .5\,Y_s + Y_l - Y_u\,,$$
$$E = Y_s - Y_l - .5\,Y_u\,.$$

Solving yields $Y_s = 1.6$ E, $Y_l = .5$ E and $Y_u = .33$ E. The radiative equilibrium temperatures (blackbody at the surface and gray body in the atmosphere) are readily computed.

$$T_s = [1.6E / \sigma]^{1/4} = 287\text{ K}\,,$$
$$T_l = [0.5E / 0.5\sigma]^{1/4} = 255\text{ K}\,,$$
$$T_u = [0.33E / 0.5\sigma]^{1/4} = 231\text{ K}\,.$$

Thus, a crude temperature profile emerges for this simple two-layer model of the atmosphere.

Transmittance

Transmission through an absorbing medium for a given wavelength is governed by the number of intervening absorbing molecules (path length u) and their absorbing power (k_λ) at that wavelength. Beer's law indicates that transmittance decays exponentially with increasing path length

$$\tau_\lambda (z \rightarrow \infty) = e^{-k_\lambda u(z)}$$

where the path length is given by $u(z) = \int_z^\infty \rho \, dz$.

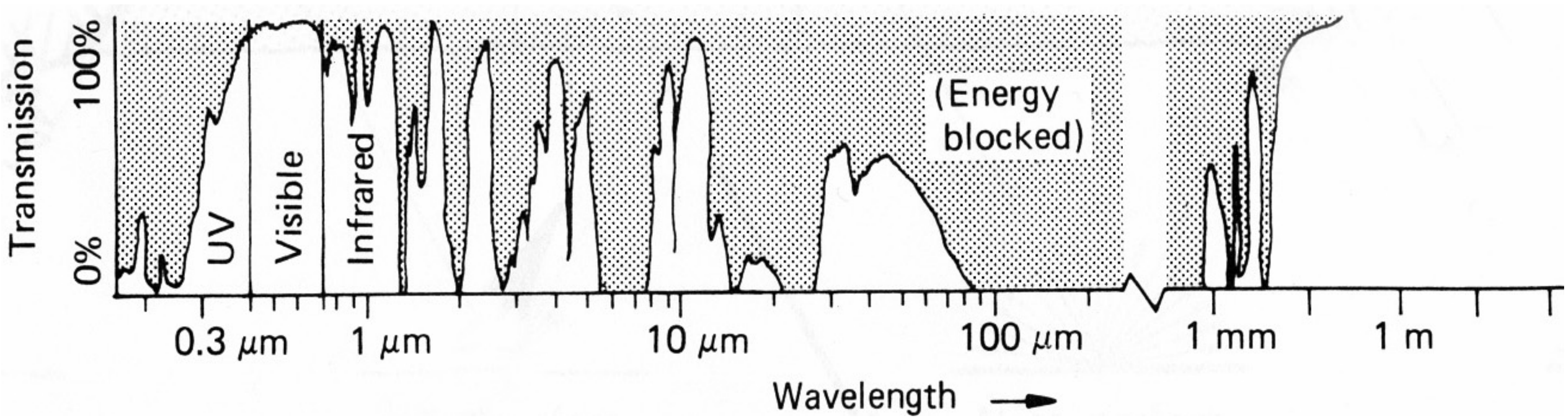
$k_\lambda u$ is a measure of the cumulative depletion that the beam of radiation has experienced as a result of its passage through the layer and is often called the optical depth σ_λ.

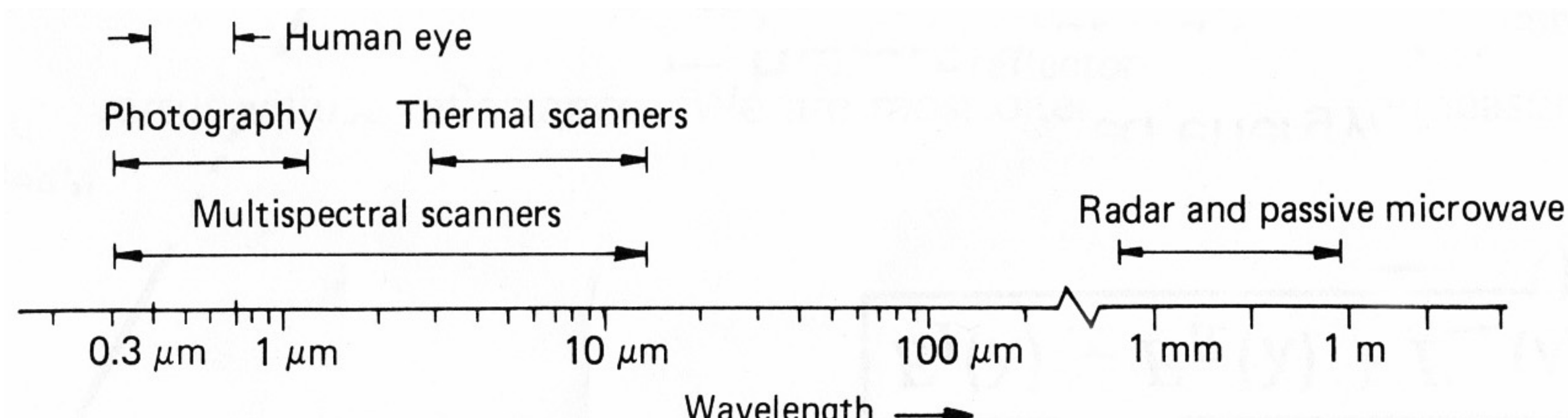
Realizing that the hydrostatic equation implies $g \rho \, dz = - q \, dp$

where q is the mixing ratio and ρ is the density of the atmosphere, then

$$u(p) = \int_o^p q \, g^{-1} \, dp \quad \text{and} \quad \tau_\lambda (p \rightarrow o) = e^{-k_\lambda u(p)} \quad .$$

Spectral Characteristics of Atmospheric Transmission and Sensing Systems



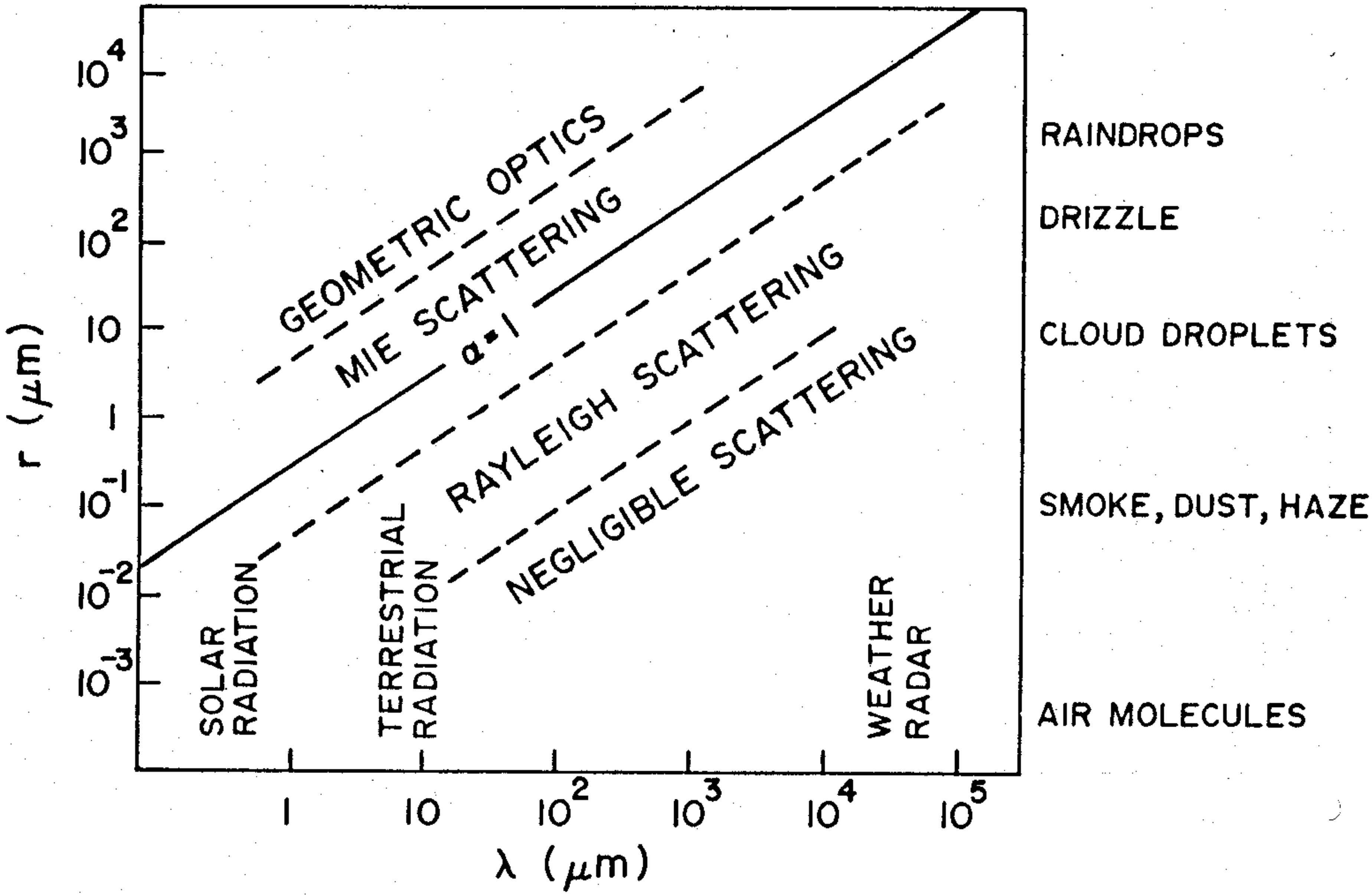


Relative Effects of Radiative Processes

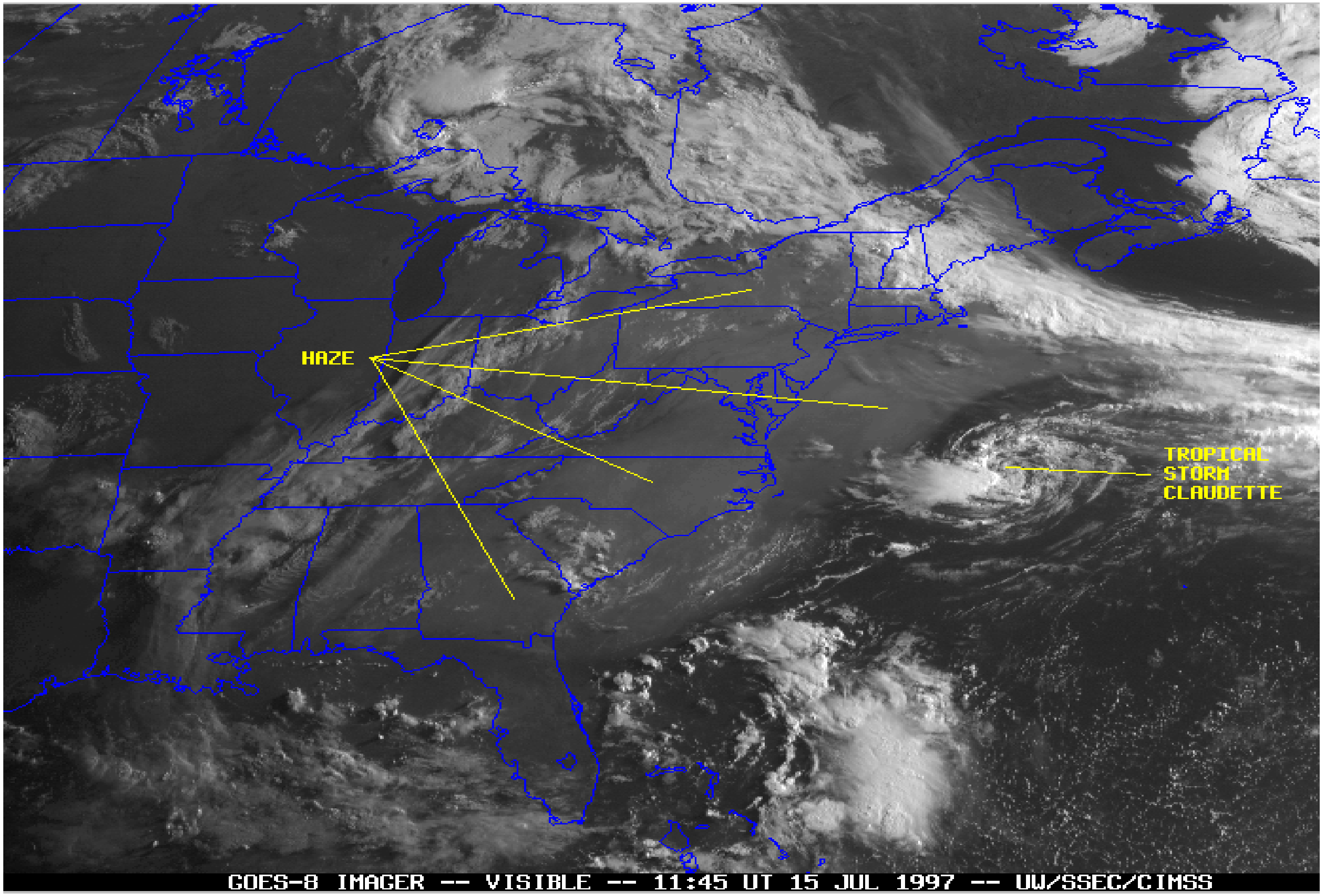
Sun - Earth - Atmosphere Energy System

		Solar Radiation		Terrestrial Radiation	
		Absorption / Emission	Scattering	Absorption / Emission	Scattering
Clouds	Water	✓ Small	✓ Large	✓ Moderate	✓ Negligible
	Ice	✓ Variable	✓ Moderate	✓ Small	✓ Negligible
Molecules in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
Aerosols in the Atmosphere		✓ Small	✓ Moderate	✓ Variable	✓ Negligible
Earth's Surface	Land	✓ Large	✓ Moderate	✓ Large	✓ Negligible
	Water	✓ Large	✓ Small	✓ Large	✓ Negligible
	Snow / Ice	✓ Variable	✓ Large	✓ Variable	✓ Negligible

Earth


GEOMETRIC OPTICS
MIE SCATTERING
α=1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES
r (μm)
λ (μm)
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
1
10
10^2
10^3
10^4
10^5

Scattering of early morning sun light from haze



Schwarzchild's equation

At wavelengths of terrestrial radiation, absorption and emission are equally important and must be considered simultaneously. Absorption of terrestrial radiation along an upward path through the atmosphere is described by the relation

$$-dL_\lambda^{abs} = L_\lambda \, k_\lambda \, \rho \sec \varphi \, dz \, .$$

Making use of Kirchhoff's law it is possible to write an analogous expression for the emission,

$$dL_\lambda^{em} = B_\lambda \, d\varepsilon_\lambda = B_\lambda \, da_\lambda = B_\lambda \, k_\lambda \, \rho \sec \varphi \, dz \, ,$$

where B_λ is the blackbody monochromatic radiance specified by Planck's law. Together

$$dL_\lambda = -(L_\lambda - B_\lambda) \, k_\lambda \, \rho \sec \varphi \, dz \, .$$

This expression, known as Schwarzchild's equation, is the basis for computations of the transfer of infrared radiation.

Schwarzschild to RTE

$$dL_\lambda = -(L_\lambda - B_\lambda)\, k_\lambda\, \rho\, dz$$

but

$$d\tau_\lambda = \tau_\lambda\, k\, \rho\, dz \quad \text{since} \quad \tau_\lambda = \exp\,[-\, k_\lambda \int_z^\infty \rho\, dz].$$

so

$$\tau_\lambda\, dL_\lambda = -(L_\lambda - B_\lambda)\, d\tau_\lambda$$

$$\tau_\lambda\, dL_\lambda + L_\lambda\, d\tau_\lambda = B_\lambda d\tau_\lambda$$

$$d\,(L_\lambda\, \tau_\lambda\,) = B_\lambda d\tau_\lambda$$

Integrate from 0 to ∞

$$L_\lambda(\infty)\, \tau_\lambda(\infty) - L_\lambda(0)\, \tau_\lambda(0) = \int_0^\infty B_\lambda\, [d\tau_\lambda / dz]\, dz.$$

and

$$L_\lambda(\text{sat}) = L_\lambda(\text{sfc})\, \tau_\lambda(\text{sfc}) + \int_0^\infty B_\lambda\, [d\tau_\lambda / dz]\, dz.$$

Radiative Transfer Equation

The radiance leaving the earth-atmosphere system sensed by a satellite borne radiometer is the sum of radiation emissions from the earth-surface and each atmospheric level that are transmitted to the top of the atmosphere. Considering the earth's surface to be a blackbody emitter (emissivity equal to unity), the upwelling radiance intensity, I_λ, for a cloudless atmosphere is given by the expression

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T_{sfc})\, \tau_\lambda(sfc - top) + \sum_{layers} \varepsilon_\lambda^{layer} B_\lambda(T_{layer})\, \tau_\lambda(layer - top)$$

where the first term is the surface contribution and the second term is the atmospheric contribution to the radiance to space.

In standard notation,

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) + \sum_p \varepsilon_\lambda(\Delta p) B_\lambda(T(p)) \tau_\lambda(p)$$

The emissivity of an infinitesimal layer of the atmosphere at pressure p is equal to the absorptance (one minus the transmittance of the layer). Consequently,

$$\varepsilon_\lambda(\Delta p) \tau_\lambda(p) = [1 - \tau_\lambda(\Delta p)] \tau_\lambda(p)$$

Since transmittance is an exponential function of depth of absorbing constituent,

$$\tau_\lambda(\Delta p) \tau_\lambda(p) = \exp [- \int_p^{p+\Delta p} k_\lambda q g^{-1} dp] * \exp [- \int_o^p k_\lambda q g^{-1} dp] = \tau_\lambda(p + \Delta p)$$

Therefore

$$\varepsilon_\lambda(\Delta p) \tau_\lambda(p) = \tau_\lambda(p) - \tau_\lambda(p + \Delta p) = - \Delta\tau_\lambda(p) .$$

So we can write

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) - \sum_p B_\lambda(T(p)) \Delta\tau_\lambda(p) .$$

which when written in integral form reads

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p)) [d\tau_\lambda(p) / dp] dp .$$

When reflection from the earth surface is also considered, the Radiative Transfer Equation for infrared radiation can be written

$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T_s)\, \tau_\lambda(p_s) + \int_{p_s}^{o} B_\lambda(T(p))\, F_\lambda(p)\, [d\tau_\lambda(p)/ dp]\, dp$$

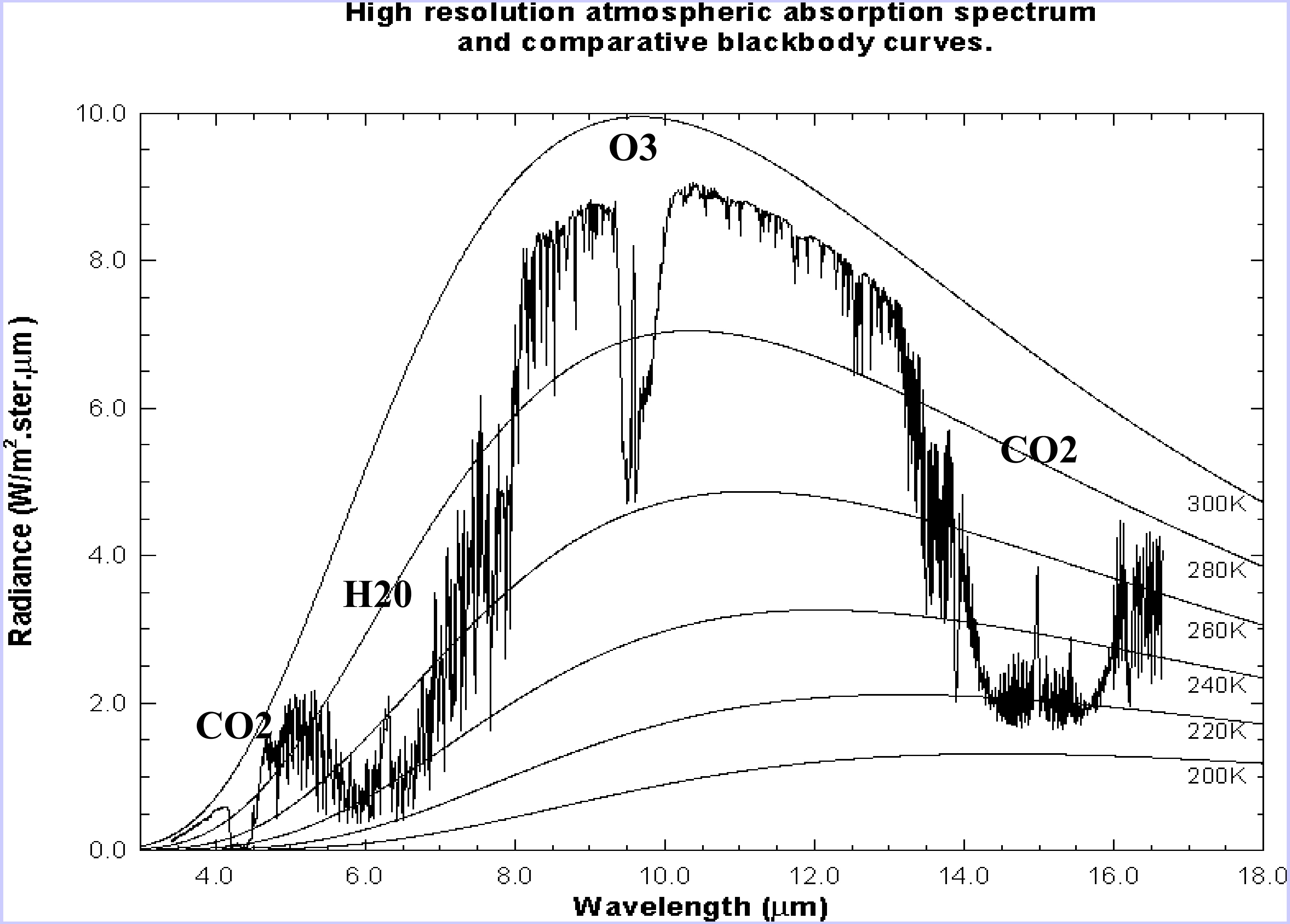
where

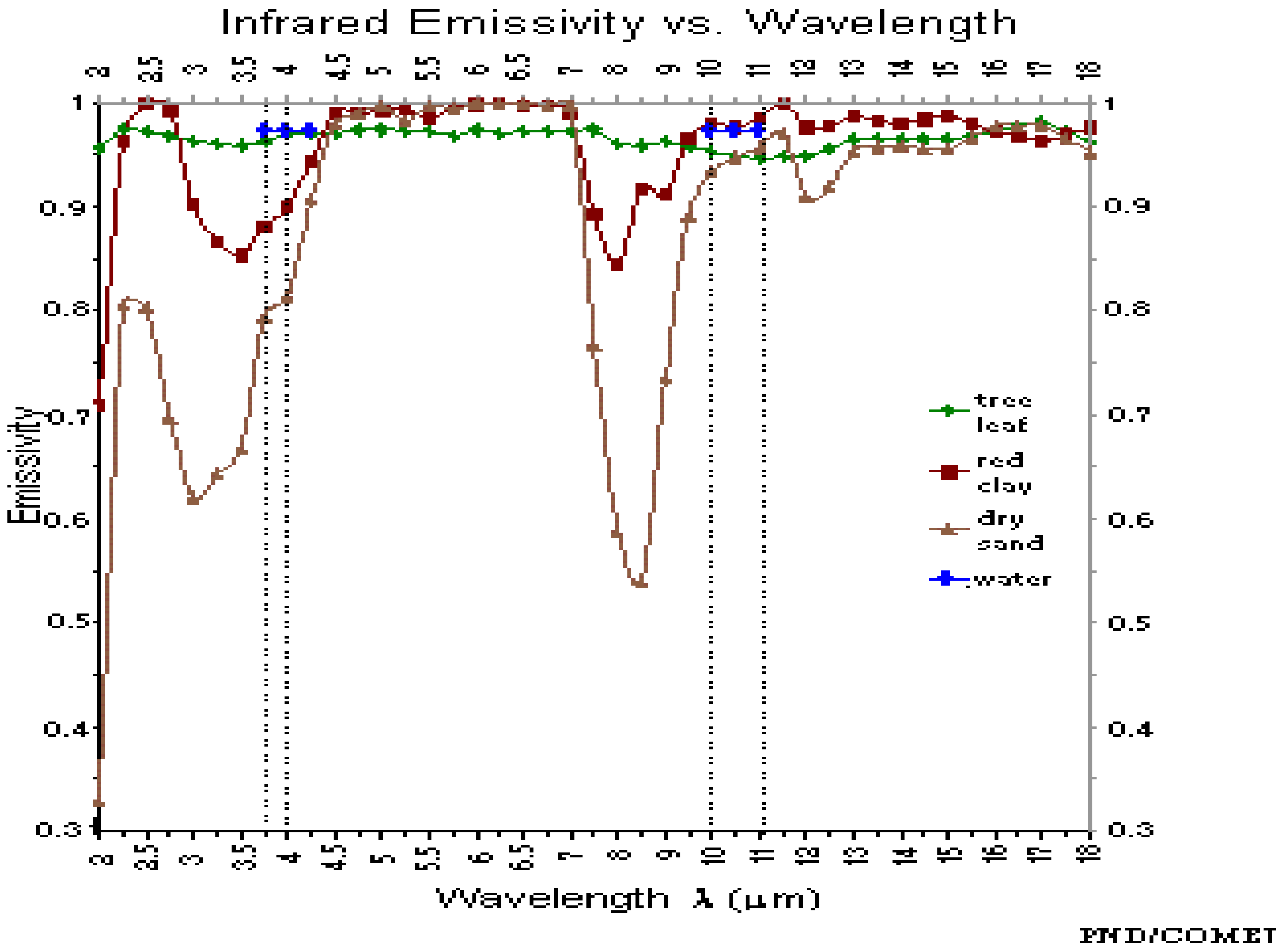
$$F_\lambda(p) = \{ 1 + (1 - \varepsilon_\lambda) [\tau_\lambda(p_s) / \tau_\lambda(p)]^2 \}$$

The first term is the spectral radiance emitted by the surface and attenuated by the atmosphere, often called the boundary term and the second term is the spectral radiance emitted to space by the atmosphere directly or by reflection from the earth surface.

The atmospheric contribution is the weighted sum of the Planck radiance contribution from each layer, where the weighting function is [$d\tau_\lambda(p) / dp$]. This weighting function is an indication of where in the atmosphere the majority of the radiation for a given spectral band comes from.

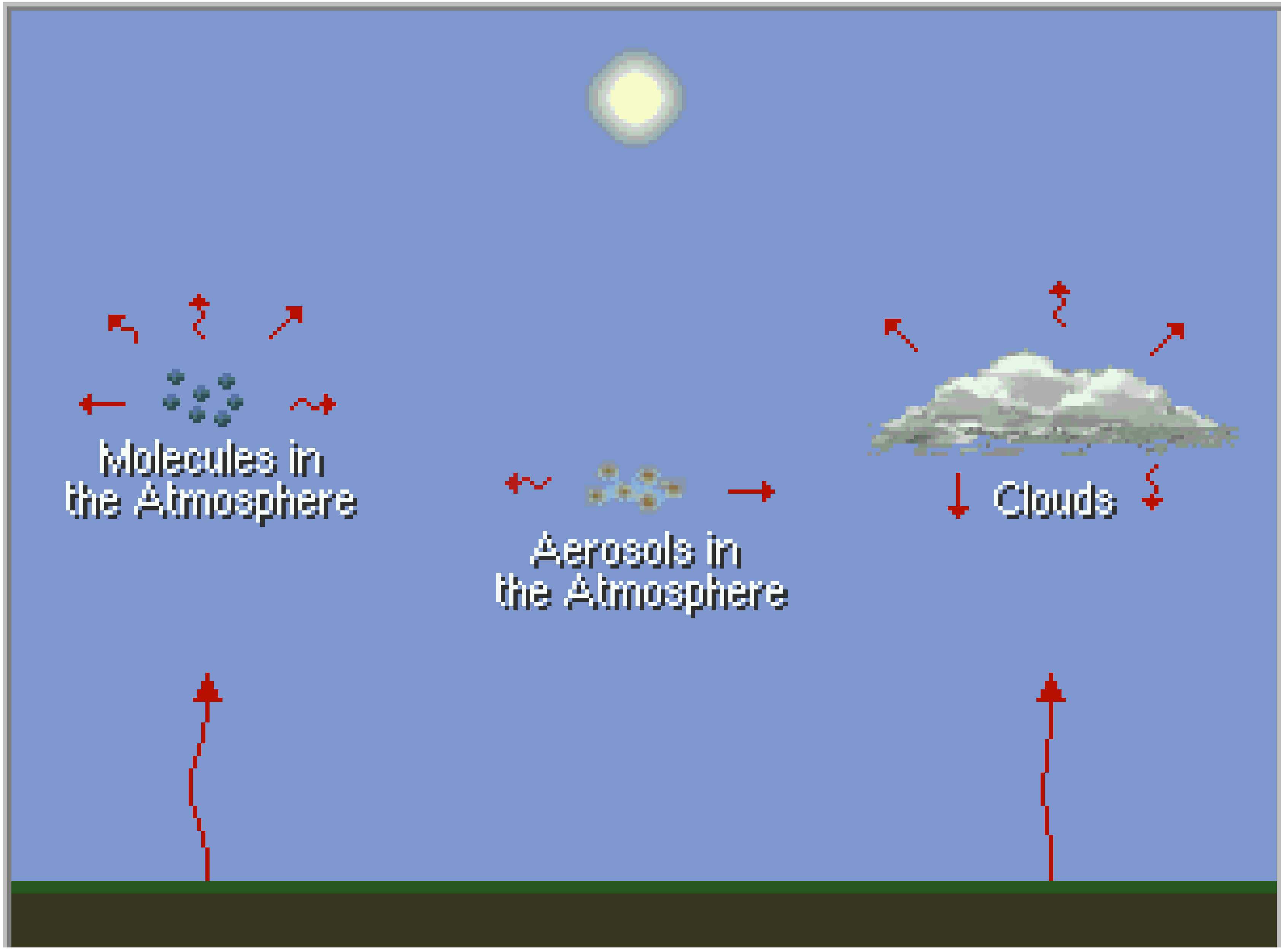
Earth emitted spectra overlaid on Planck function envelopes



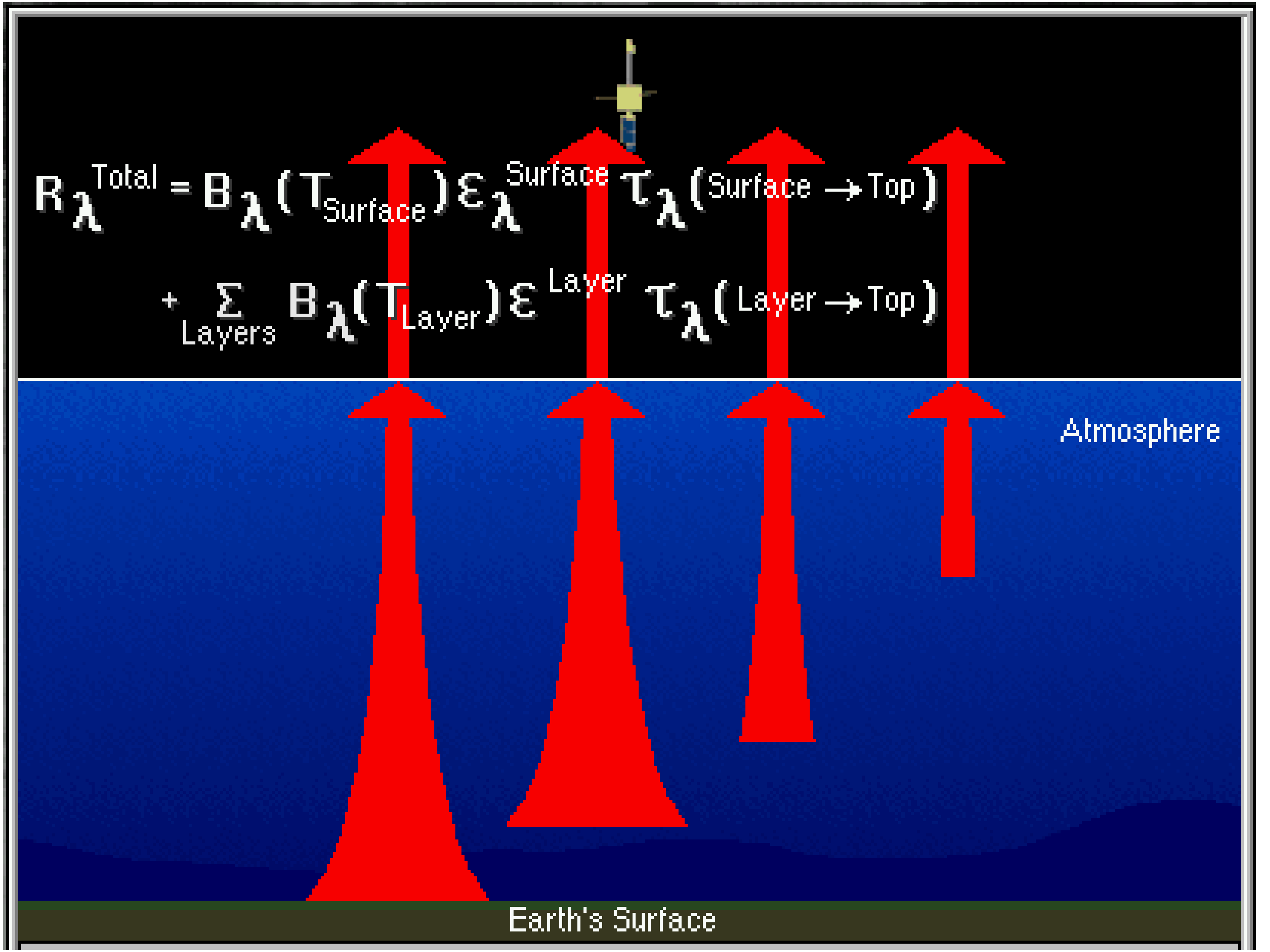


Infrared Emissivity vs. Wavelength
Emissivity
Wavelength λ (μm)
tree leaf
red clay
dry sand
water
PND/COMET

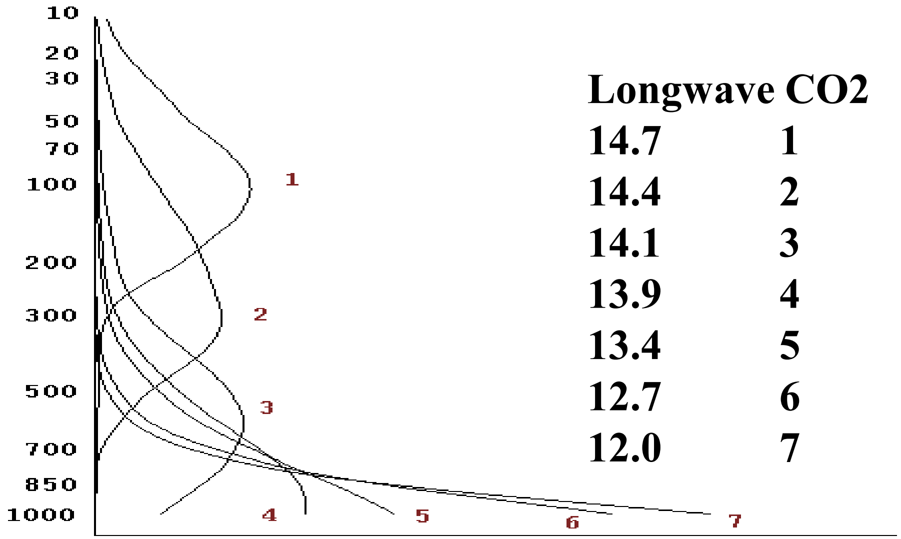
Re-emission of Infrared Radiation



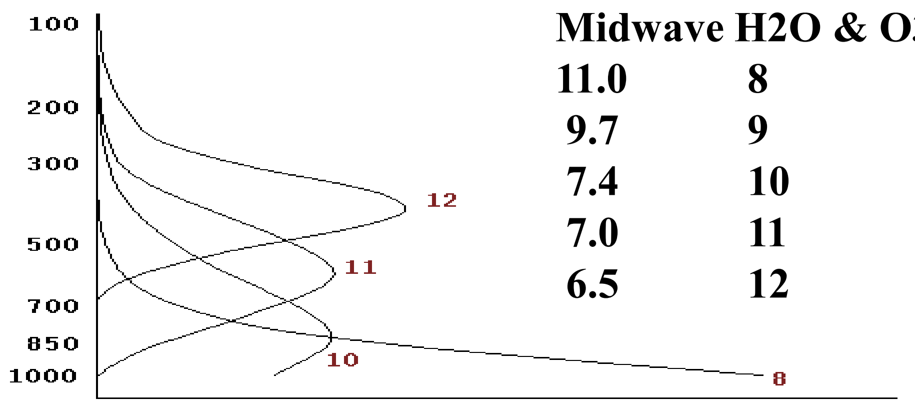
Radiative Transfer through the Atmosphere



Weighting Functions



Longwave CO_2			
14.7	1	680	CO_2, strat temp
14.4	2	696	CO_2, strat temp
14.1	3	711	CO_2, upper trop temp
13.9	4	733	CO_2, mid trop temp
13.4	5	748	CO_2, lower trop temp
12.7	6	790	H_2O, lower trop moisture
12.0	7	832	H_2O, dirty window



Midwave H_2O & O_3			
11.0	8	907	window
9.7	9	1030	O_3, strat ozone
7.4	10	1345	H_2O, lower mid trop moisture
7.0	11	1425	H_2O, mid trop moisture
6.5	12	1535	H_2O, upper trop moisture

Characteristics of RTE

* Radiance arises from deep and overlapping layers

* The radiance observations are not independent

* There is no unique relation between the spectrum of the outgoing radiance and T(p) or Q(p)

* T(p) is buried in an exponent in the denominator in the integral

* Q(p) is implicit in the transmittance

* Boundary conditions are necessary for a solution; the better the first guess the better the final solution

To investigate the RTE further consider the atmospheric contribution to the radiance to space of an infinitesimal layer of the atmosphere at height z, $dI_\lambda(z) = B_\lambda(T(z))\, d\tau_\lambda(z)$.

Assume a well-mixed isothermal atmosphere where the density drops off exponentially with height $\rho = \rho_o \exp(-\gamma z)$, and assume k_λ is independent of height, so that the optical depth can be written for normal incidence

$$\sigma_\lambda = \int_z^\infty k_\lambda \rho \, dz = \gamma^{-1} k_\lambda \rho_o \exp(-\gamma z)$$

and the derivative with respect to height

$$\frac{d\sigma_\lambda}{dz} = - k_\lambda \rho_o \exp(-\gamma z) = -\gamma \sigma_\lambda \, .$$

Therefore, we may obtain an expression for the detected radiance per unit thickness of the layer as a function of optical depth,

$$\frac{dI_\lambda(z)}{dz} = B_\lambda(T_{const}) \frac{d\tau_\lambda(z)}{dz} = B_\lambda(T_{const})\, \gamma\, \sigma_\lambda \exp(-\sigma_\lambda) \, .$$

The level which is emitting the most detected radiance is given by

$$\frac{d}{dz} \left\{ \frac{dI_\lambda(z)}{dz} \right\} = 0 \text{ , or where } \sigma_\lambda = 1.$$

Most of monochromatic radiance detected is emitted by layers near level of unit optical depth.

Profile Retrieval from Sounder Radiances

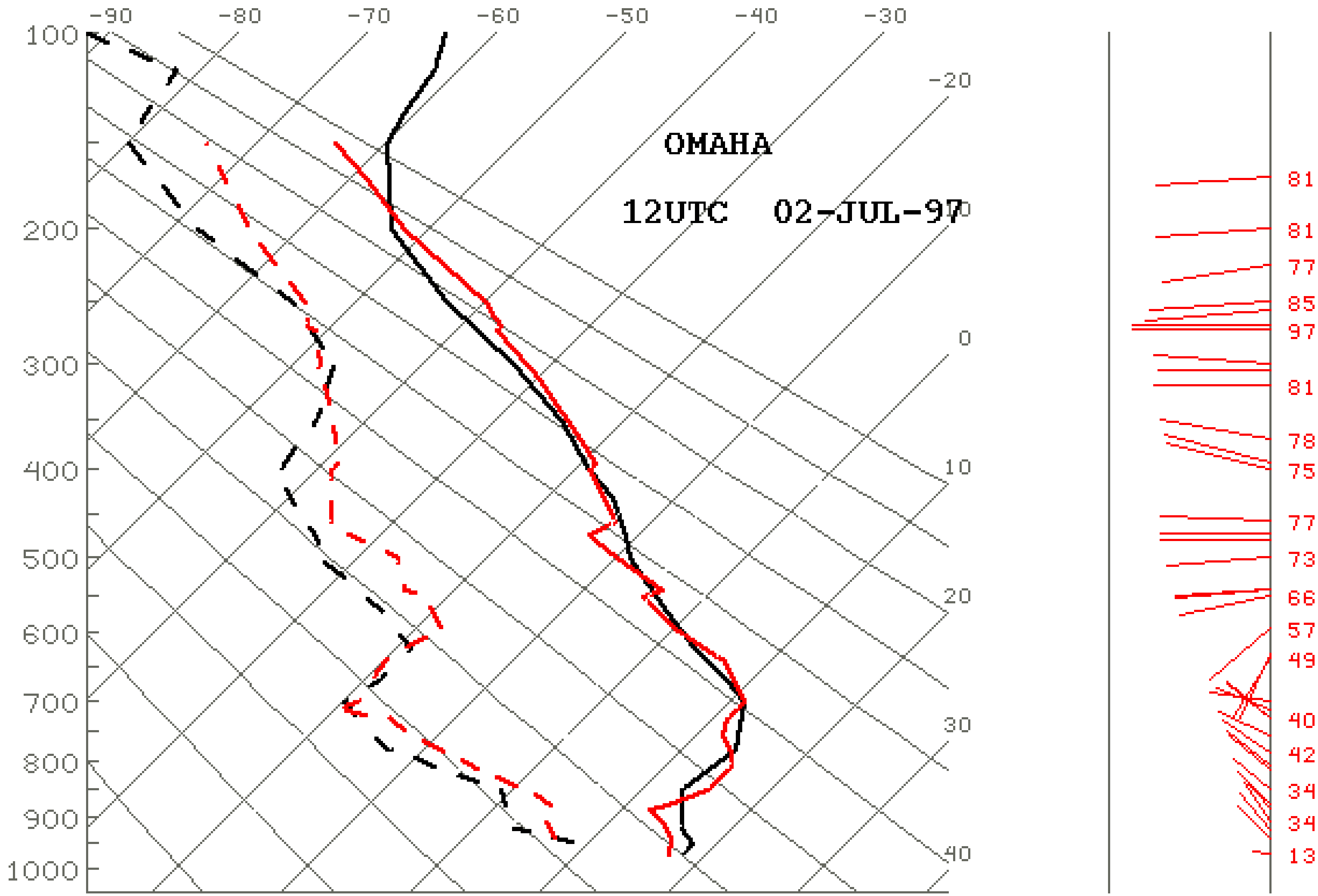
$$I_\lambda = \varepsilon_\lambda^{sfc} B_\lambda(T(p_s)) \tau_\lambda(p_s) - \int_o^{p_s} B_\lambda(T(p)) F_\lambda(p) [d\tau_\lambda(p) / dp] dp .$$

I1, I2, I3, ... , In are measured with the sounder
P(sfc) and T(sfc) come from ground based conventional observations
$\tau_\lambda(p)$ are calculated with physics models (using for CO_2 and O_3)
$\varepsilon_\lambda^{sfc}$ is estimated from a priori information (or regression guess)

First guess solution is inferred from (1) in situ radiosonde reports,
(2) model prediction, or (3) blending of (1) and (2)

Profile retrieval from perturbing guess to match measured sounder radiances

Example GOES Sounding



GMT	ID	TOTAL	EQUIP	FMAX	CVT	L.I.	KINX	PW	
021153	267	30				11	-10	12	**GOES-8 RTVL**
021200	72558	36				10	-4	14	**RAOB**

Sounder Retrieval Products

Direct

brightness temperatures

Derived in Clear Sky

20 retrieved temperatures (at mandatory levels)
20 geo-potential heights (at mandatory levels)
11 dewpoint temperatures (at 300 hPa and below)
3 thermal gradient winds (at 700, 500, 400 hPa)
1 total precipitable water vapor
1 surface skin temperature
2 stability index (lifted index, CAPE)

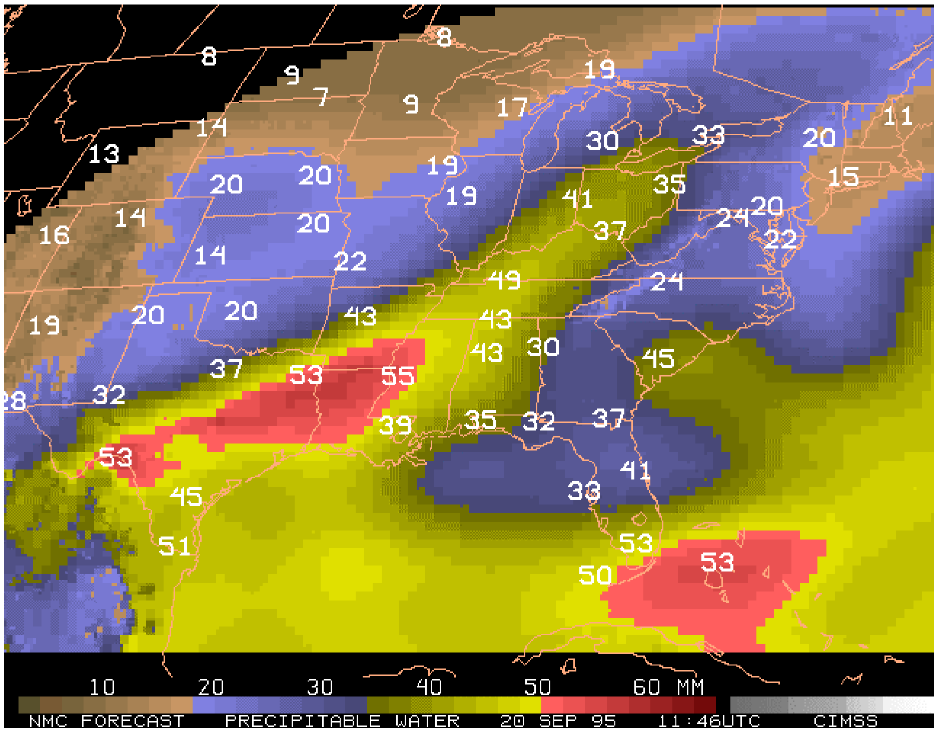
Derived in Cloudy conditions

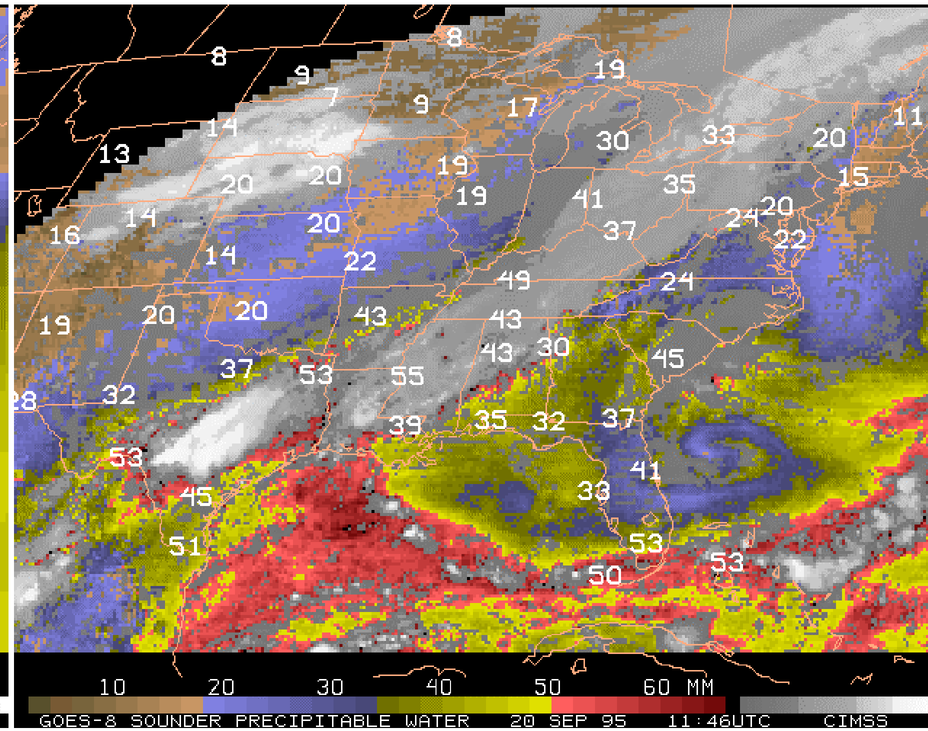
3 cloud parameters (amount, cloud top pressure, and cloud top temperature)

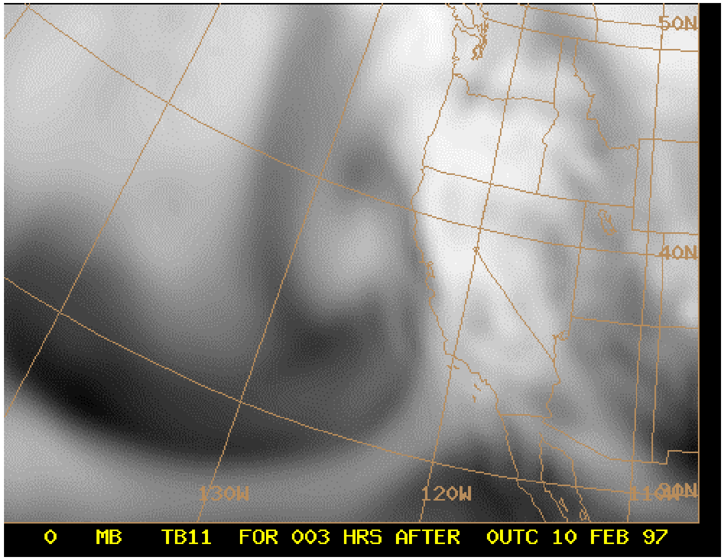
Mandatory Levels (in hPa)

sfc	780	300	70
1000	700	250	50
950	670	200	30
920	500	150	20
850	400	100	10

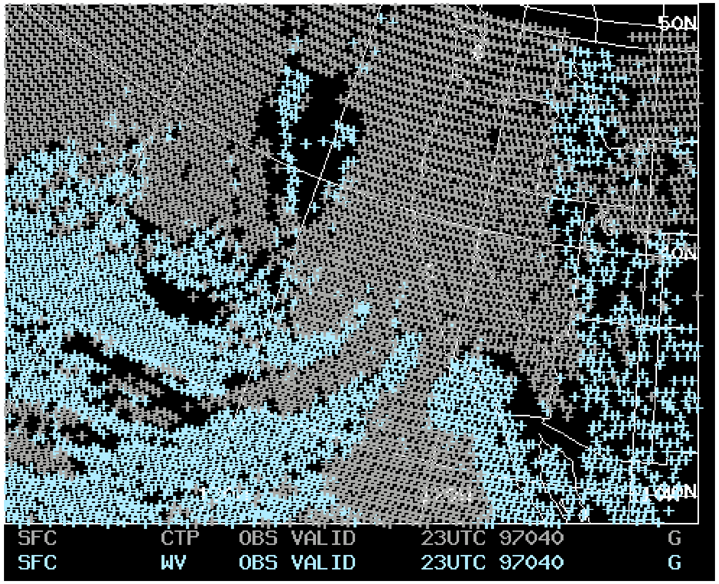
Example GOES TPW DPI



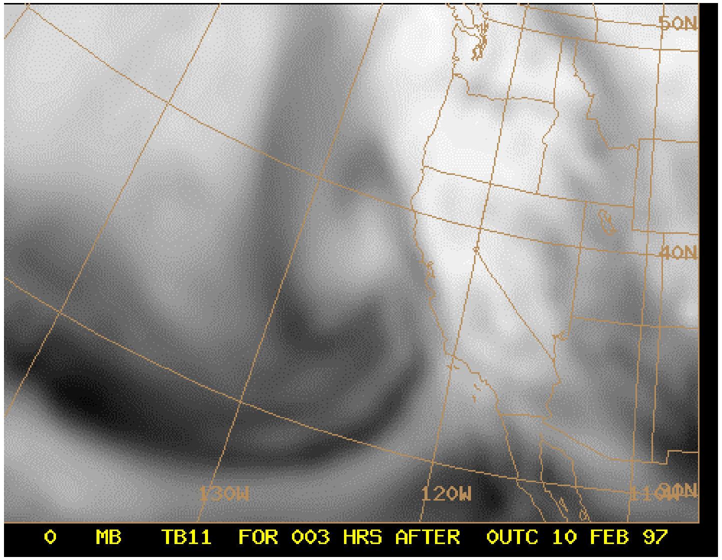




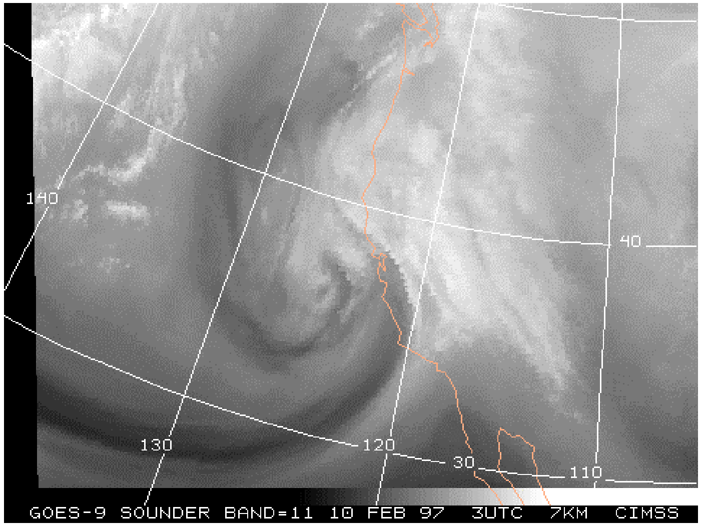
3-hour forecast: No satellite data



Coverage: Cloud Top Pressures and Total Water Vapor



3-hour forecast: With both Clouds and Water Vapor data

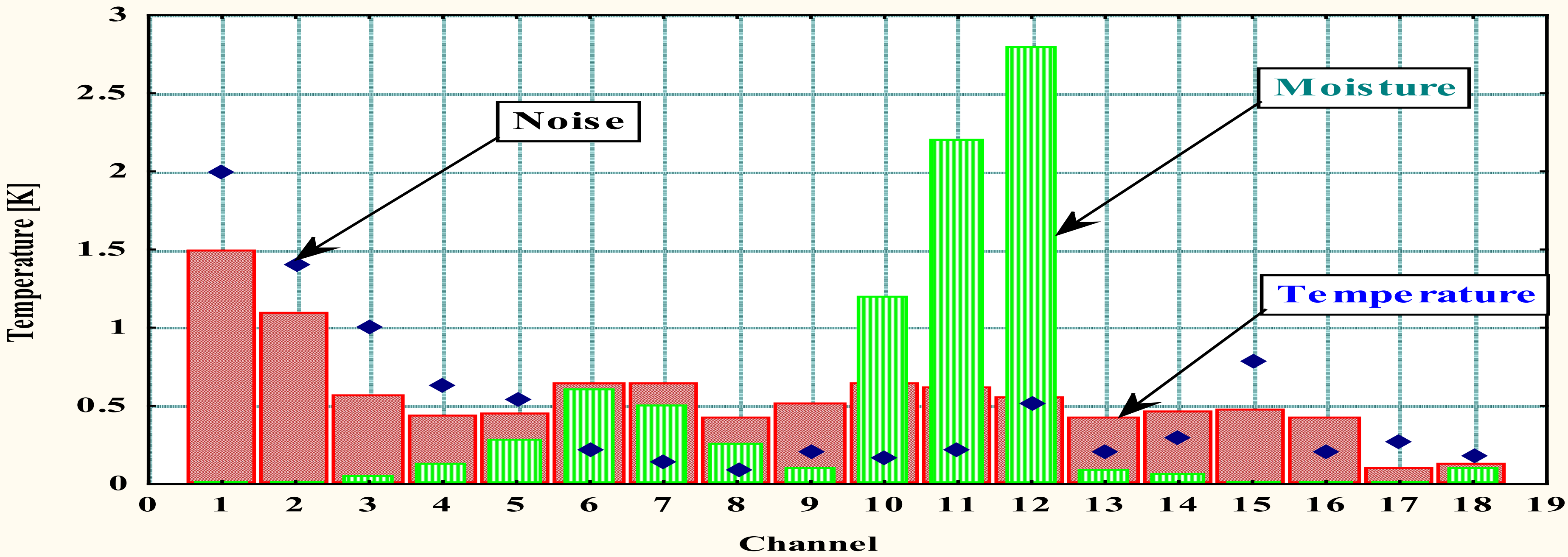


Observed GOES-9 Sounder (7 microns)

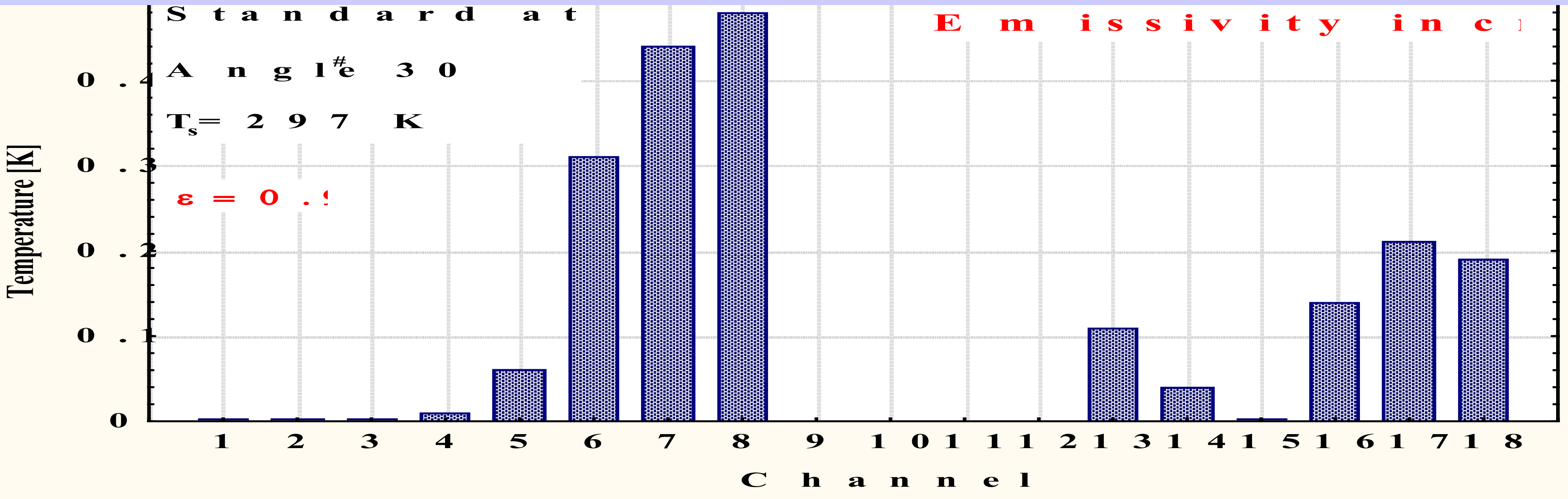
More realistic moisture forecasts with GOES Sounder Cloud and Water Vapor data

NOAA/NESDIS/ASPT

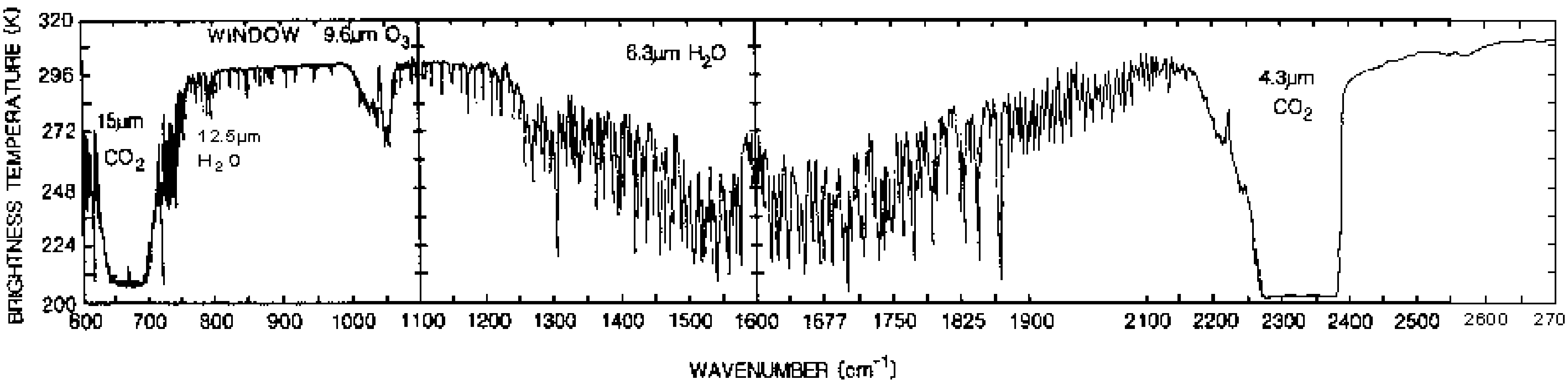
Spectral distribution of radiance contributions due to profile uncertainties

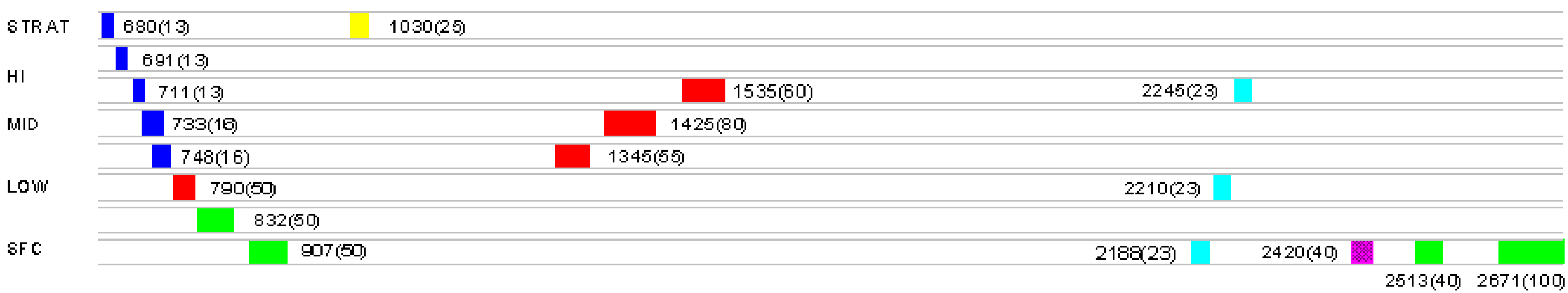


Spectral distribution of reflective changes for emissivity increments of 0.01



EARTH EMITTED SPECTRA

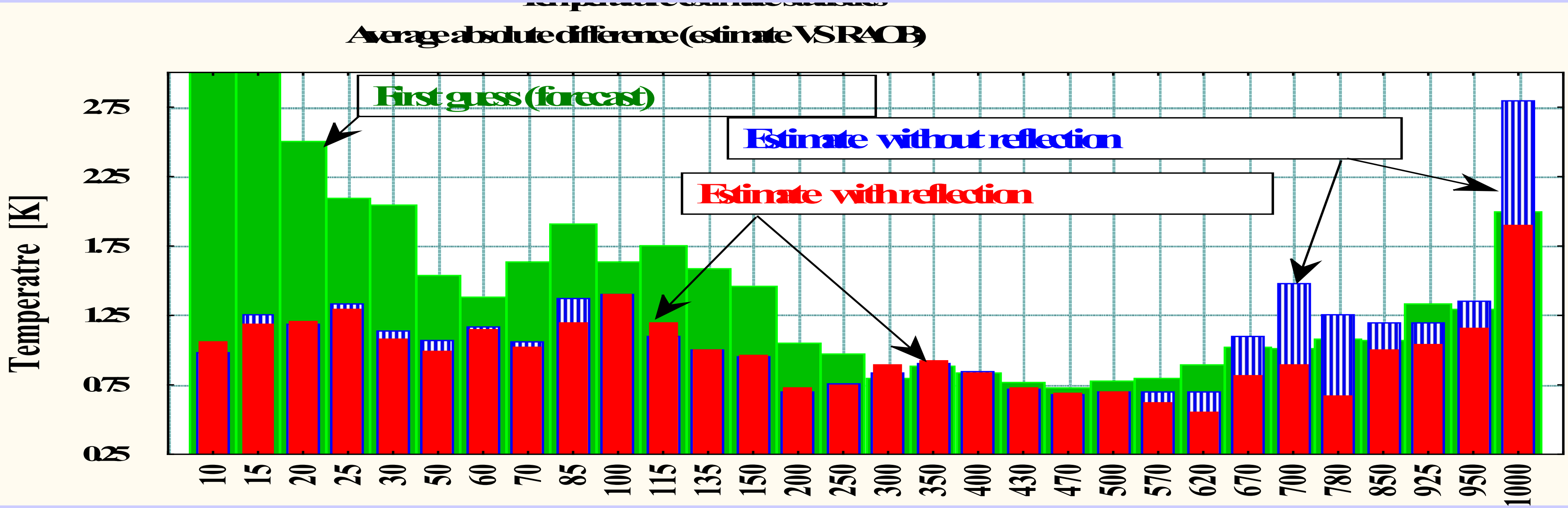




GOES-I SOUNDER SPECTRAL BANDS

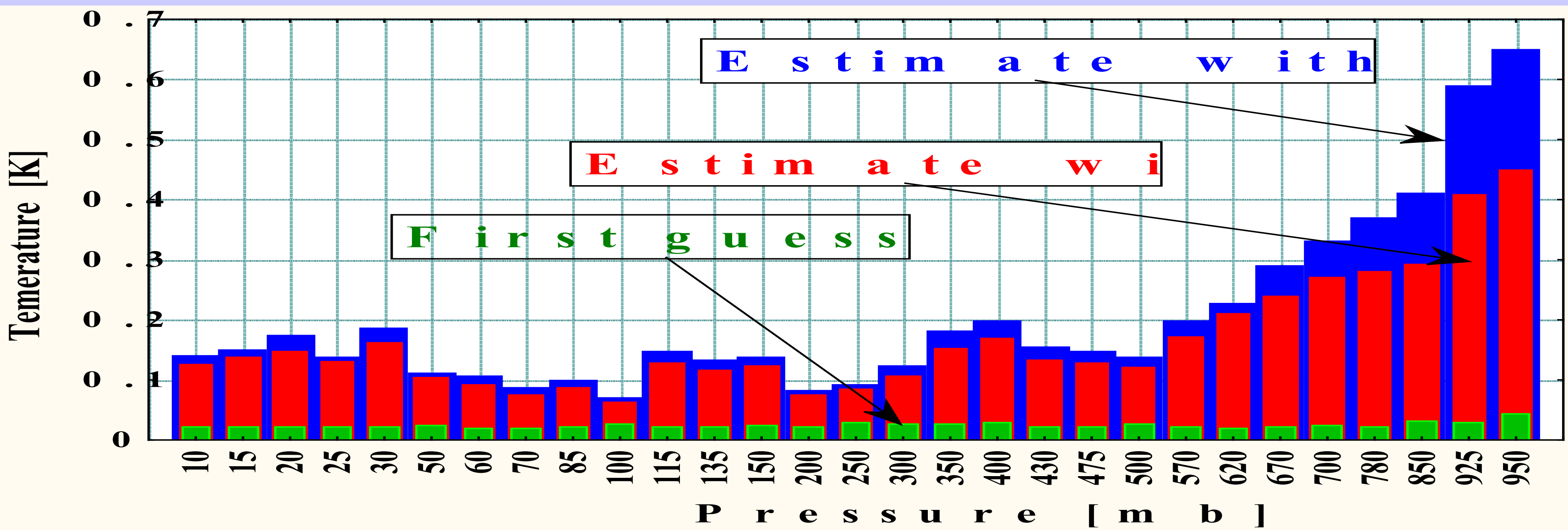
COOPERATIVE INSTITUTE FOR METEOROLOGICAL SATELLITE STUDIES

Average absolute temp diff (solution with and wo sfc reflection vs raobs)

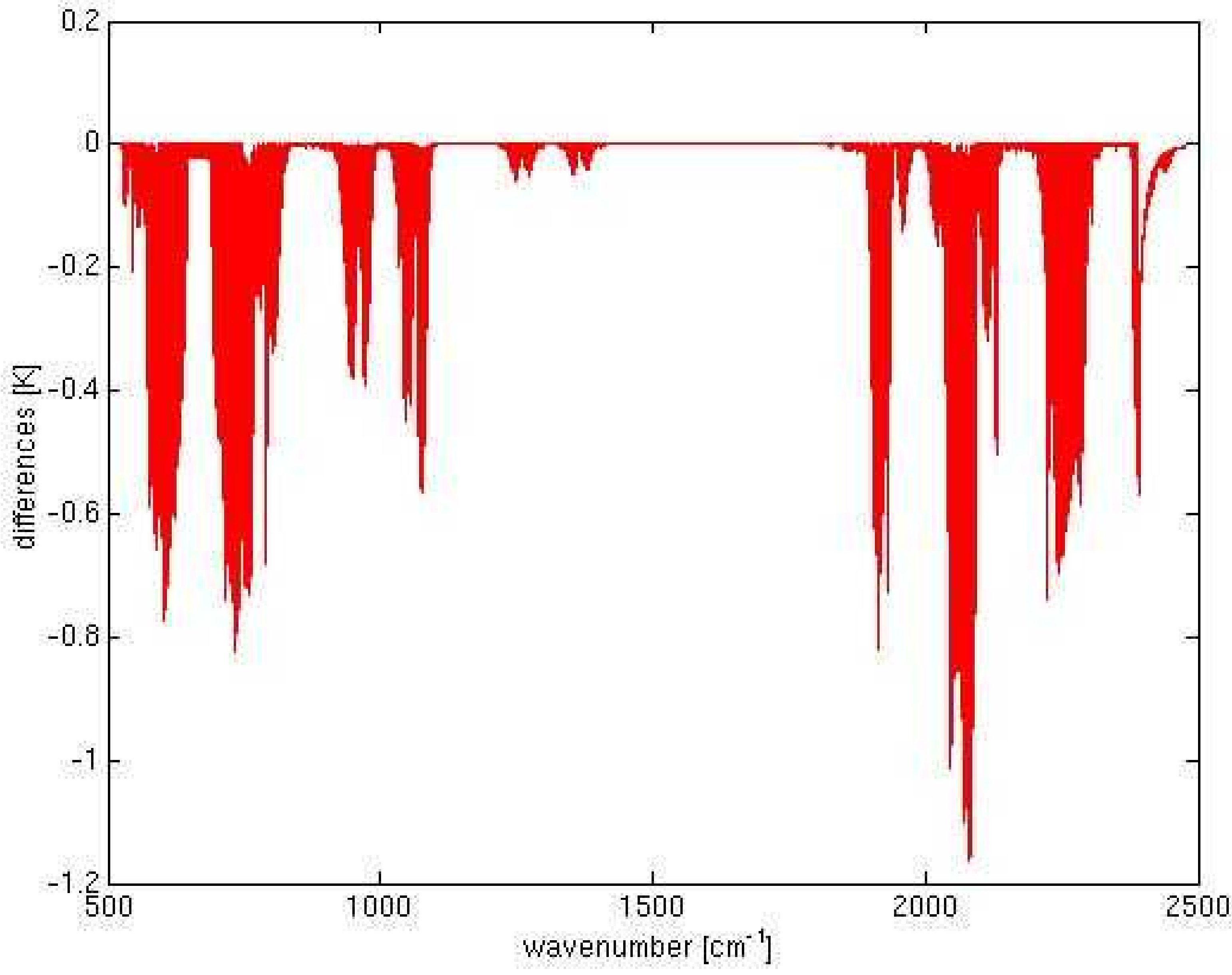


Spatial smoothness of temperature solution with and wo sfc reflection

standard deviation of second spatial derivative (multiplied by 100 * km * km)



BT differences resulting from 10 ppmv change in CO2 concentration



First Order Estimation of TPW

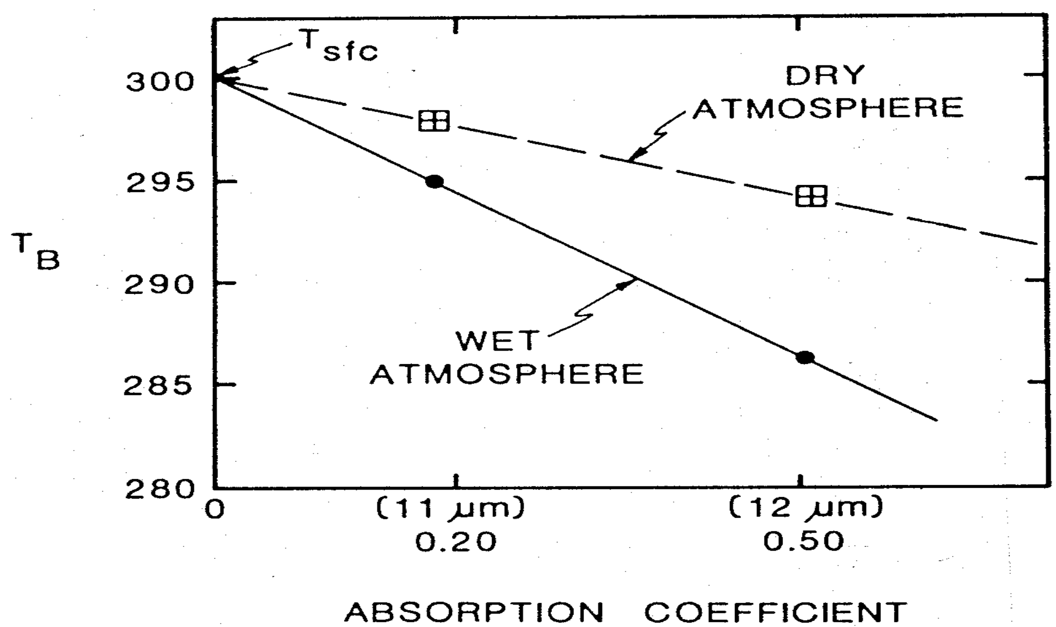
Moisture attenuation in atmospheric windows varies linearly with optical depth.

$$\tau_\lambda = e^{-k_\lambda u} = 1 - k_\lambda u$$

For same atmosphere, deviation of brightness temperature from surface temperature is a linear function of absorbing power. Thus moisture corrected SST can inferred by using split window measurements and extrapolating to zero k_λ

$$T_s = T_{bw1} + [\, k_{w1} / (k_{w2} - k_{w1}) \,] \, [T_{bw1} - T_{bw2}] \; .$$

Moisture content of atmosphere inferred from slope of linear relation.



Water vapour evaluated in multiple infrared window channels where absorption is weak, so that

$$\tau_w = \exp[- k_w u] \sim 1 - k_w u \text{ where w denotes window channel}$$

and

$$d\tau_w = - k_w du$$

What little absorption exists is due to water vapour, therefore, u is a measure of precipitable water vapour. RTE in window region

$$I_w = B_{sw} (1-k_w u_s) + k_w \int_o^{u_s} B_w du$$

u_s represents total atmospheric column absorption path length due to water vapour, and s denotes surface. Defining an atmospheric mean Planck radiance, then

$$I_w = B_{sw} (1-k_w u_s) + k_w u_s \bar{B}_w \text{ with } \bar{B}_w = \int_o^{u_s} B_w du / \int_o^{u_s} du$$

Since B_{sw} is close to both I_w and B_w, first order Taylor expansion about the surface temperature T_s allows us to linearize the RTE with respect to temperature, so

$$T_{bw} = T_s (1-k_w u_s) + k_w u_s \bar{T}_w$$, where T_w is mean atmospheric temperature corresponding to B_w.

For two window channels (11 and 12um) the following ratio can be determined.

$$\frac{T_s - T_{bw1}}{T_s - T_{bw2}} = \frac{k_{w1} u_s (T_s - \overline{T}_{w1})}{k_{w1} u_s (T_s - \overline{T}_{w2})} = \frac{k_{w1}}{k_{w2}}$$

where the mean atmospheric temperature measured in the one window region is assumed to be comparable to that measured in the other, $T_{w1} \sim T_{w2}$,

Thus it follows that

$$T_s = T_{bw1} + \frac{k_{w1}}{k_{w2} - k_{w1}} [T_{bw1} - T_{bw2}]$$

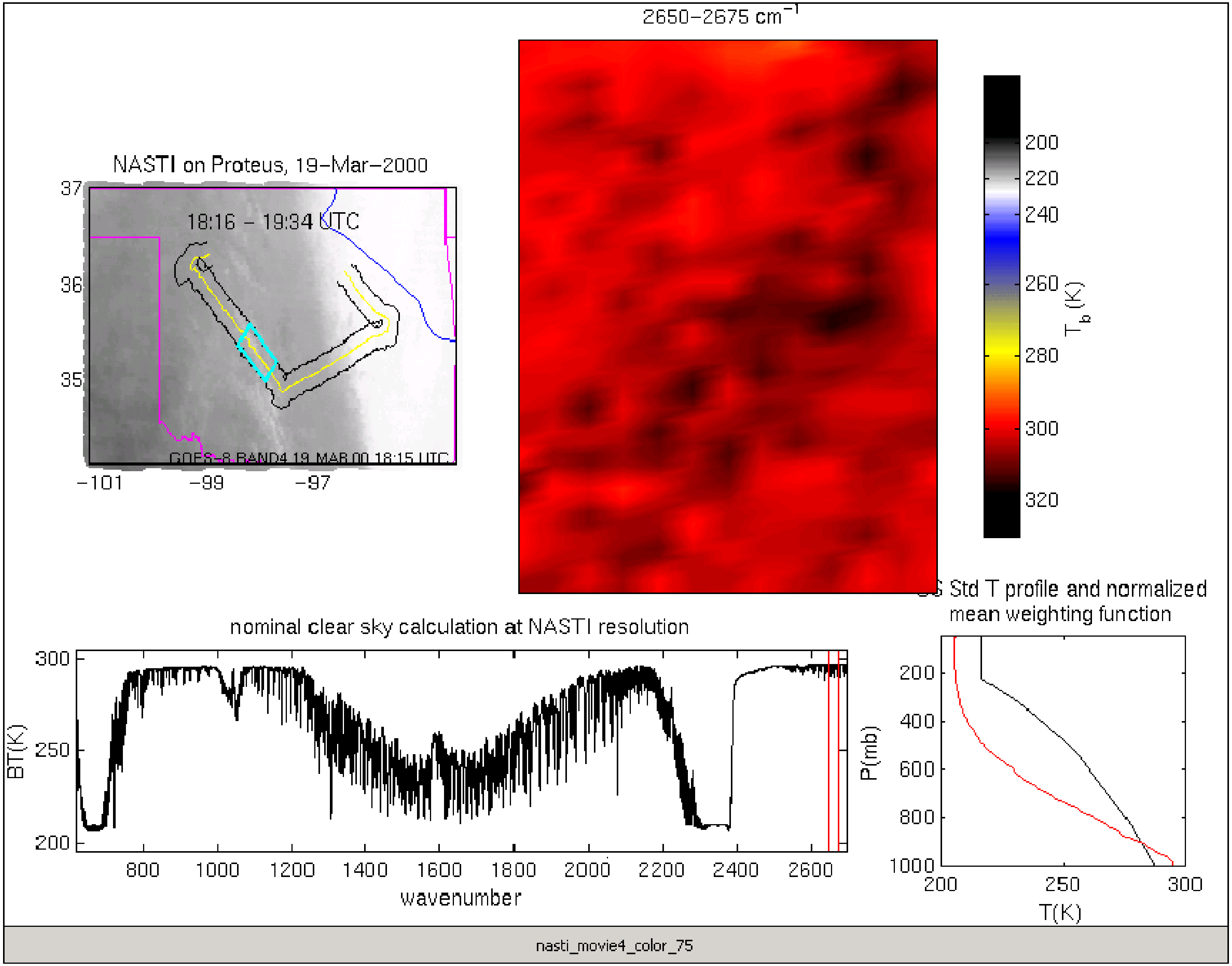
and

$$u_s = \frac{T_{bw} - T_s}{k_w (\overline{T}_w - T_s)} .$$

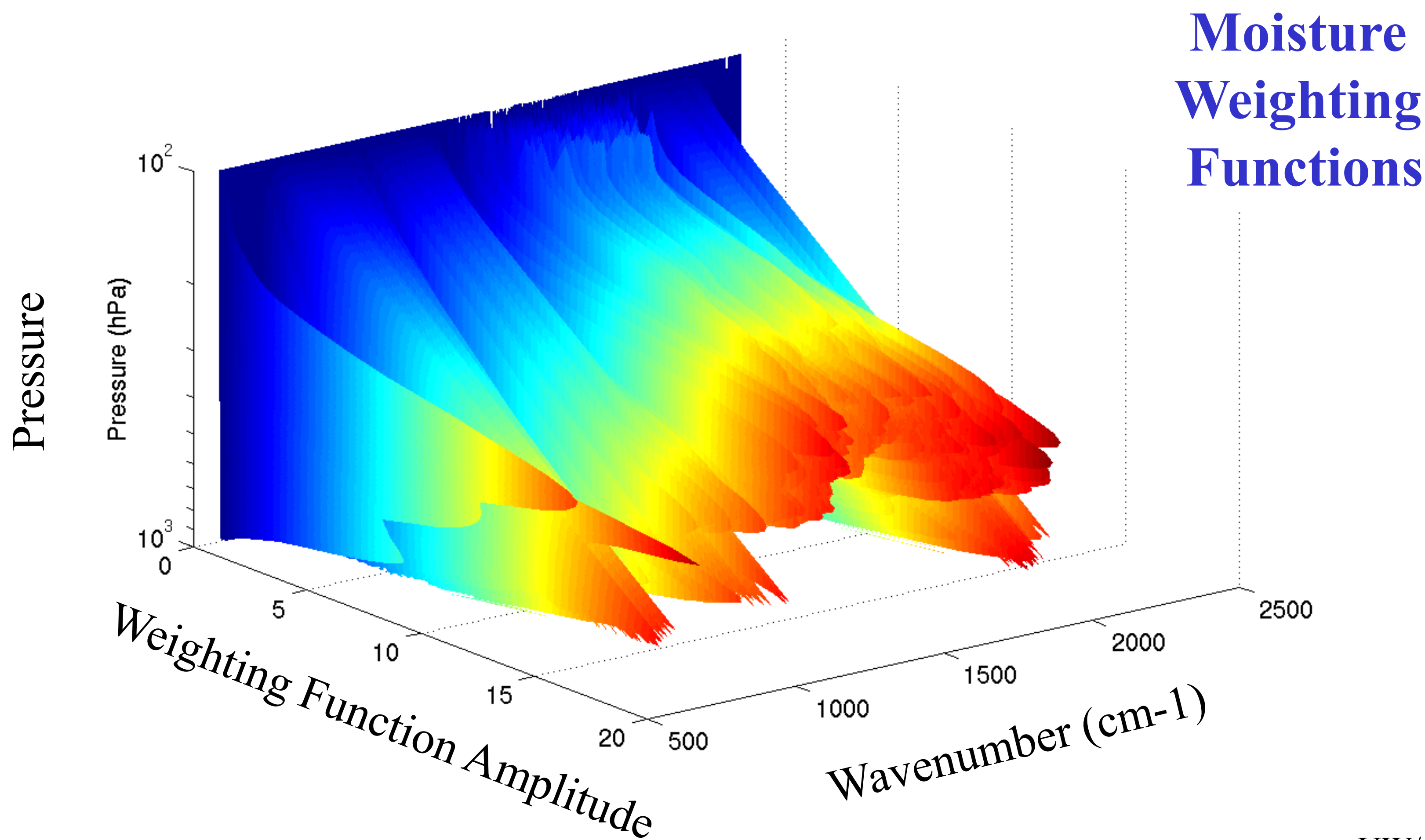
Obviously, the accuracy of the determination of the total water vapour concentration depends upon the contrast between the surface temperature, T_s, and

the effective temperature of the atmosphere $\overline{T}_w$

Improvements with Hyperspectral IR Data



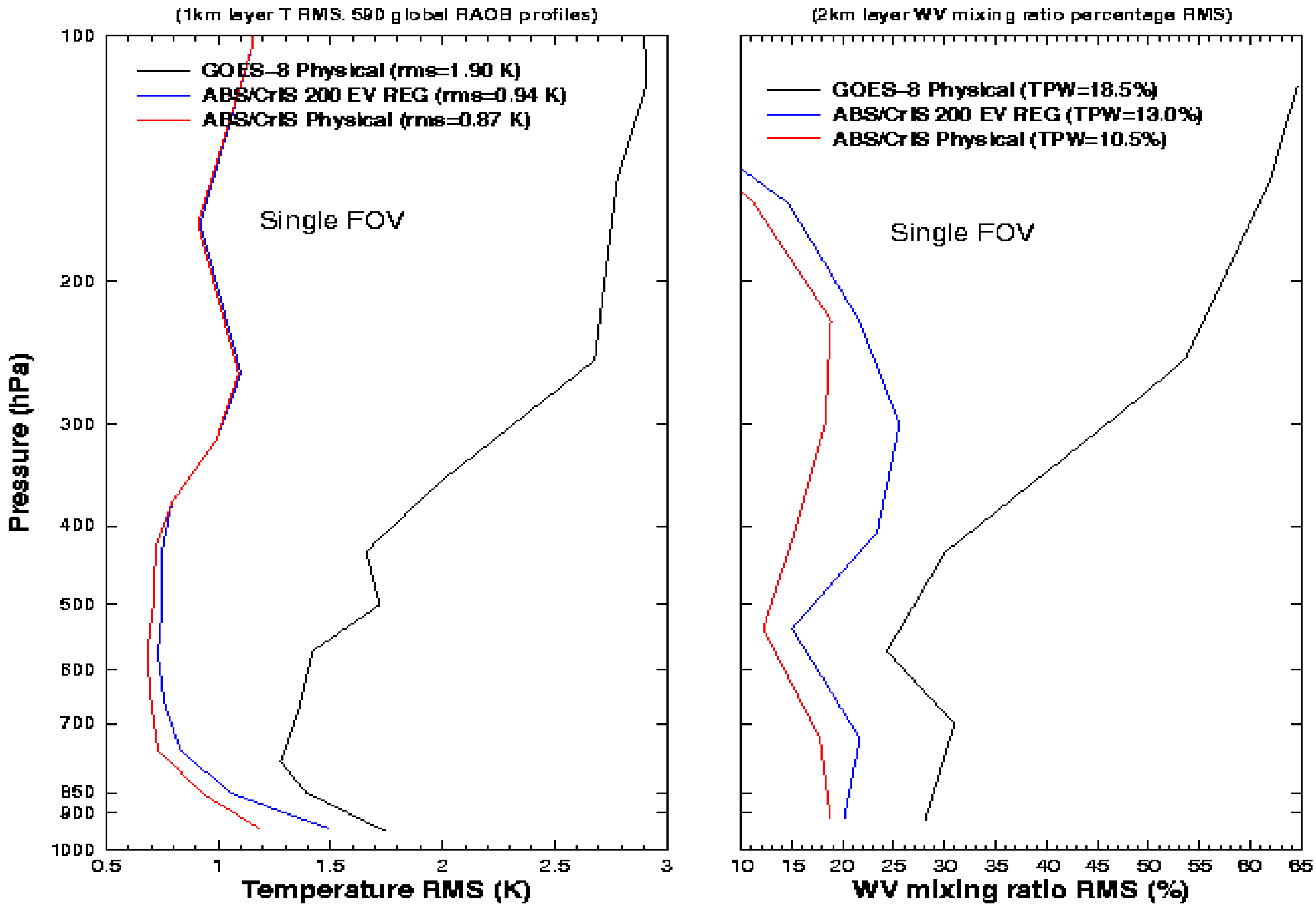
These water vapor weighting functions reflect the radiance sensitivity of the specific channels to a water vapor % change at a specific level (equivalent to dR/dlnq scaled by dlnp).



UW/CIMSS

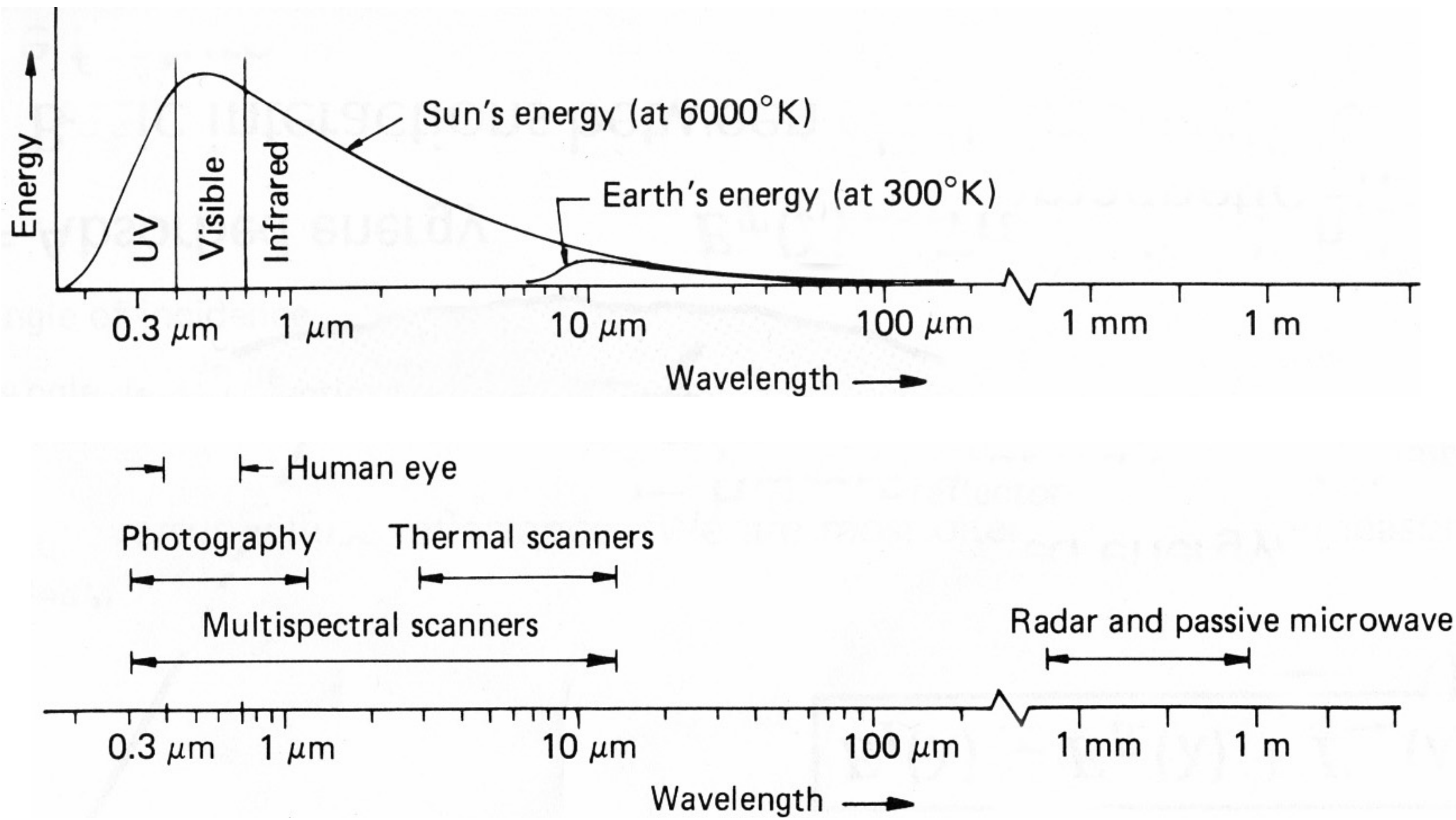
The advanced sounder has more and sharper weighting functions

1-km temperature rms and 2 km water vapor mixing ratio % rms from simulated hyperspectral IR retrievals



Hyperspectral IR gets 1 K for 1 km T(p) and 15% for 2 km Q(p)

Spectral Characteristics of Energy Sources and Sensing Systems



WAVELENGTH			FREQUENCY		WAVENUMBER
cm	μm	Å	Hz	GHz	cm^{-1}
10^{-5} Near Ultraviolet (UV)	0.1	1,000	$3x10^{15}$		
$4x10^{-5}$ Visible	0.4	4,000	$7.5x10^{14}$		
$7.5x10^{-5}$ Near Infrared (IR)	0.75	7,500	$4x10^{14}$		13,333
$2x10^{-3}$ Far Infrared (IR)	20	$2x10^{5}$	$1.5x10^{13}$		500
0.1 Microwave (MW)	10^{3}		$3x10^{11}$	300	10

Radiation is governed by Planck's Law

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

In microwave region $c_2/\lambda T << 1$ so that

$$e^{c_2/\lambda T} = 1 + c_2/\lambda T + \text{second order}$$

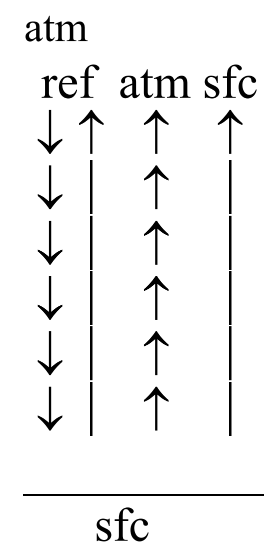
And classical Rayleigh Jeans radiation equation emerges

$$B_\lambda(T) \approx [c_1 / c_2] [T / \lambda^4]$$

Radiance is linear function of brightness temperature.

Microwave Form of RTE

$$I_\lambda^{sfc} = \varepsilon_\lambda B_\lambda(T_s)\,\tau_\lambda(p_s) + (1-\varepsilon_\lambda)\,\tau_\lambda(p_s)\int_o^{p_s} B_\lambda(T(p))\,\frac{\partial \tau'_\lambda(p)}{\partial \ln p}\,d\ln p$$



$$I_\lambda = \varepsilon_\lambda B_\lambda(T_s)\,\tau_\lambda(p_s) + (1-\varepsilon_\lambda)\,\tau_\lambda(p_s)\int_o^{p_s} B_\lambda(T(p))\,\frac{\partial \tau'_\lambda(p)}{\partial \ln p}\,d\ln p$$

$$+ \int_{p_s}^{o} B_\lambda(T(p))\,\frac{\partial \tau_\lambda(p)}{\partial \ln p}\,d\ln p$$

In the microwave region $c_2/\lambda T << 1$, so the Planck radiance is linearly proportional to the temperature

$$B_\lambda(T) \approx [c_1 / c_2]\,[T / \lambda^4]$$

So

$$T_{b\lambda} = \varepsilon_\lambda T_s(p_s)\,\tau_\lambda(p_s) + \int_{p_s}^{o} T(p)\,F_\lambda(p)\,\frac{\partial \tau_\lambda(p)}{\partial \ln p}\,d\ln p$$

where

$$F_\lambda(p) = \left\{ 1 + (1-\varepsilon_\lambda)\left[\frac{\tau_\lambda(p_s)}{\tau_\lambda(p)}\right]^2 \right\} .$$

The transmittance to the surface can be expressed in terms of transmittance to the top of the atmosphere by remembering

$$\tau'_\lambda(p) = \exp\left[- \frac{1}{g} \int_p^{p_s} k_\lambda(p)\, g(p)\, dp \right]$$

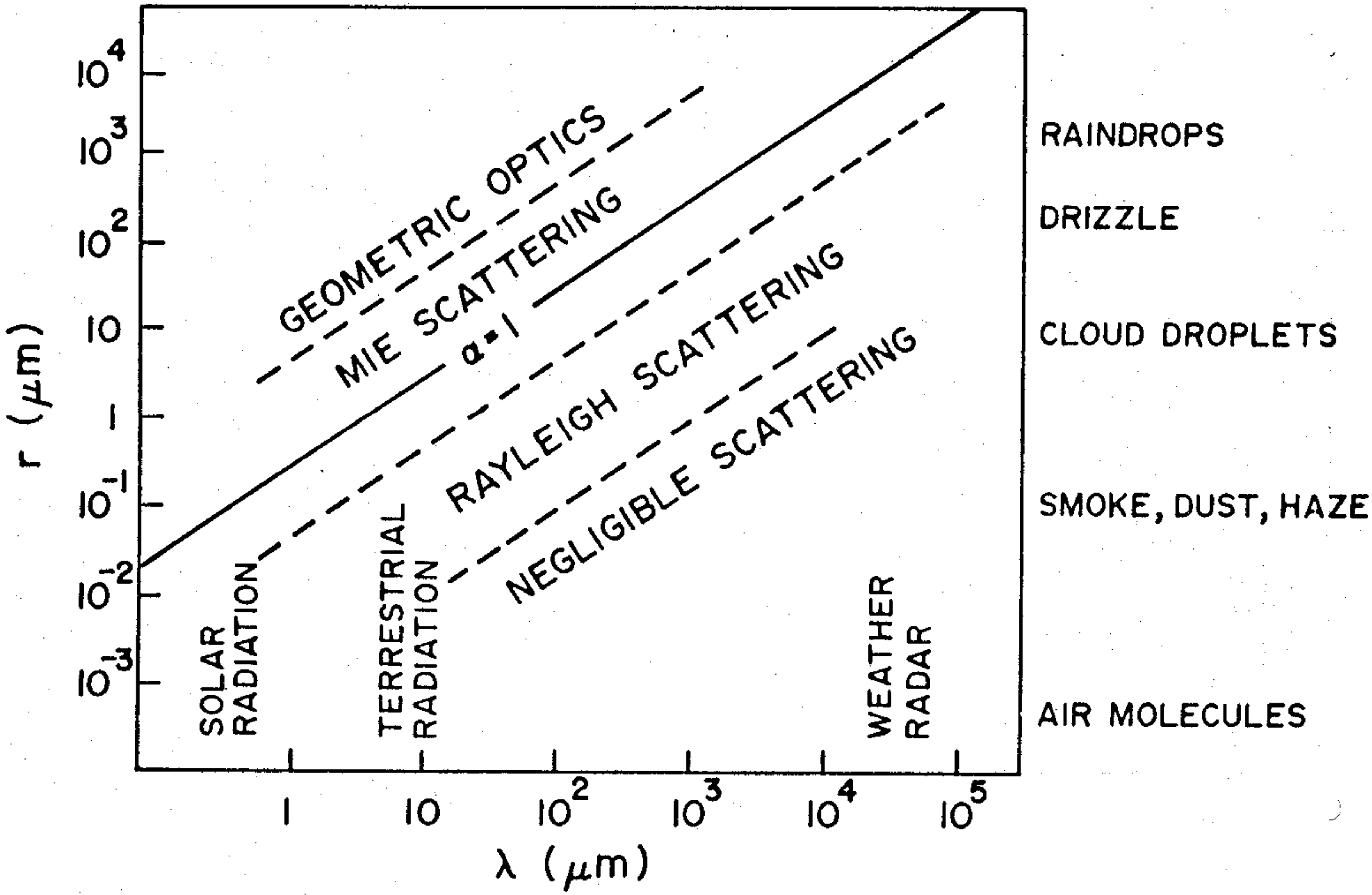
$$= \exp\left[- \int_o^{p_s} + \int_o^{p} \right]$$

$$= \tau_\lambda(p_s) \,/\, \tau_\lambda(p) \;.$$

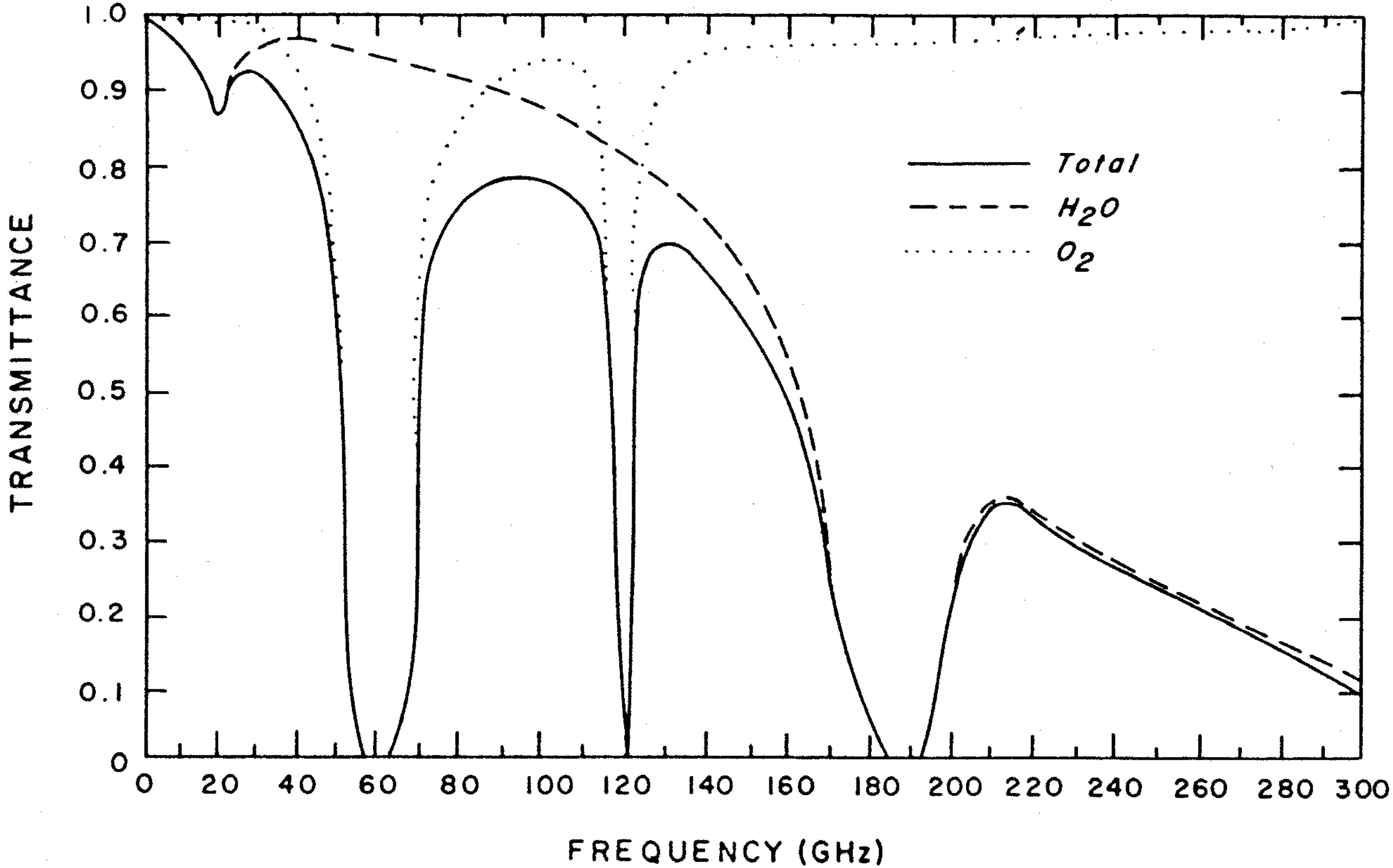
So

$$\frac{\partial \tau'_\lambda(p)}{\partial \ln p} = - \frac{\tau_\lambda(p_s)}{(\tau_\lambda(p))^2} \frac{\partial \tau_\lambda(p)}{\partial \ln p} \;.$$

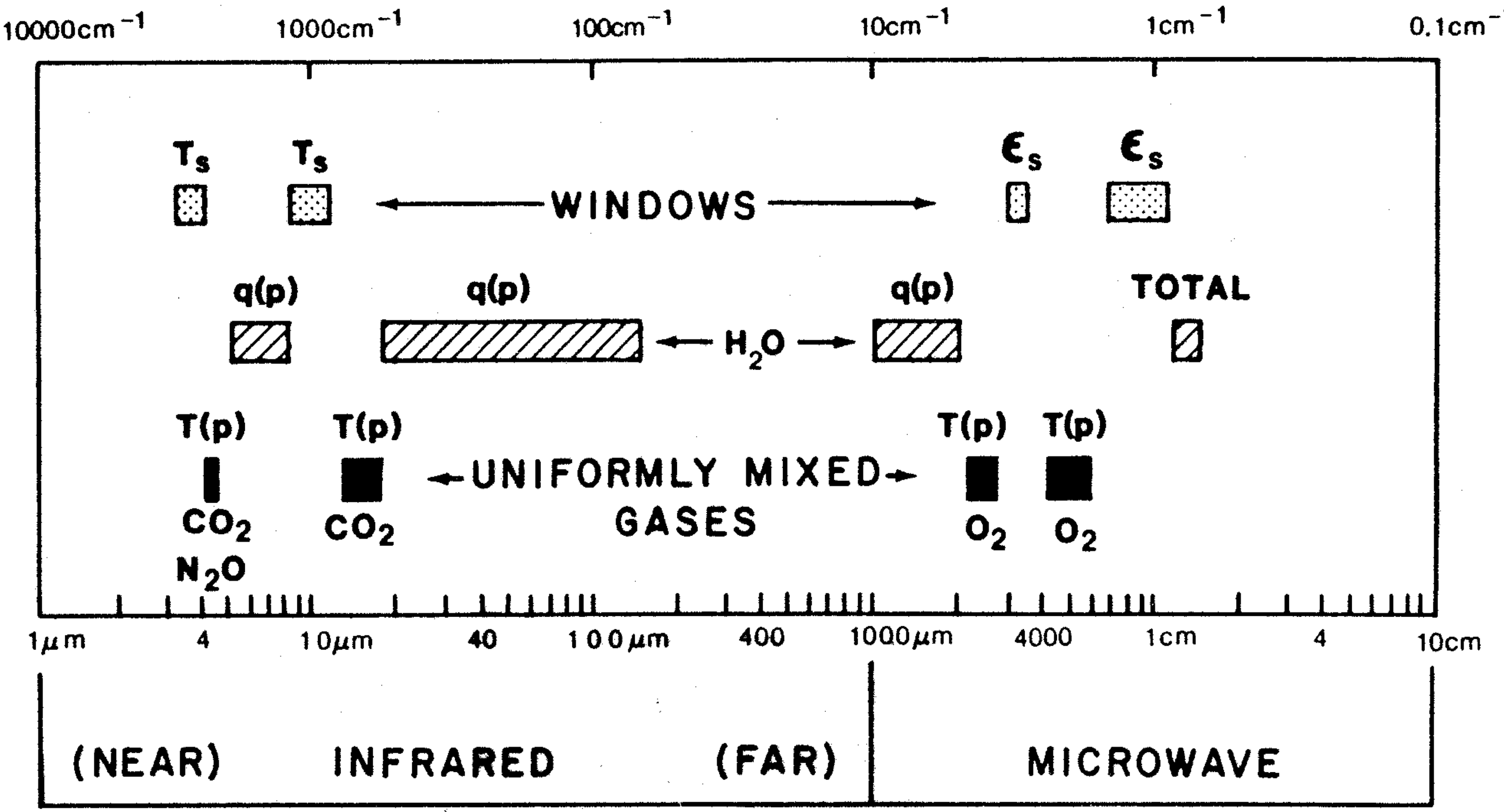
[remember that $\tau_\lambda(p_s, p)\, \tau_\lambda(p, 0) = \tau_\lambda(p_s, 0)$ and $\tau_\lambda(p_s, p) = \tau_\lambda(p, p_s)$]



r (μm)
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
GEOMETRIC OPTICS
MIE SCATTERING
α=1
RAYLEIGH SCATTERING
NEGLIGIBLE SCATTERING
SOLAR RADIATION
TERRESTRIAL RADIATION
WEATHER RADAR
1
10
10^2
10^3
10^4
10^5
λ (μm)
RAINDROPS
DRIZZLE
CLOUD DROPLETS
SMOKE, DUST, HAZE
AIR MOLECULES



TRANSMITTANCE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
FREQUENCY (GHz)
Total
H_2O
O_2



Spectral regions used for remote sensing of the earth atmosphere and surface from satellites. ε indicates emissivity, q denotes water vapour, and T represents temperature.

Direct Physical Solution to RTE

To solve for temperature and moisture profiles simultaneously, a simplified form of RTE is considered,

$$R = B_o + \int_o^{p_s} \tau \, dB$$

which comes integrating the atmospheric term by parts in the more familiar form of the RTE. Then in perturbation form, where δ represents a perturbation with respect to an a priori condition

$$\delta R = \int_o^{p_s} (\delta\tau) \, dB + \int_o^{p_s} \tau \, d(\delta B)$$

Integrating by parts,

$$\int_o^{p_s} \tau \, d(\delta B) = \tau \, \delta B \Big|_o^{p_s} - \int_o^{p_s} \delta B \, d\tau = \tau_s \, \delta B_s - \int_o^{p_s} \delta B \, d\tau \; ,$$

yields

$$\delta R = \int_o^{p_s} (\delta\tau) \, dB + \tau_s \, \delta B_s - \int_o^{p_s} \delta B \, d\tau$$

Write the differentials with respect to temperature and pressure

$$\delta R = \delta T_b \frac{\partial B}{\partial T_b}, \quad \delta B = \delta T \frac{\partial B}{\partial T}, \quad dB = \frac{\partial B}{\partial T} \frac{\partial T}{\partial p} dp\,, \quad d\tau = \frac{\partial \tau}{\partial p} dp\,.$$

Substituting

$$\delta T_b = \int_o^{p_s} \delta\tau \frac{\partial T}{\partial p} \left[\frac{\partial B}{\partial T} / \frac{\partial B}{\partial T_b} \right] dp - \int_o^{p_s} \delta T \frac{\partial \tau}{\partial p} \left[\frac{\partial B}{\partial T} / \frac{\partial B}{\partial T_b} \right] dp$$

$$+ \delta T_s \left[\frac{\partial B_s}{\partial T_s} / \frac{\partial B}{\partial T_b} \right] \tau_s$$

where T_b is the brightness temperature. Finally, assume that the transmittance perturbation is dependent only on the uncertainty in the column of precipitable water density weighted path length u according to the relation $\delta\tau = [\partial\tau / \partial u]\, \delta u$. Thus

$$\delta T_b = \int_o^{p_s} \delta u \frac{\partial T}{\partial p} \frac{\partial \tau}{\partial u} \left[\frac{\partial B}{\partial T} / \frac{\partial B}{\partial T_b} \right] dp - \int_o^{p} \delta T \frac{\partial \tau}{\partial p} \left[\frac{\partial B}{\partial T} / \frac{\partial B}{\partial T_b} \right] dp + \delta T_s \left[\frac{\partial B_s}{\partial T_s} / \frac{\partial B}{\partial T_b} \right] \tau_s$$

$$= f\,[\,\delta u, \delta T, \delta T_s\,]$$

CD Tutorial on GOES Sounder