Beyond optimal estimation: sensitivity of analysis error to mis-specification of background error

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Beyond optimal estimation: sensitivity of analysis error to mis-specification of background error

- Motivation
- Theory of analysis/retrieval error
  - optimal estimation
  - sub-optimal case
- Illustration
  - scalar case
  - IASI example
- Conclusions and further work

Motivation (1)

What improvements are needed to exploit advanced IR sounder data more fully in NWP?

- Efficient processing of the full spectrum
- Observation errors, including correlations
- Residual biases
- Surface properties over land and ice
- Background error statistics
- Treatment of cloud

Motivation (2)

- Optimal estimation (OE) theory
  - … assumes the error covariances are known.
  - In practice, they are not known.

→ 2 ways forward:
  - improve estimates of covariances – continuing work
  - make assimilation/retrieval robust against our inevitable lack of knowledge

- Applies to both background and obs error covs, B and R
  - in this presentation, only B considered

Motivation (3)

- Why is B inevitably in error?
  - global averages can be estimated quite accurately
  - … but large spatial/temporal variability.

- We need to understand our sensitivity to B and its inevitable mis-specification,
  - particularly for satellite radiances …
    - non-local observations (→ Fiona Hilton's paper)

Motivation (4)

- Advanced IR sounders have vertical resolution ~ 1 km
  - sensitive with low error to scales »1 km
  - not sensitive to scales «1 km
  - sensitive to scales ~ 1 km, but with errors comparable to background errors

→ Need to understand B - its magnitude on different scales

- determines how measurements and prior information are weighted on each scale

→ … and effects of mis-specifying B on each scale

Theory (1)

GENERAL CASE

Analysis equation (linearised): $x^a = x^b + K \,.\, (y^o - H[x^b])$

Analysis error equation: $\varepsilon^a = \varepsilon^b + K \,.\, (\varepsilon^o - H.\,\varepsilon^b)$

$\varepsilon^a = (I - K.H).\,\varepsilon^b + K.\,\varepsilon^o$

Analysis error covariance: $A = (I - K.H).B.(I - K.H)^T + K.R.K^T$

OPTIMAL CASE assumed value B_A = true value B

$$K = B_A.H^T.(H.B_A.H^T + R)^{-1}$$

$$A_{OPT} = (I - K.H).\,B_A.(I - K.H)^T + K.R.K^T$$

$$= (I - K.H).B_A$$

$$A_{OPT}^{-1} = B_A^{-1} + H^T.R^{-1}.H$$

Theory (2)

Projecting on to the eigenvectors of B_A :

V = eigenvectors of B_{A}; Λ = eigenvalues of B_A

$$A_{OPT}^{-1} = B_A^{-1} + H^T.R^{-1}.H$$

$$V^T.A_{OPT}^{-1}.V = V^T.B_A^{-1}.V + V^T.H^T.R^{-1}.H.V$$

$$V^T.A_{OPT}^{-1}.V = \Lambda^{-1} + V^T.H^T.R^{-1}.H.V$$

Why B_A?

- because this is what we use – the “filter” within the DA system
    - Met Office 4D-Var performs vertical analysis in this eigen-space

Theory (3)

OPTIMAL

$$\mathbf{A_{OPT}(B) = (I-K.H).\ B\ .(I-K.H)^T + K.R.K^T}$$

$$K(B) = B.H^T.(H.B.H^T+R)^{-1}$$

GENERAL / SUB-OPTIMAL, which means $B \neq B_A$, $K=K(B_A)$

$$A(B) = (I-K(B_A).H).B.(I-K(B_A).H)^T + K(B_A).R.K(B_A)^T$$

$$\mathbf{A(B) = A_{OPT}(B_A) + (I-K(B_A).H).(B-B_A).(I-K(B_A).H)^T}$$

Note: linear in B

Illustration – scalar case (1)

$H = 1$
$R = 1$
$B_A = 1$

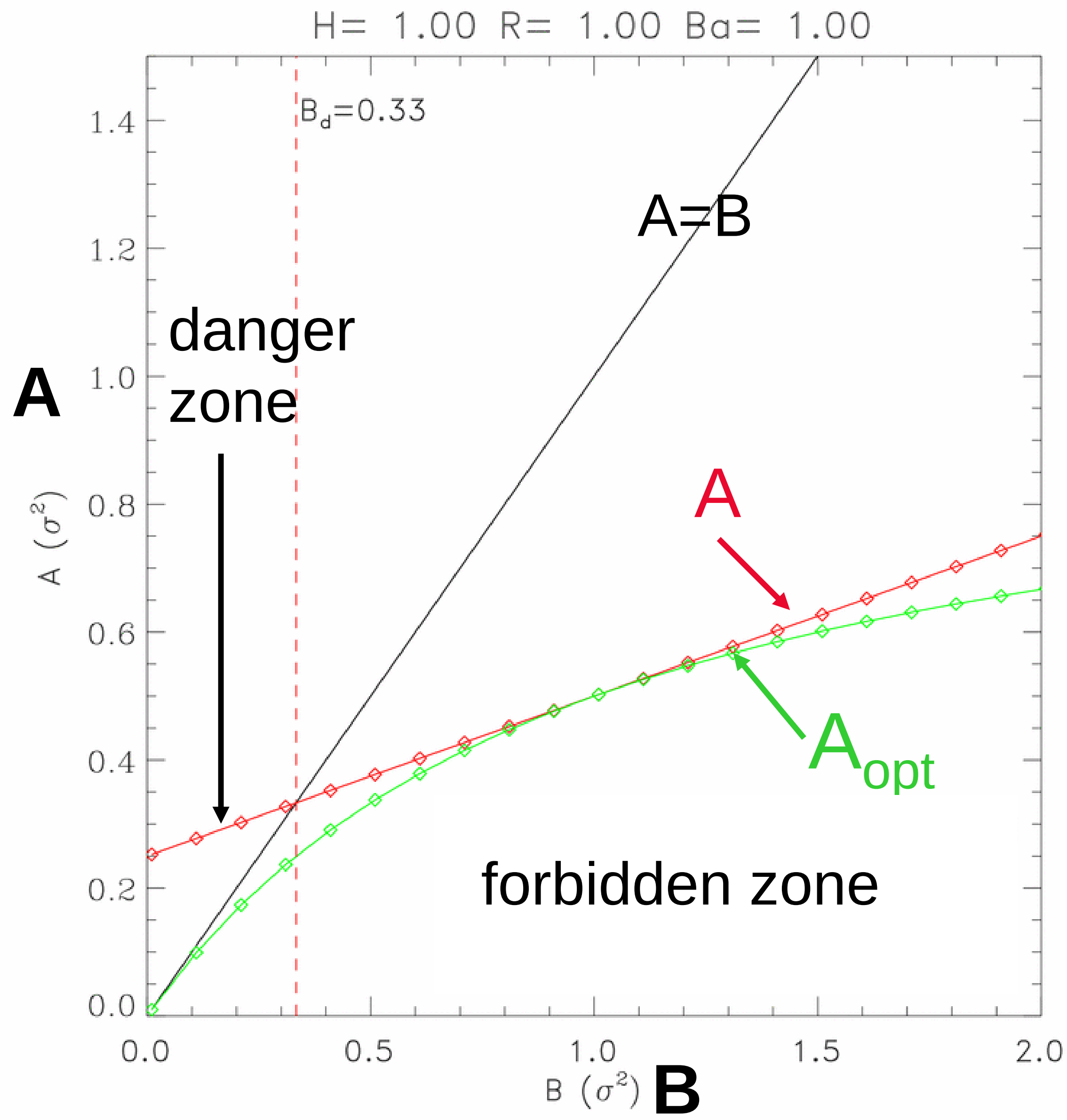


Illustration – scalar case (2)

H = 1
R = 2.72
B_A = 1

higher observation error

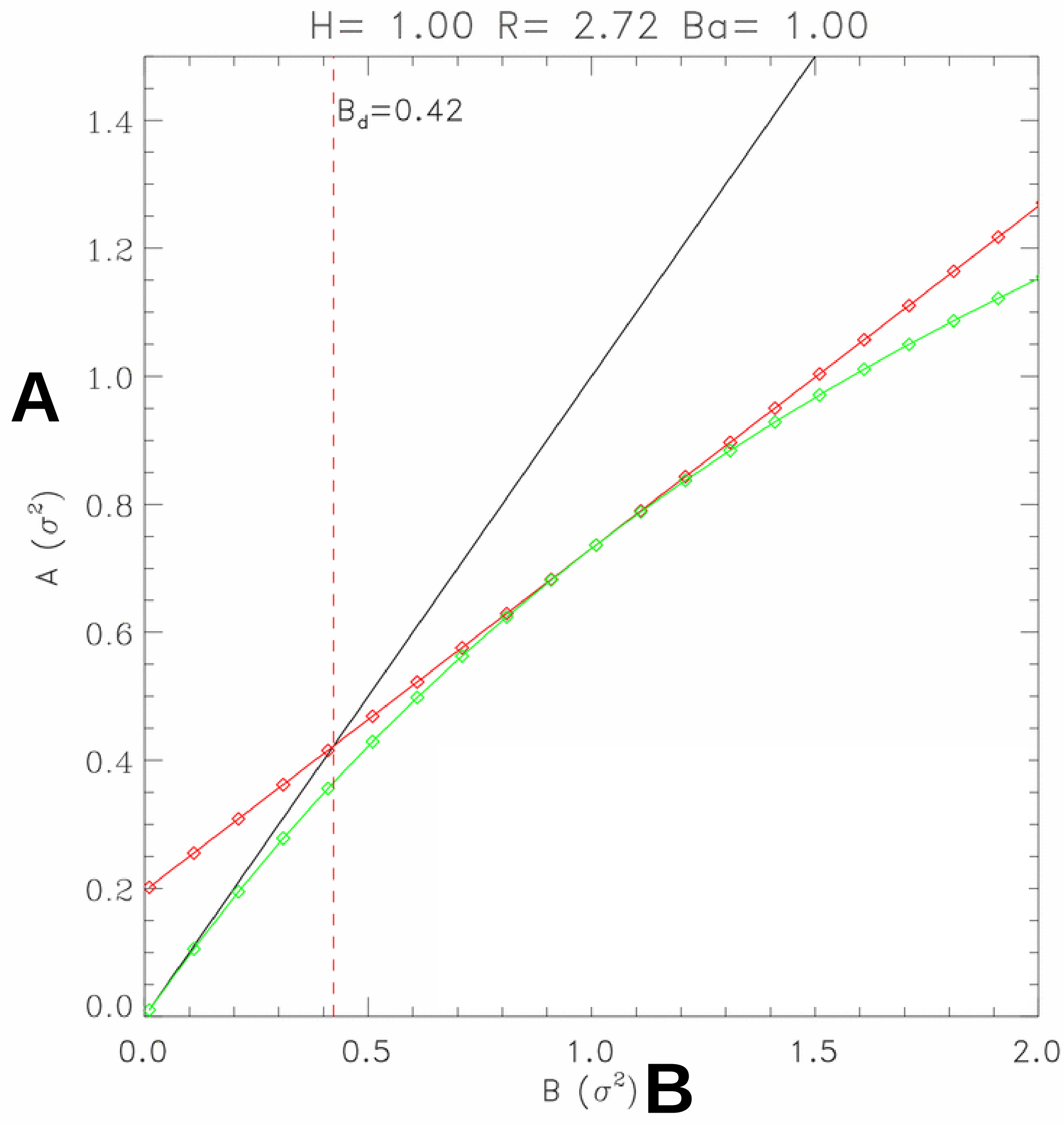
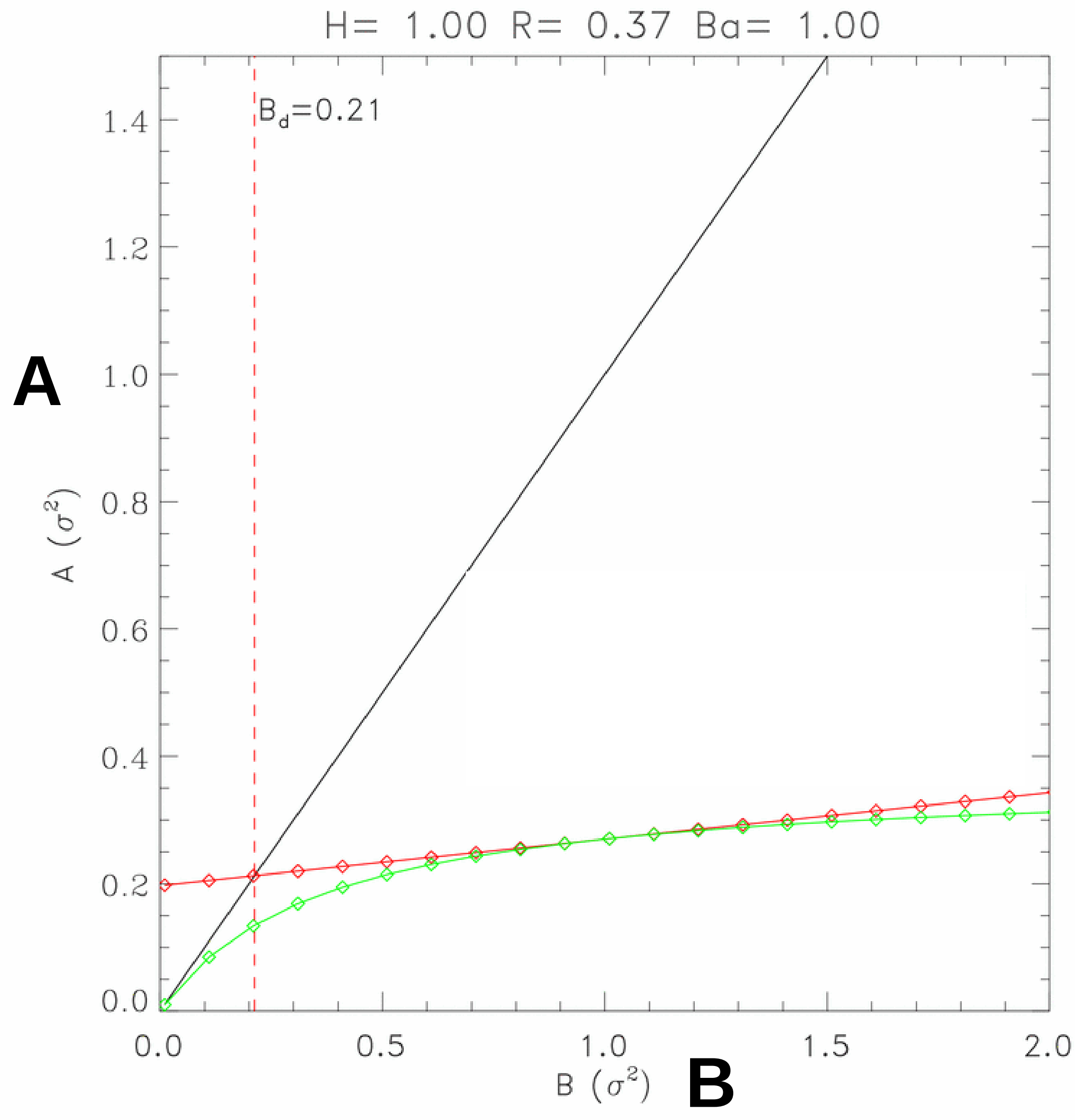


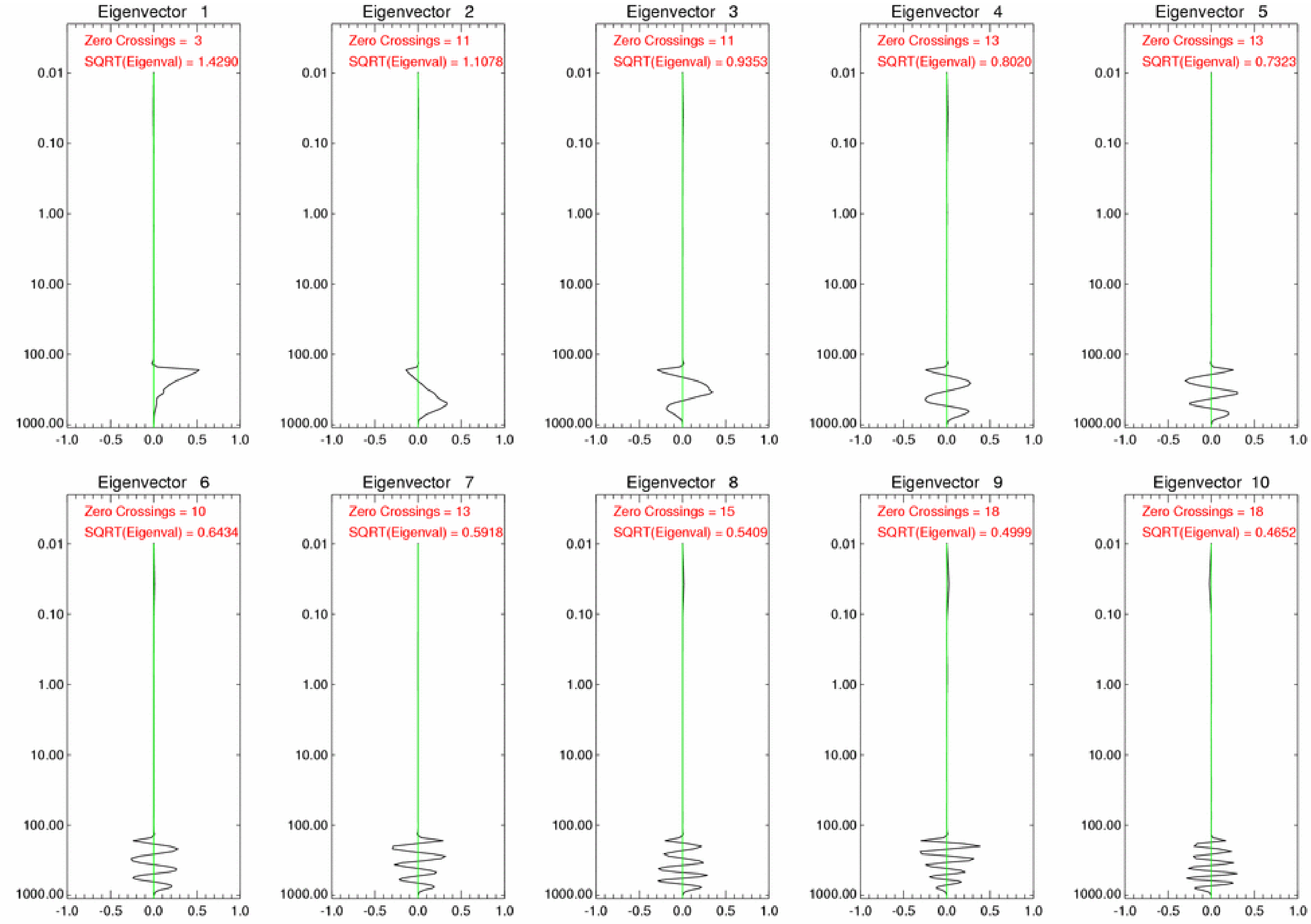
Illustration – scalar case (3)

H = 1
R = 0.37
$B_A = 1$

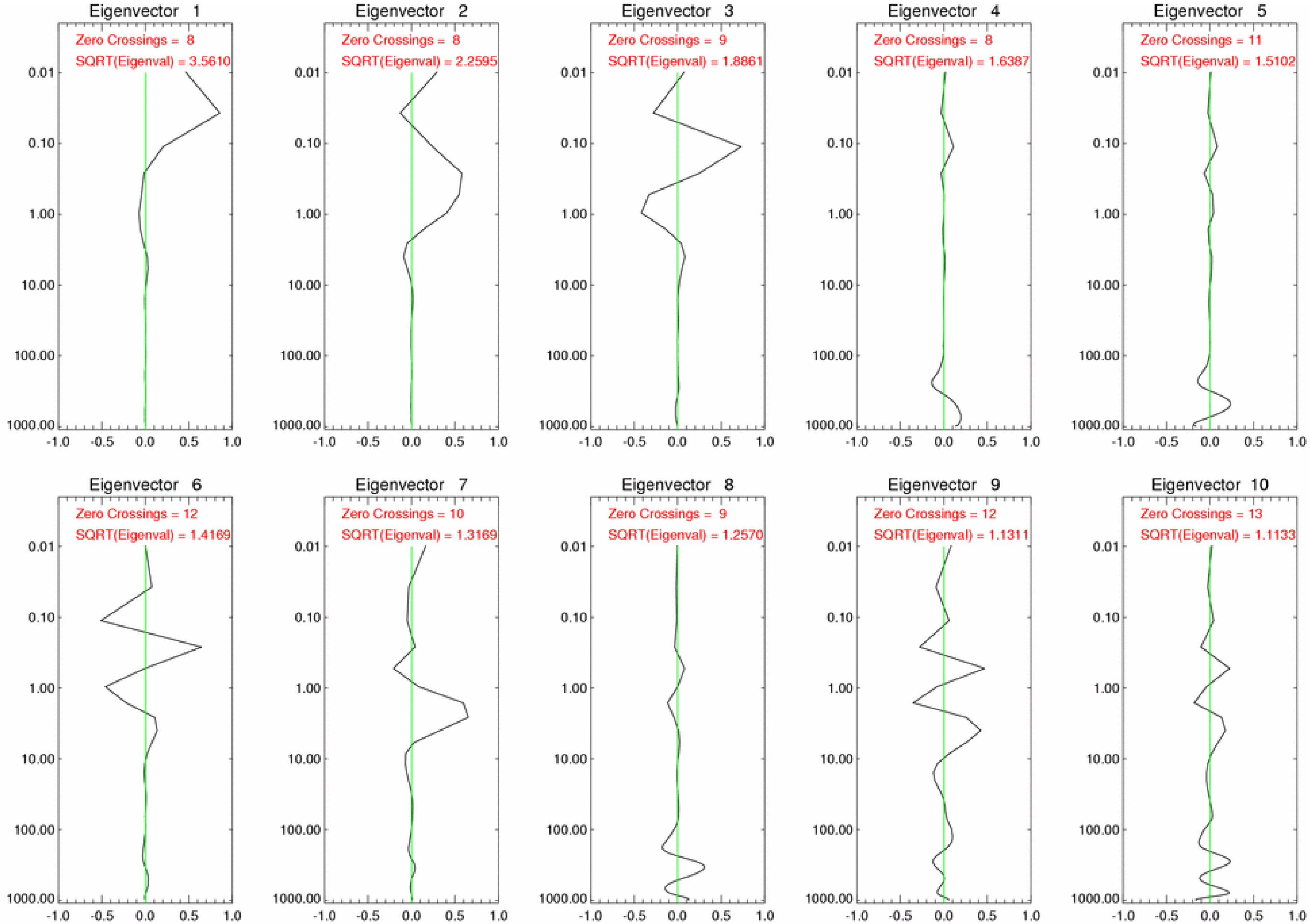
lower observation error



Leading eigenvectors of B_A MetO 70-level model, ln(q) (vectors 1-10)



Leading eigenvectors of B_A
MetO 70-level model, temp. (vectors 1-10)



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Leading eigenvectors of B_A
MetO 70-level model, temp. (vectors 11-20)

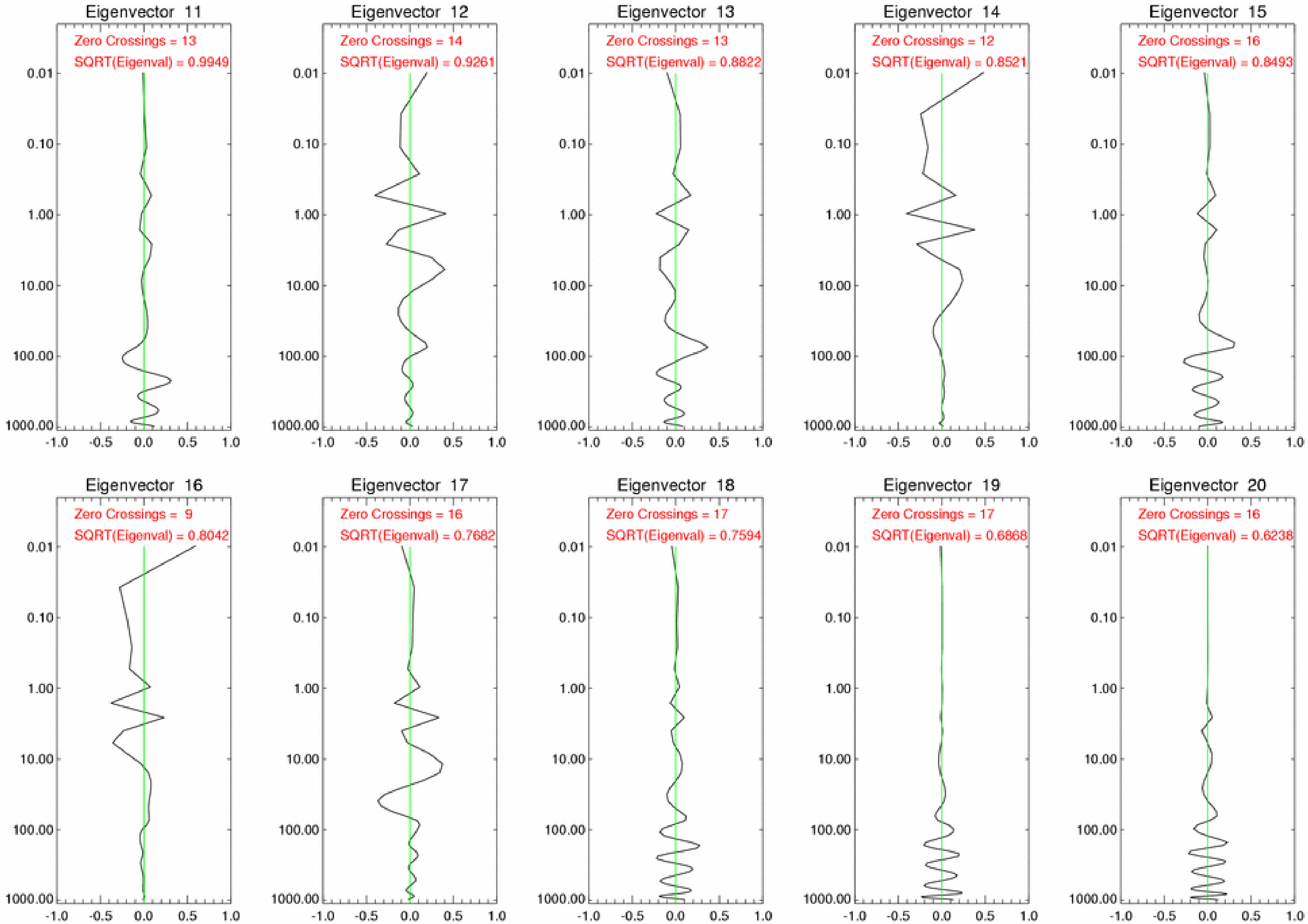
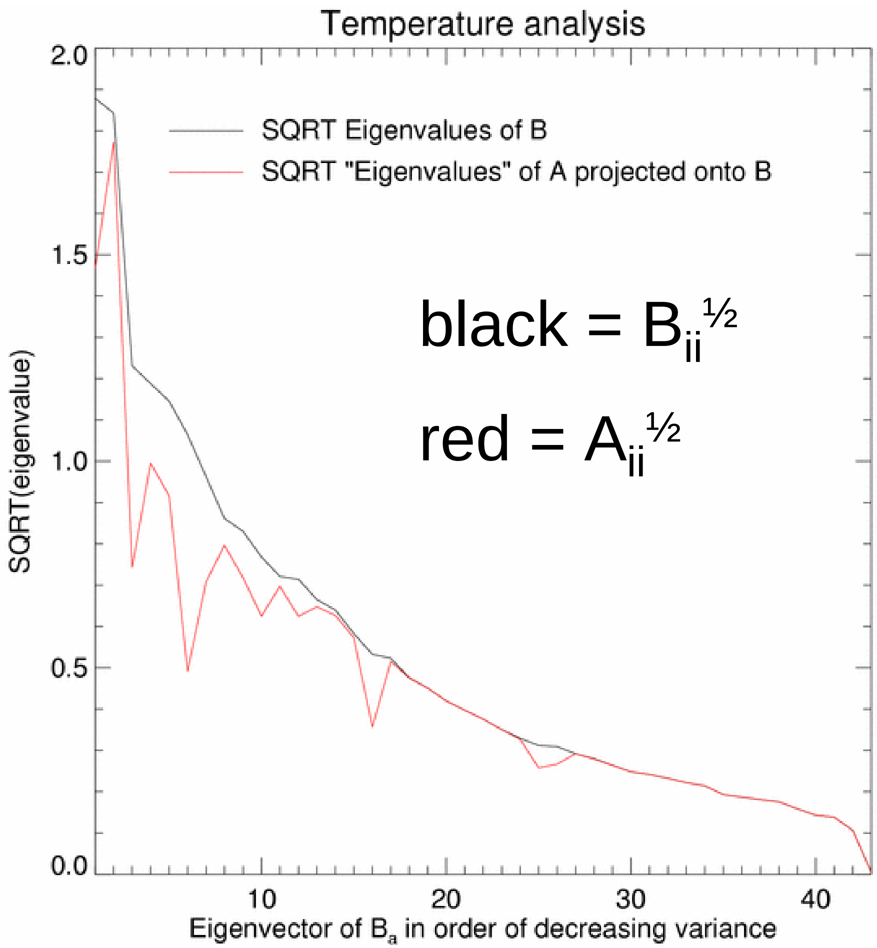


Illustration – IASI (1)

- Met Office operational 1D-var channel selection
  - 183 channels, of which 31 in water vapour band
- Observation error
  - instrument noise, or
  - instrument noise + forward model error of 0.2K + extra for unmodelled trace gas
- Analysis on 43 RTTOV levels using Jacobians from US standard atmosphere

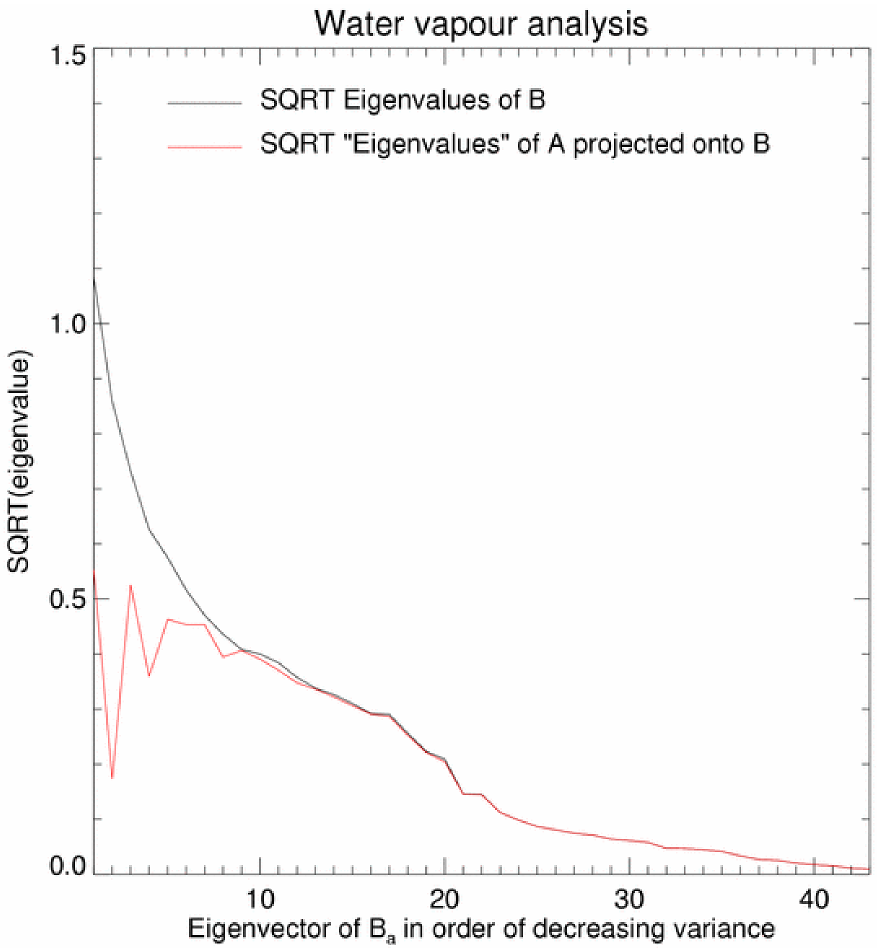
Diagonal of analysis error mapped to eigenvectors of B_A

observation error = instrument noise + forward model error



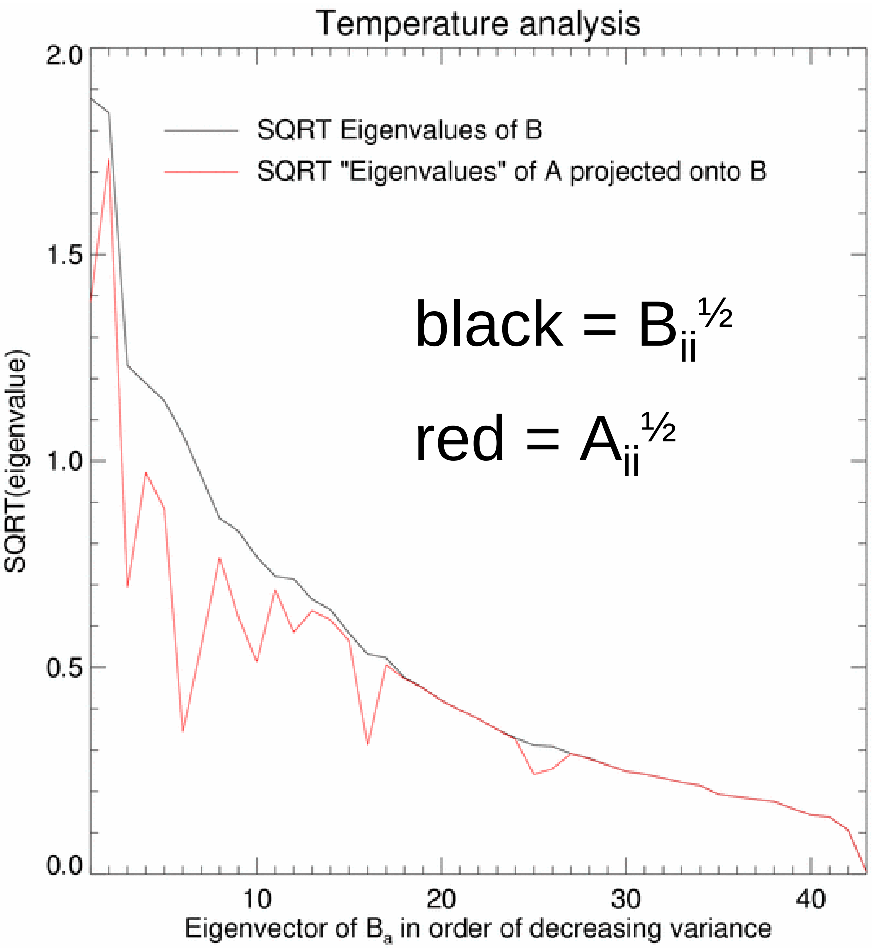
black = $B_{ii}^{½}$

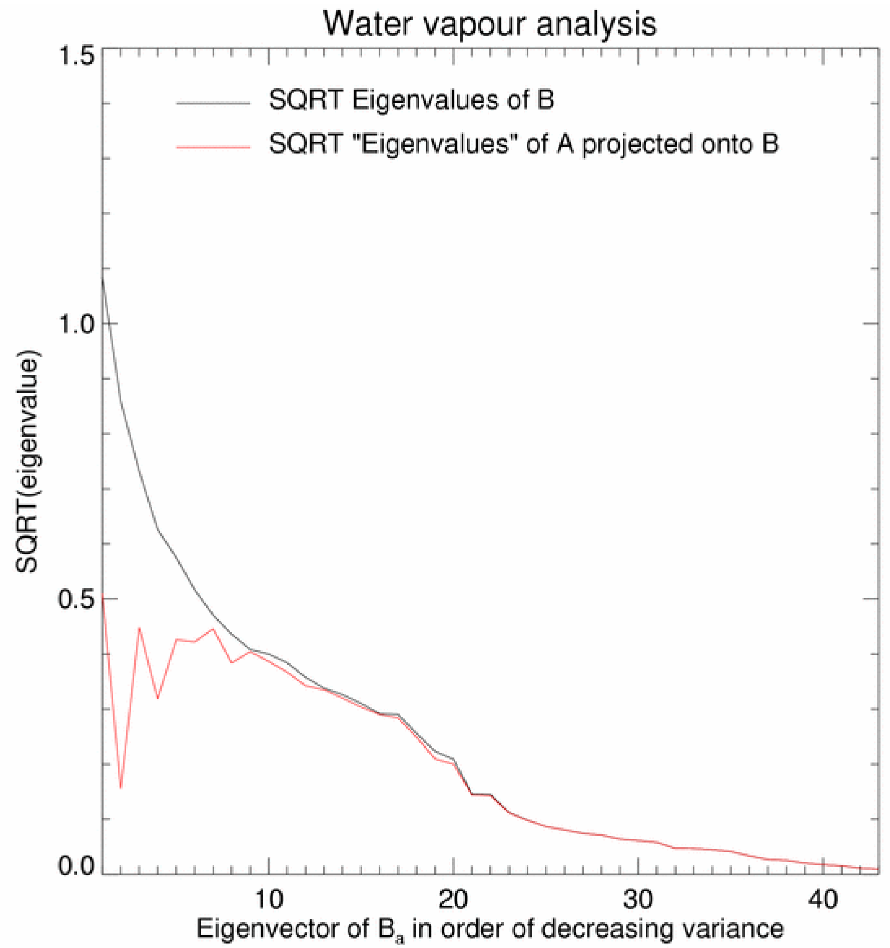
red = $A_{ii}^{½}$



Diagonal of analysis error mapped to eigenvectors of B_A

observation error = instrument noise





Conclusions so far …

- Goal: make retrievals/analyses robust against inevitable errors in the background error covariance
- … particularly for effective assimilation of satellite sounder data
- What is crucial for NWP? - structure of B assumed by the DA system, B_A
- Beware the danger zone! – analysis errors higher than background errors
- Current problems with Met Office 4D-Var B-matrix for temperature
- (provisional result) Some real IASI information is currently filtered out by the assimilation system

Further work

- Further work needed:
  - to perform a more complete error analysis for IASI
  - to understand B_A on each scale | good idea, in general
  - … and to improve it
  - to make B_A robust against inevitable errors

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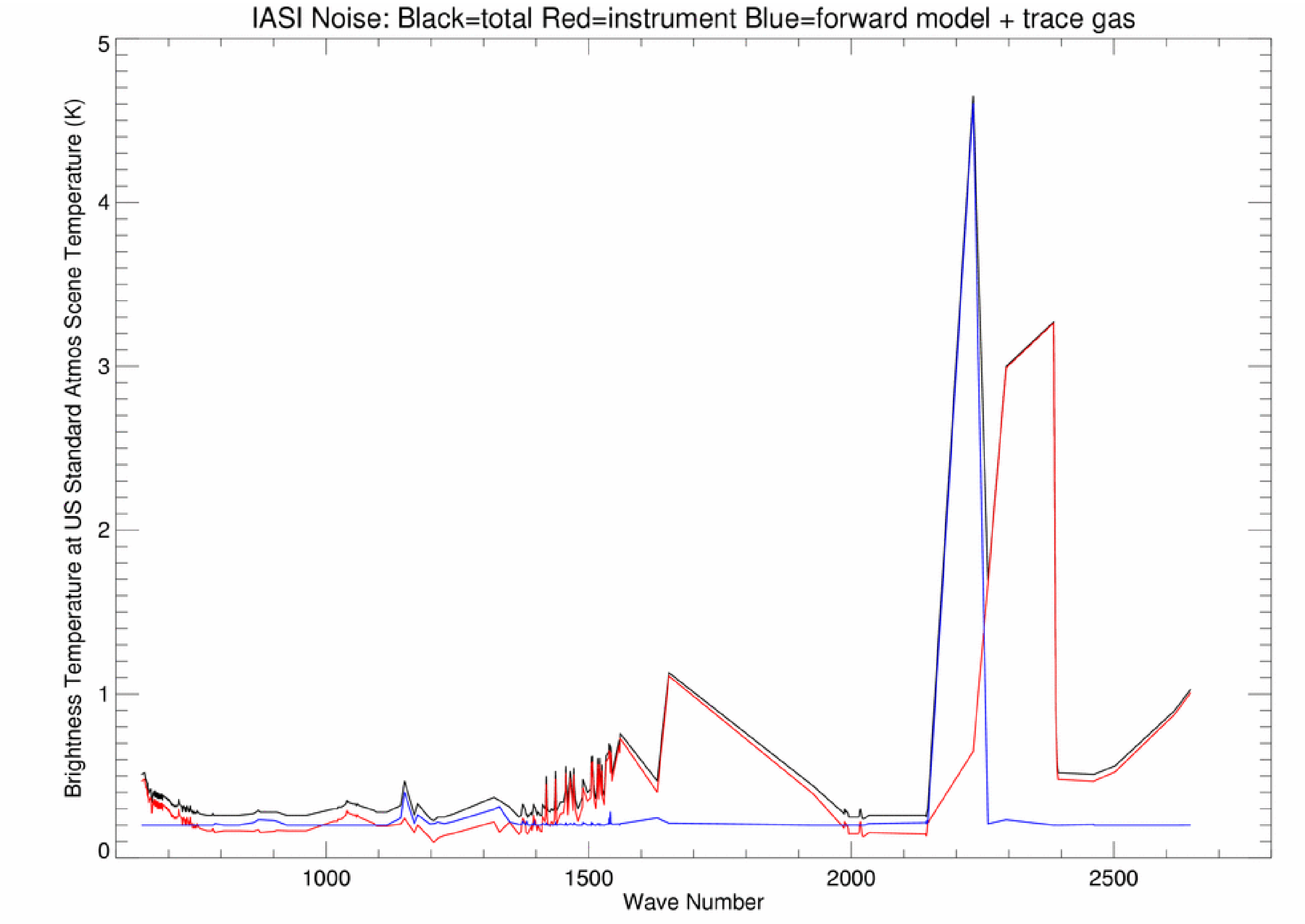
Thank you! Questions?

IASI noise

black: total

red: instrument

blue: forward model + “trace gas noise”



Goal

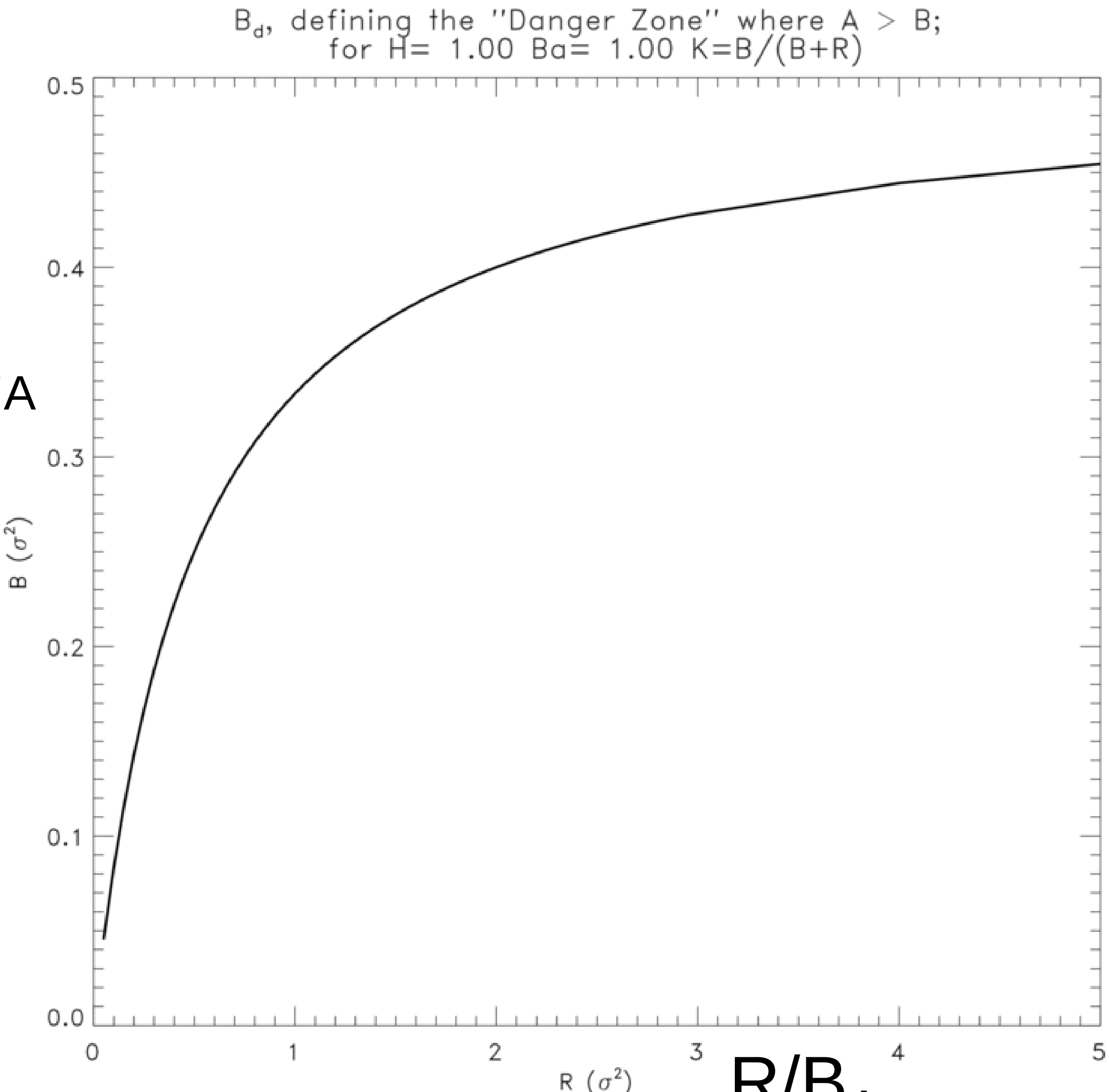
- To exploit the improved vertical resolution of advanced IR sounders
- … whilst retaining the (usually accurate) information from the NWP model on sharp vertical structures
  - e.g. PBL top, tropopause

Prior work on mis-specification of errors

- O.N.Strand   1977   The Annals of Statistics
- R.Daley   1991   Atmospheric Data Assimilation
  - R.Seaman   1977   MWR
  - R.Seaman et al.   1983   Aus. Met. Mag.
  - R.Franke   1985   MWR
- P.Watts and A.McNally   1988   Proc. ITSC-IV
- A.McNally   2000   QJRMS
- Others?

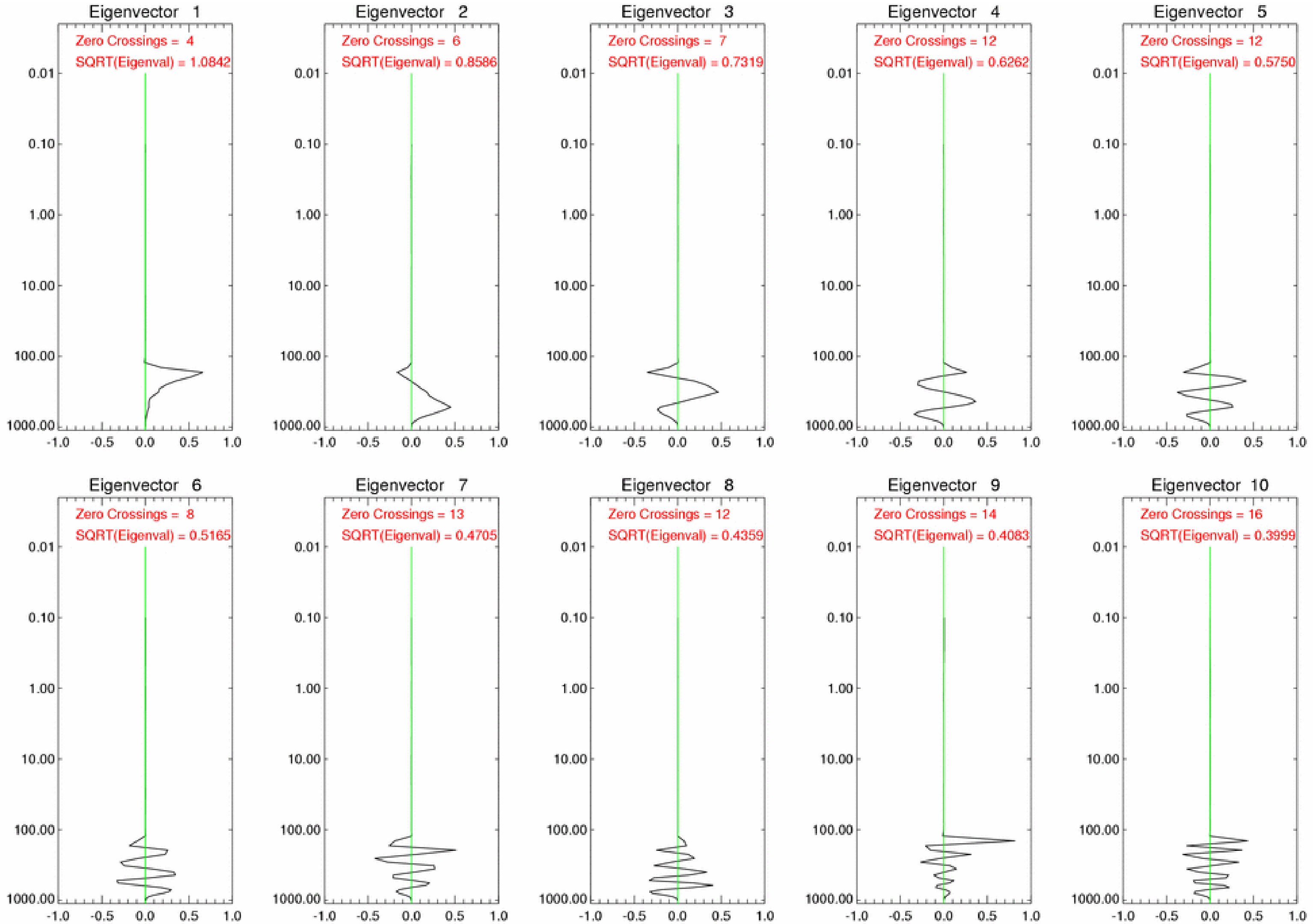
Scalar case – the danger zone

B(danger)/B_A

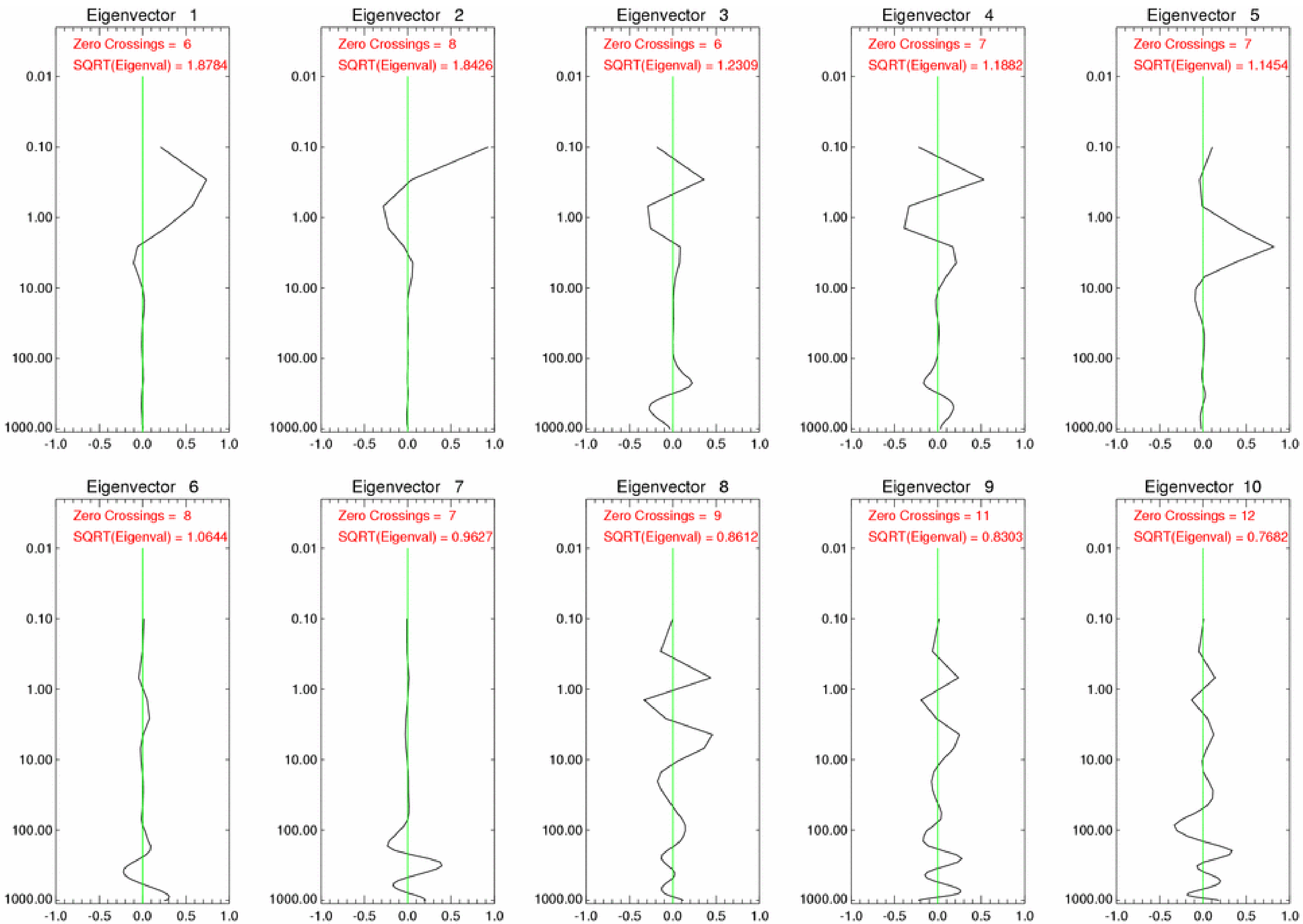


R/B_A

Leading eigenvectors of B_A
43 RTTOV levels ln(q) (vectors 1-10)

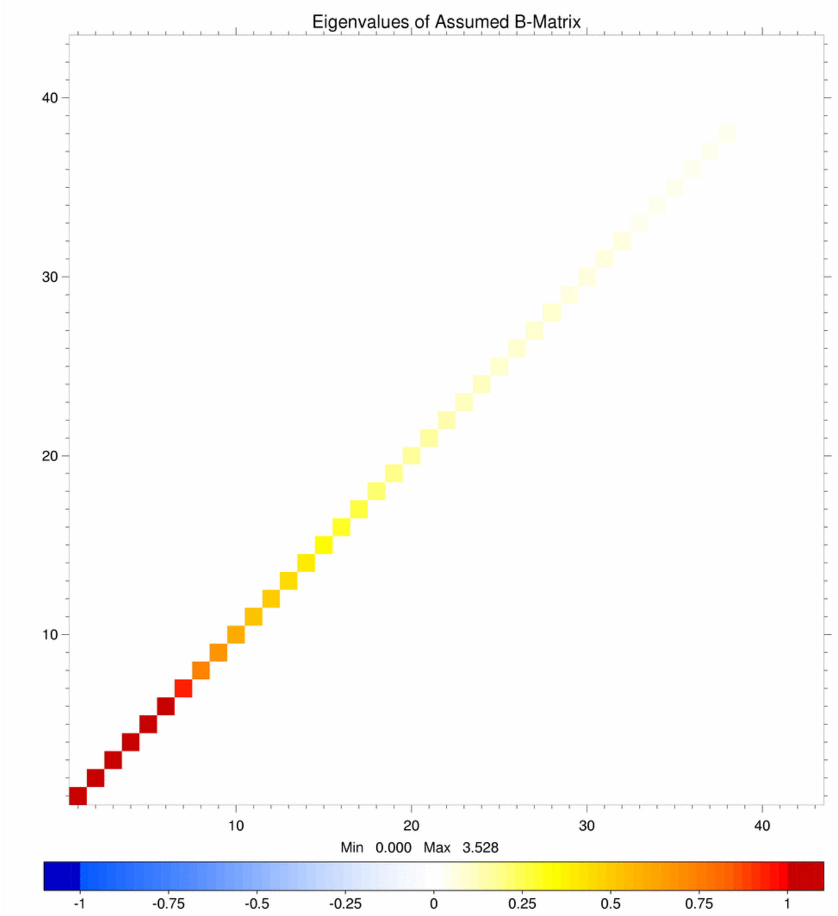


Leading eigenvectors of B_A
43 RTTOV levels temperature (vectors 1-10)

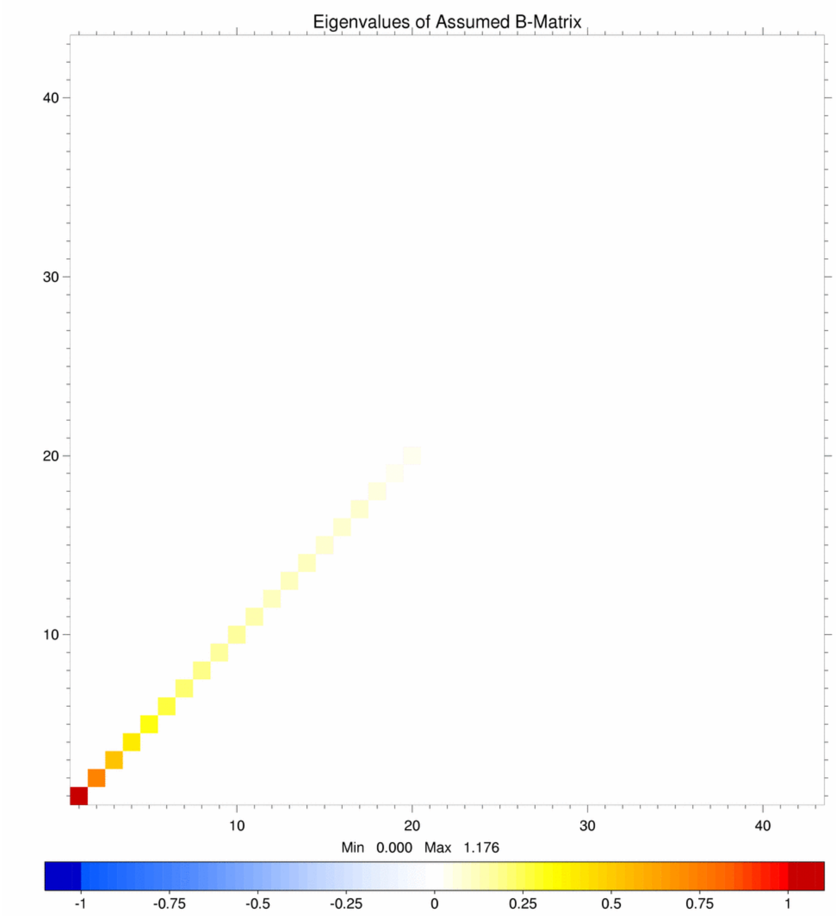


Eigenvalues of B_A:
background errors in the eigenspace of B_A

Temperature

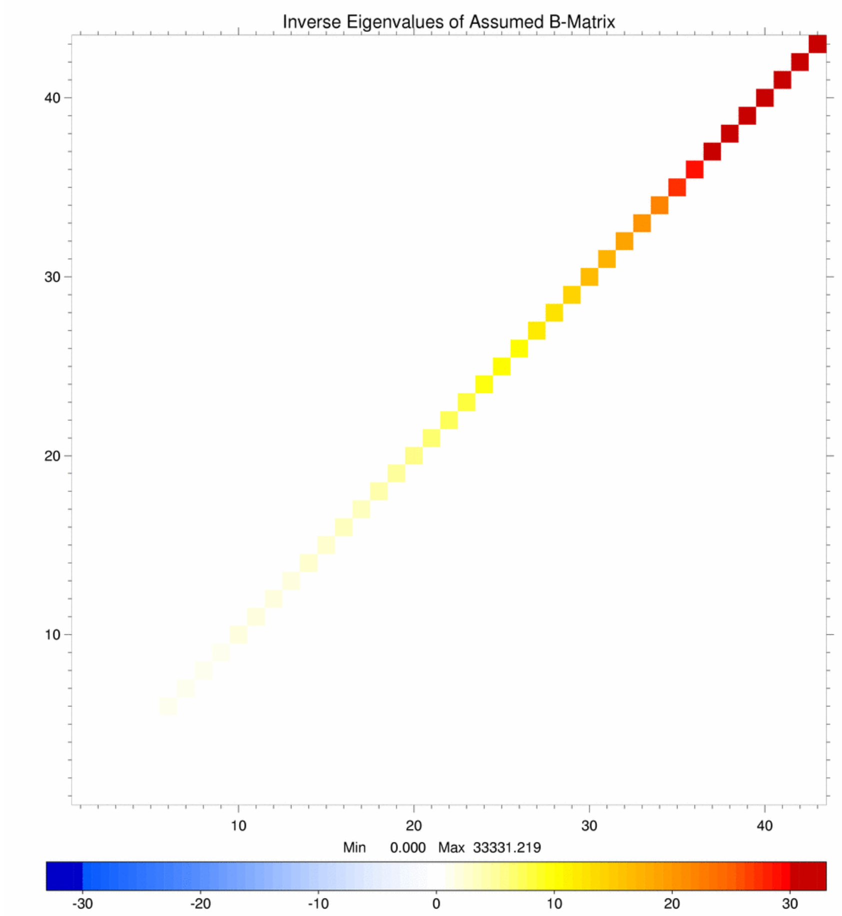


ln(q)

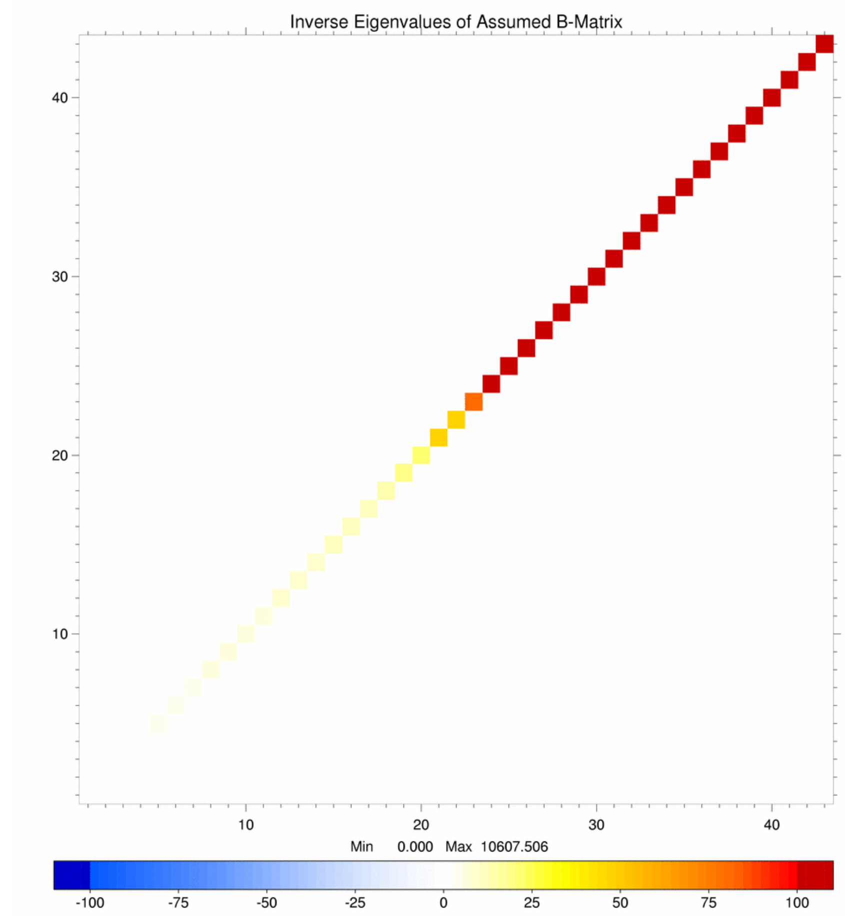


Inverse eigenvalues of B_A

Temperature



ln(q)

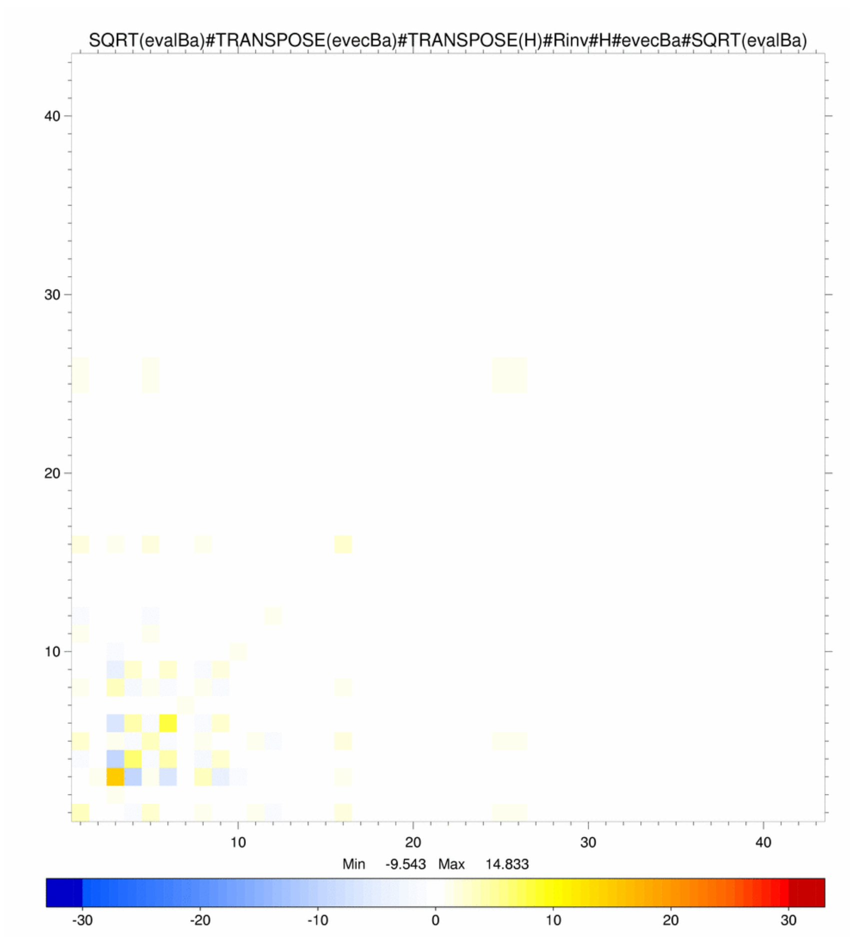


IASI information mapped to B_A eigenvectors and normalised by B_A eigenvalues:

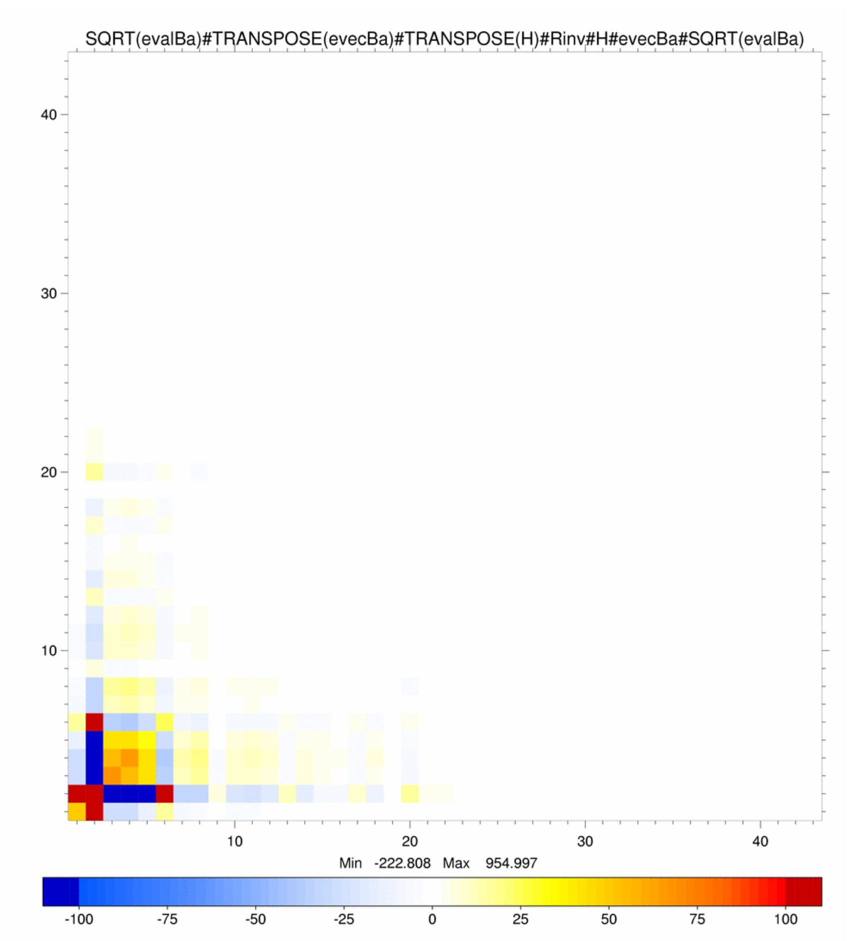
$$\Lambda^{½}.V^T.H^T.R^{-1}.H.V.\Lambda^{½}$$

Instrument noise + forward model error

Temperature



ln(q)

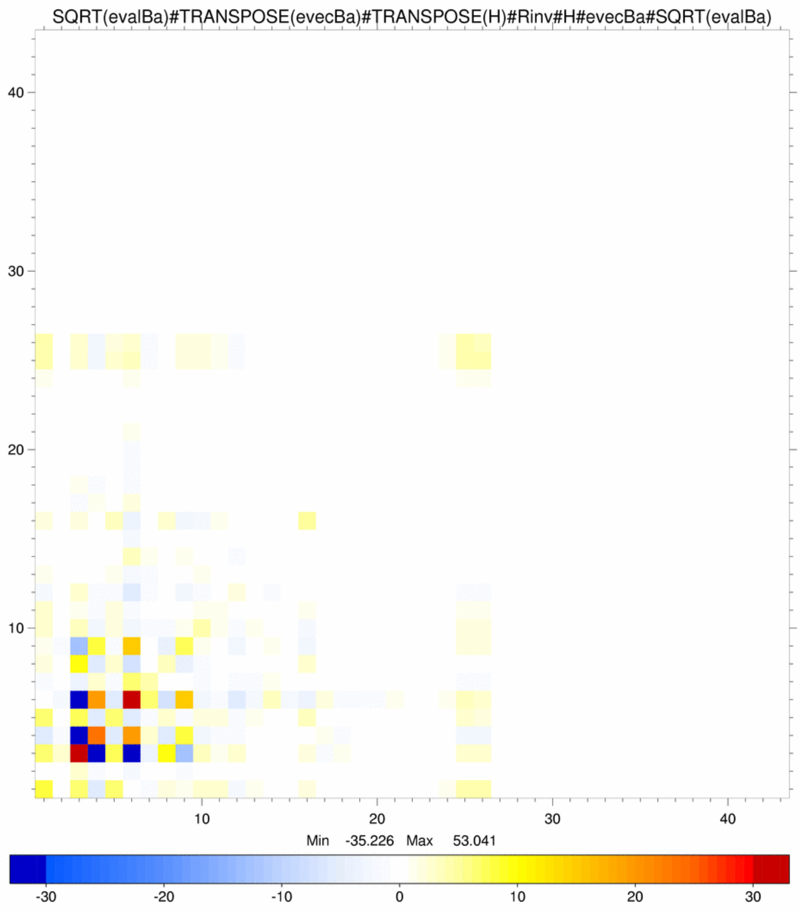


IASI information mapped to B_A eigenvectors and normalised by B_A eigenvalues:

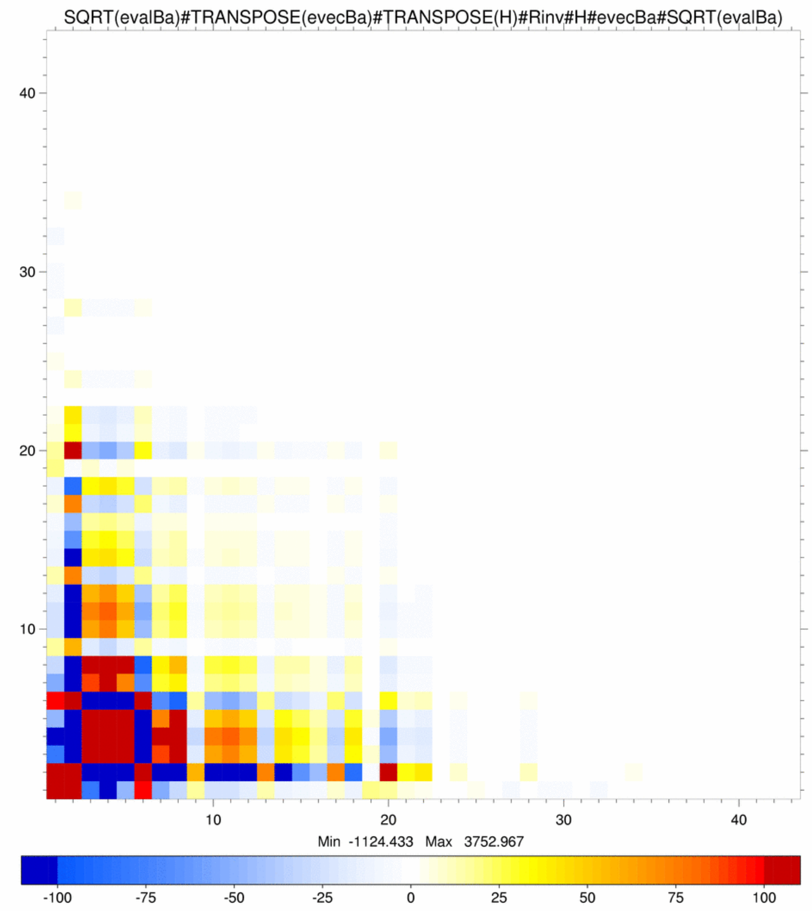
$\Lambda^{1/2}.V^T.H^T.R^{-1}.H.V.\Lambda^{1/2}$

Instrument noise only

Temperature



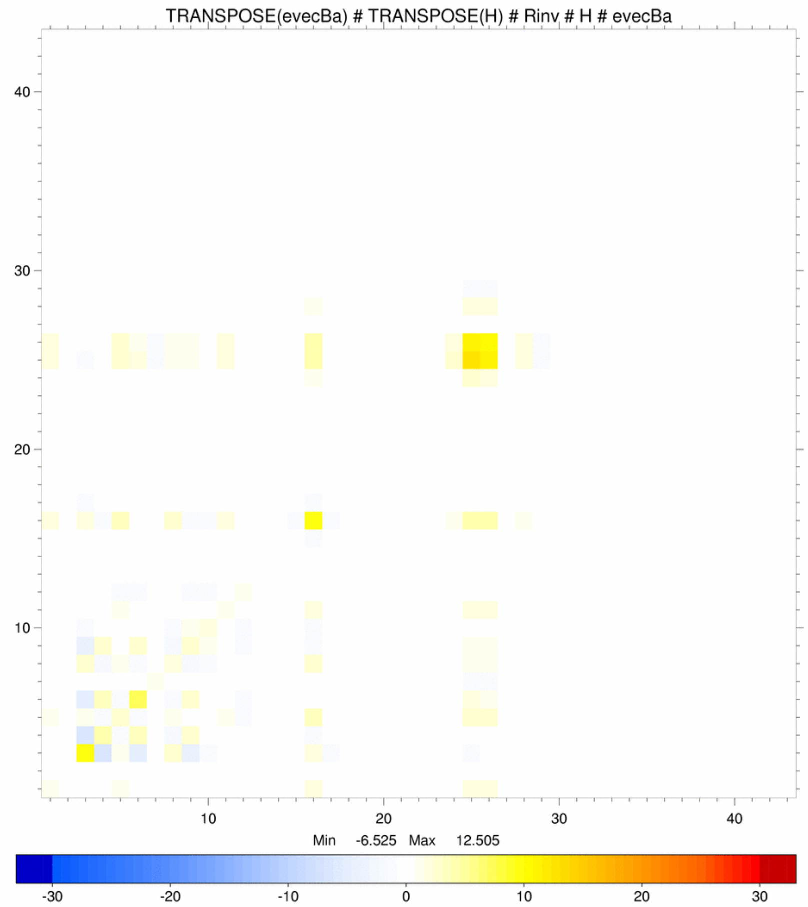
ln(q)



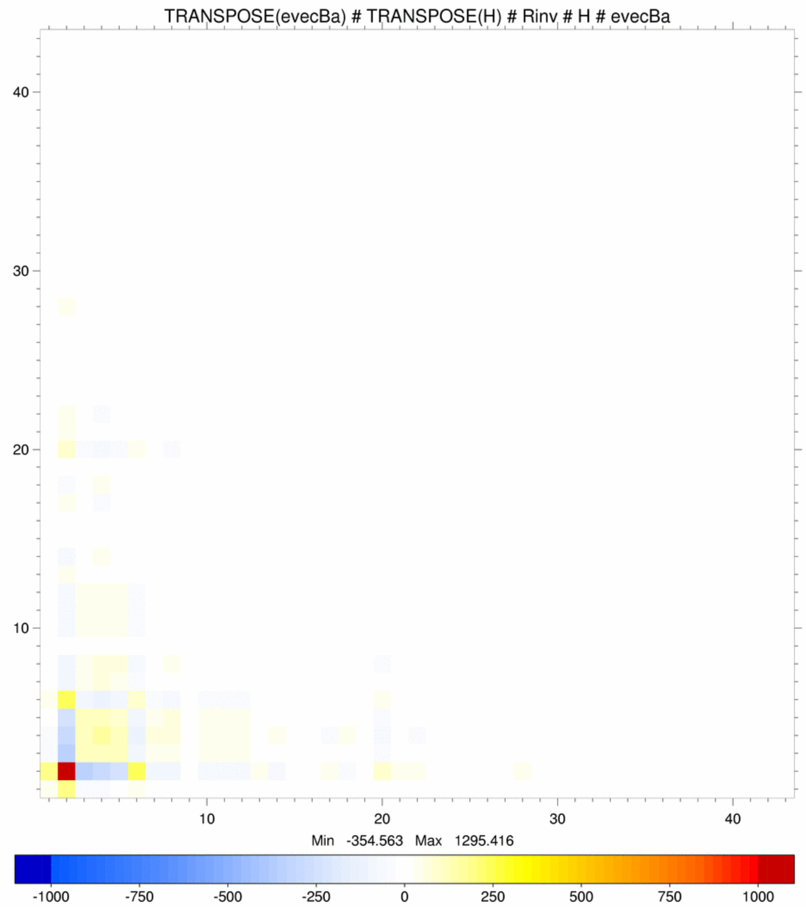
IASI observation error mapped to eigenvectors of B_A: $V^T.H^T.R^{-1}.H.V$

observation error = instrument noise + forward model error

temperature



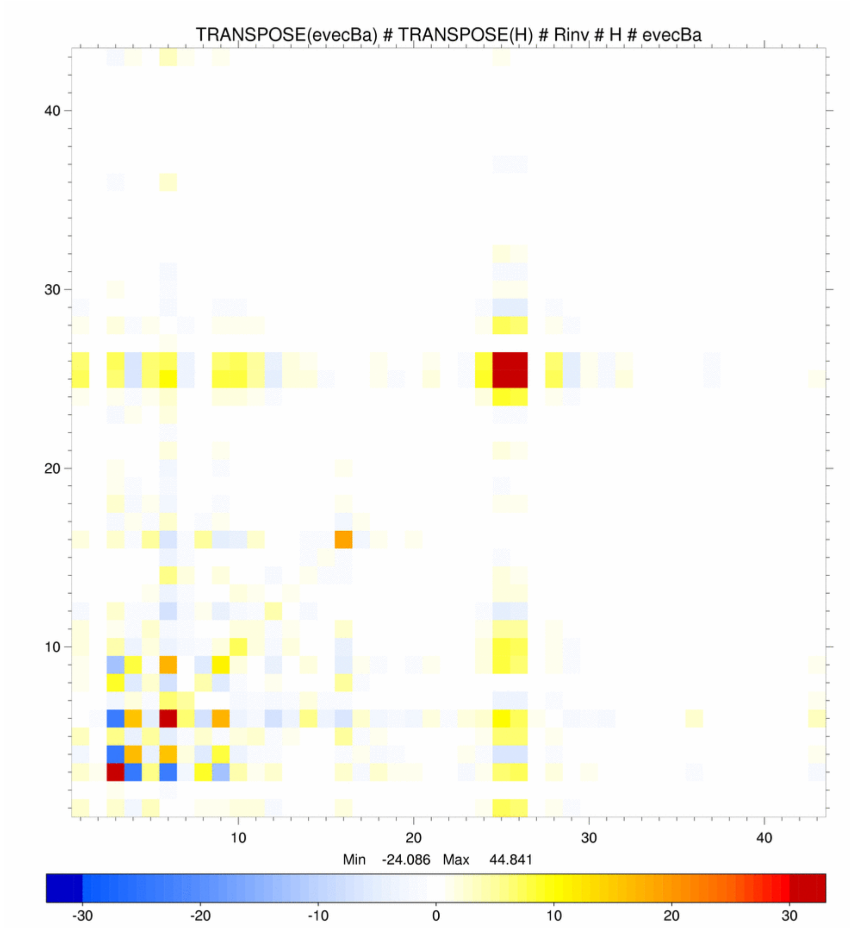
ln(q)



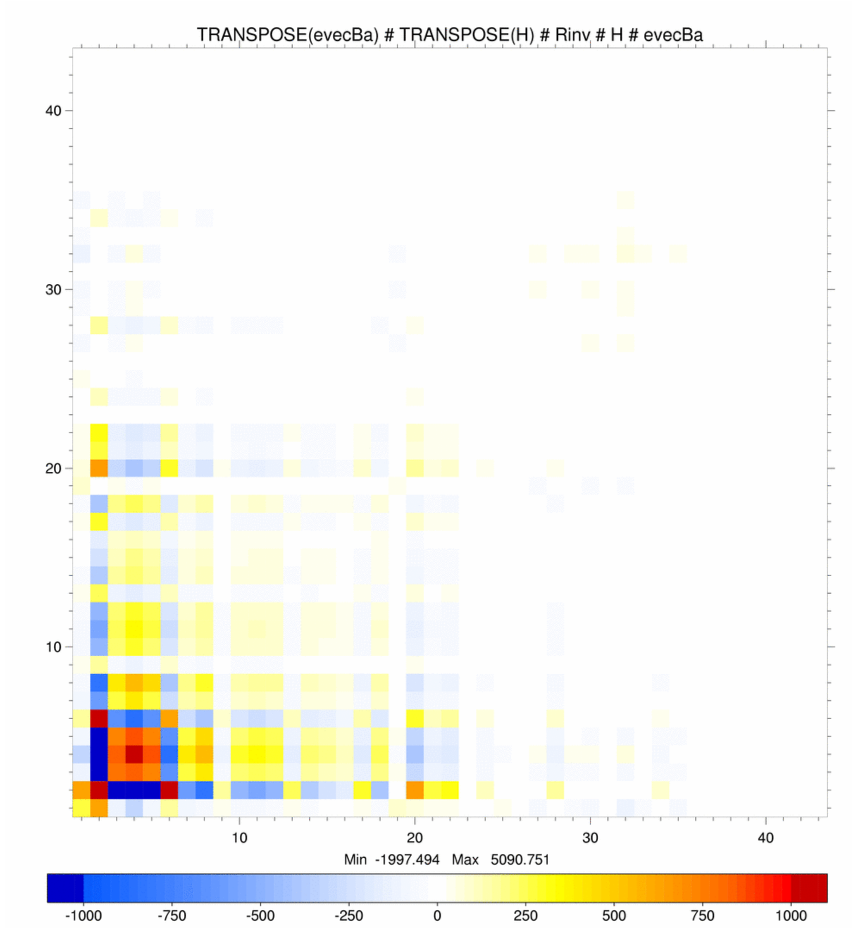
IASI observation error mapped to eigenvectors of B_A

observation error = instrument noise only

temperature



ln(q)



International TOVS Study Conference, 17th, ITSC-17, Monterey, CA, 14-20 April 2010. Madison, WI, University of Wisconsin-Madison, Space Science and Engineering Center, Cooperative Institute for Meteorological Satellite Studies, 2011.