# Radiation and the Planck Function

Lectures in Benevento
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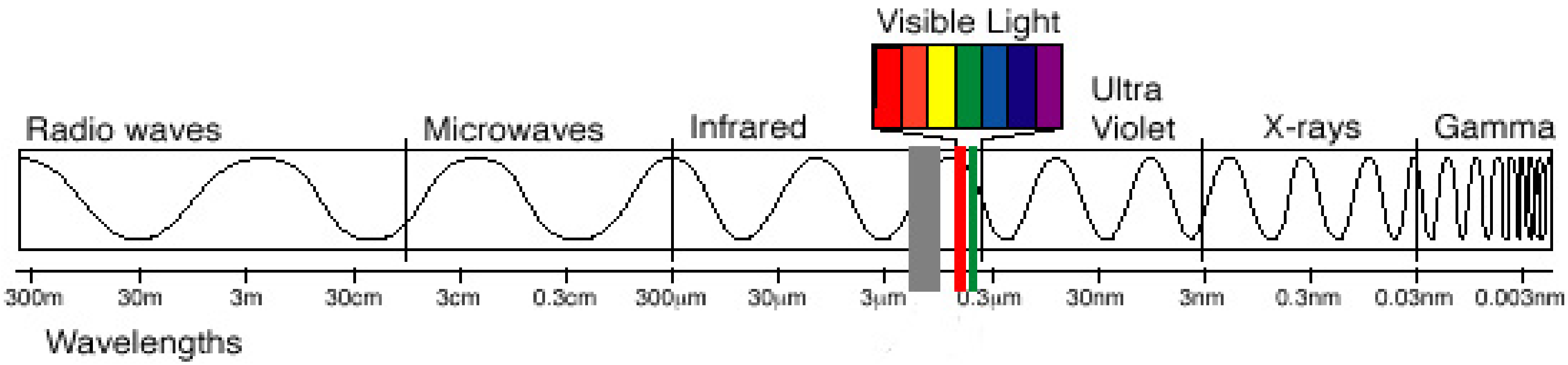
**All satellite remote sensing systems involve the measurement of electromagnetic radiation.**

**Electromagnetic radiation has the properties of both waves and discrete particles, although the two are never manifest simultaneously.**

**Electromagnetic radiation is usually quantified according to its wave-like properties; for many applications it considered to be a continuous train of sinusoidal shapes.**

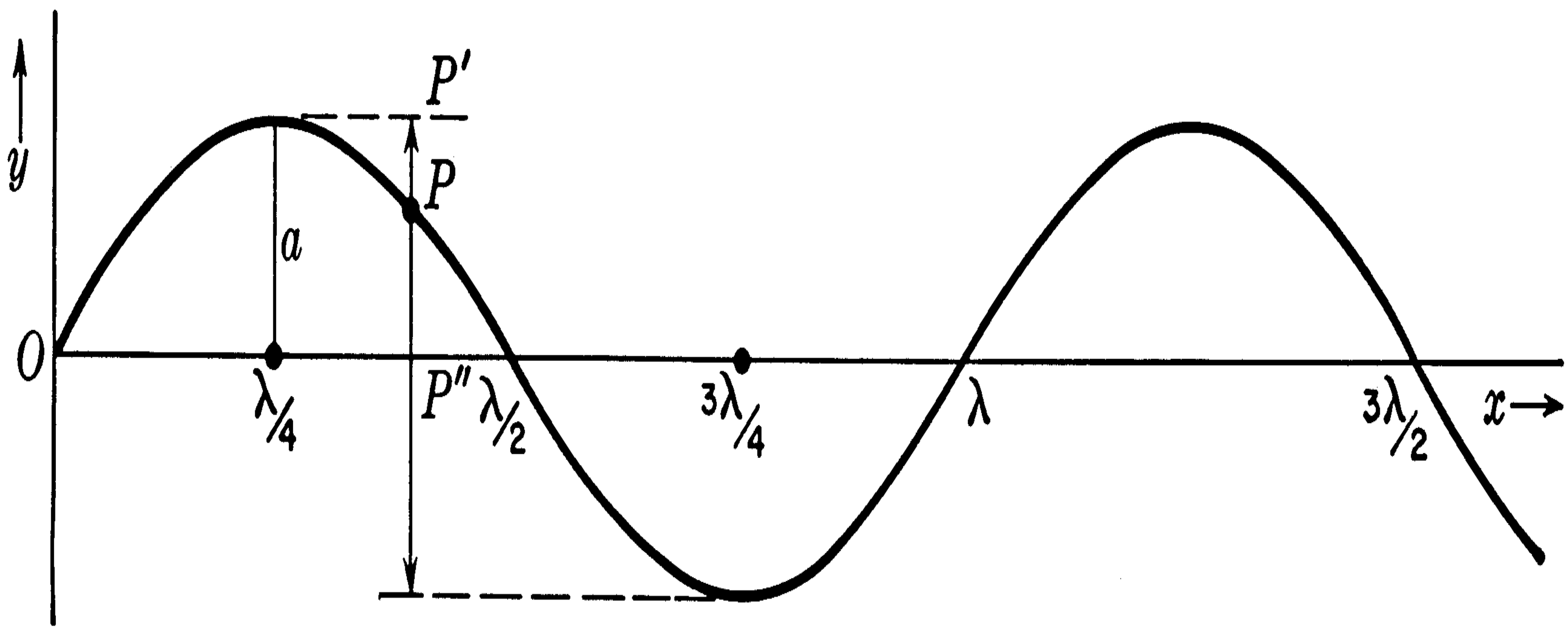
# The Electromagnetic Spectrum




Remote sensing uses radiant energy that is reflected and emitted from Earth at various "wavelengths" of the electromagnetic spectrum

Our eyes are sensitive to the visible portion of the EM spectrum

# Radiation is characterized by wavelength λ and amplitude a



# Terminology of radiant energy

**Energy from the Earth Atmosphere**

over time is

**Flux**

which strikes the detector area

**Irradiance**

at a given wavelength interval

**Monochromatic Irradiance**

over a solid angle on the Earth

**Radiance observed by satellite radiometer**

is described by

**The Planck function**

can be inverted to

**Brightness temperature**

# Definitions of Radiation

| QUANTITY | SYMBOL | UNITS |
| --- | --- | --- |
| Energy | dQ | Joules |
| Flux | dQ/dt | Joules/sec = Watts |
| Irradiance | dQ/dt/dA | Watts/meter$^2$ |
| Monochromatic Irradiance | dQ/dt/dA/dλ | W/m$^2$/micron |
| | or | |
| | dQ/dt/dA/dν | W/m$^2$/cm$^{-1}$ |
| Radiance | dQ/dt/dA/dλ/dΩ | W/m$^2$/micron/ster |
| | or | |
| | dQ/dt/dA/dν/dΩ | W/m$^2$/cm$^{-1}$/ster |

## Radiation from the Sun

The rate of energy transfer by electromagnetic radiation is called the radiant flux, which has units of energy per unit time. It is denoted by

$$F = dQ / dt$$

and is measured in joules per second or watts. For example, the radiant flux from the sun is about 3.90 x 10**26 W.

The radiant flux per unit area is called the irradiance (or radiant flux density in some texts). It is denoted by

$$E = dQ / dt / dA$$

and is measured in watts per square metre. The irradiance of electromagnetic radiation passing through the outermost limits of the visible disk of the sun (which has an approximate radius of 7 x 10**8 m) is given by

$$E \text{ (sun sfc)} = \frac{3.90 \times 10^{26}}{4\pi \, (7 \times 10^{8})^{2}} = 6.34 \times 10^{7} \text{ W m}^{-2} .$$

The solar irradiance arriving at the earth can be calculated by realizing that the flux is a constant, therefore

$$E\ (\text{earth sfc}) \times 4\pi R_{es}^2 = E\ (\text{sun sfc}) \times 4\pi R_s^2,$$

where $R_{es}$ is the mean earth to sun distance (roughly $1.5 \times 10^{11}$ m) and $R_s$ is the solar radius. This yields

$$E\ (\text{earth sfc}) = 6.34 \times 10^7\ (7 \times 10^8 / 1.5 \times 10^{11})^2 = 1380\ \text{W m}^{-2}.$$

The irradiance per unit wavelength interval at wavelength λ is called the monochromatic irradiance,

$$E_\lambda = dQ / dt / dA / d\lambda\ ,$$

and has the units of watts per square metre per micrometer. With this definition, the irradiance is readily seen to be

$$E = \int_0^\infty E_\lambda\, d\lambda\ .$$

In general, the irradiance upon an element of surface area may consist of contributions which come from an infinity of different directions. It is sometimes necessary to identify the part of the irradiance that is coming from directions within some specified infinitesimal arc of solid angle $d\Omega$. The irradiance per unit solid angle is called the radiance,

$$I = dQ / dt / dA / d\lambda / d\Omega,$$

and is expressed in watts per square metre per micrometer per steradian. This quantity is often also referred to as intensity and denoted by the letter B (when referring to the Planck function).

If the zenith angle, $\theta$, is the angle between the direction of the radiation and the normal to the surface, then the component of the radiance normal to the surface is then given by $I \cos\theta$. The irradiance represents the combined effects of the normal component of the radiation coming from the whole hemisphere; that is,

$$E = \int_{\Omega} I \cos\theta \, d\Omega \qquad \text{where in spherical coordinates } d\Omega = \sin\theta \, d\theta \, d\varphi \, .$$

Radiation whose radiance is independent of direction is called isotropic radiation. In this case, the integration over $d\Omega$ can be readily shown to be equal to $\pi$ so that

$$E = \pi I \, .$$

# Radiation is governed by Planck's Law

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2 / \lambda T} - 1] \}$$

**Summing the Planck function at one temperature over all wavelengths yields the energy of the radiating source**

$$E = \sum_{\lambda} B(\lambda, T) = \sigma T^4$$

**Brightness temperature is uniquely related to radiance for a given wavelength by the Planck function.**

## Using wavelengths

**Planck's Law** $B(\lambda,T) = c_1 / \lambda^5 / [e^{c_2/\lambda T} - 1]$ (mW/m²/ster/cm)

where
$\lambda$ = wavelengths in cm
T = temperature of emitting surface (deg K)
$c_1$ = 1.191044 x 10-5 (mW/m²/ster/cm$^{-4}$)
$c_2$ = 1.438769 (cm deg K)

**Wien's Law** $dB(\lambda_{max},T) / d\lambda = 0$ where $\lambda(max) = .2897/T$
indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\lambda_{max},T) \sim T^5$.

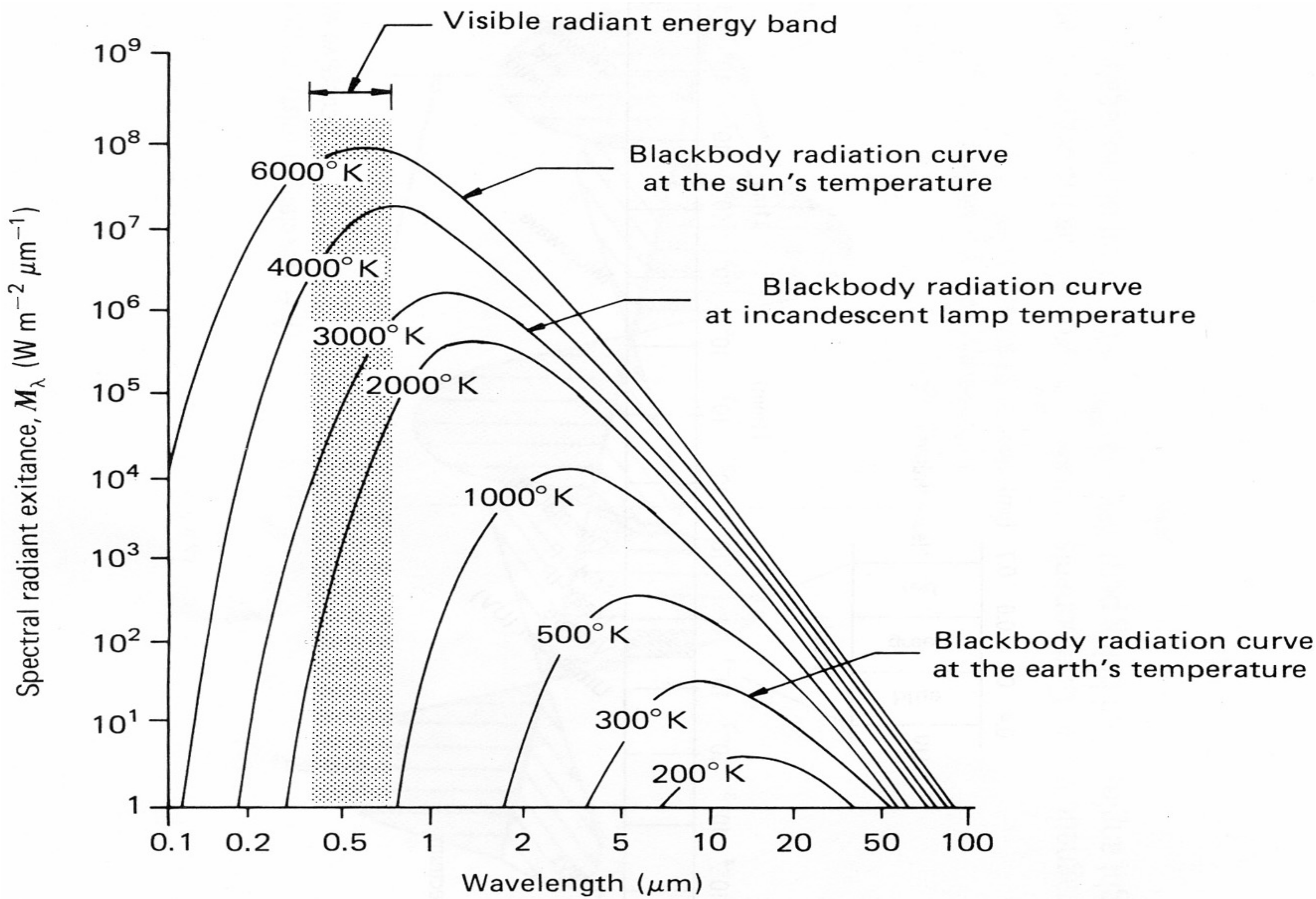
**Stefan-Boltzmann Law** $E = \pi \int_o^\infty B(\lambda,T)\, d\lambda = \sigma T^4$, where $\sigma$ = 5.67 x 10-8 W/m2/deg4.

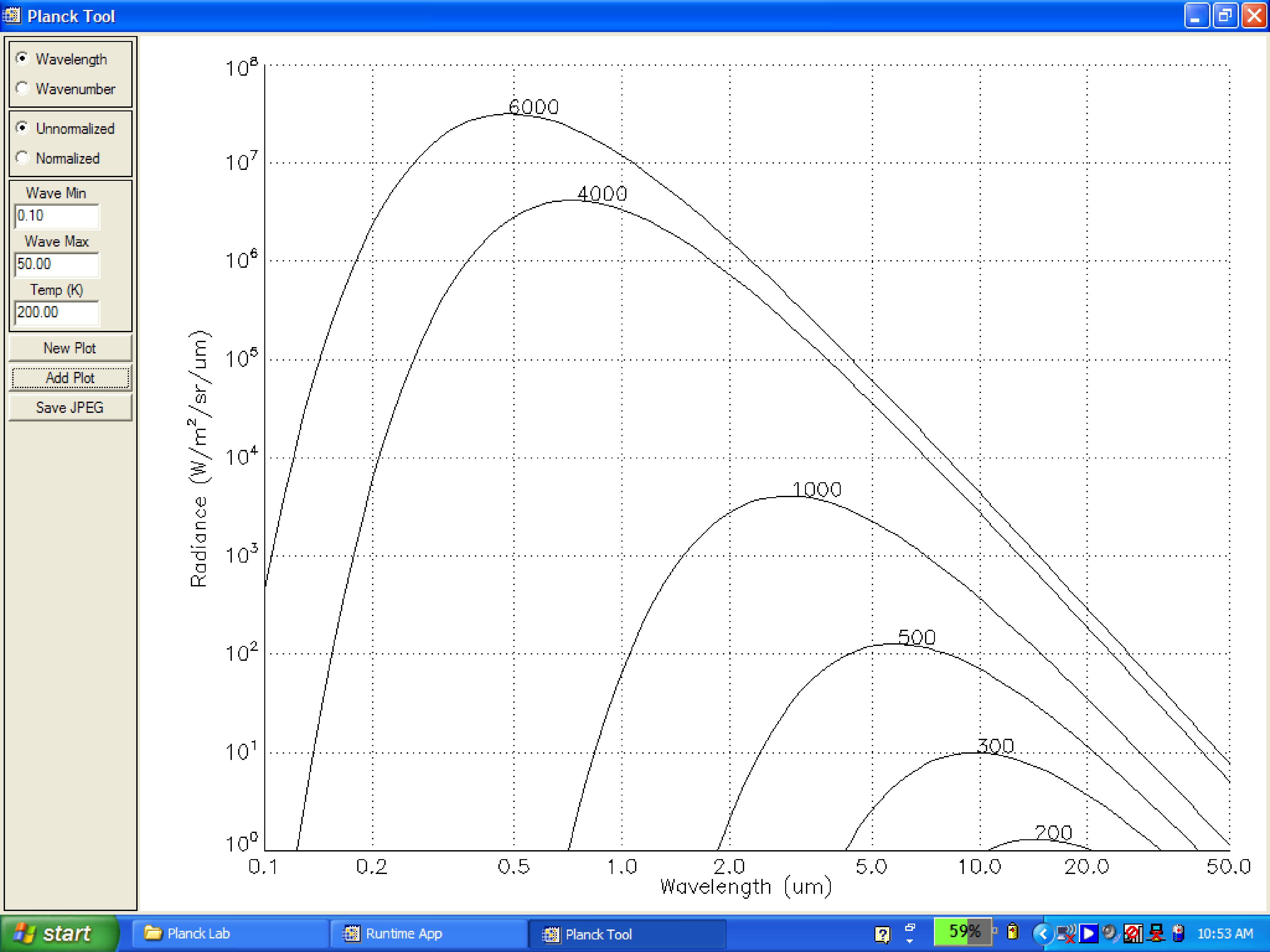
states that irradiance of a black body (area under Planck curve) is proportional to $T^4$ .

**Brightness Temperature**

$$T = c_2 / [\lambda \ln( \frac{c_1}{\lambda^5 B_\lambda} + 1)]$$ is determined by inverting Planck function

# Spectral Distribution of Energy Radiated from Blackbodies at Various Temperatures



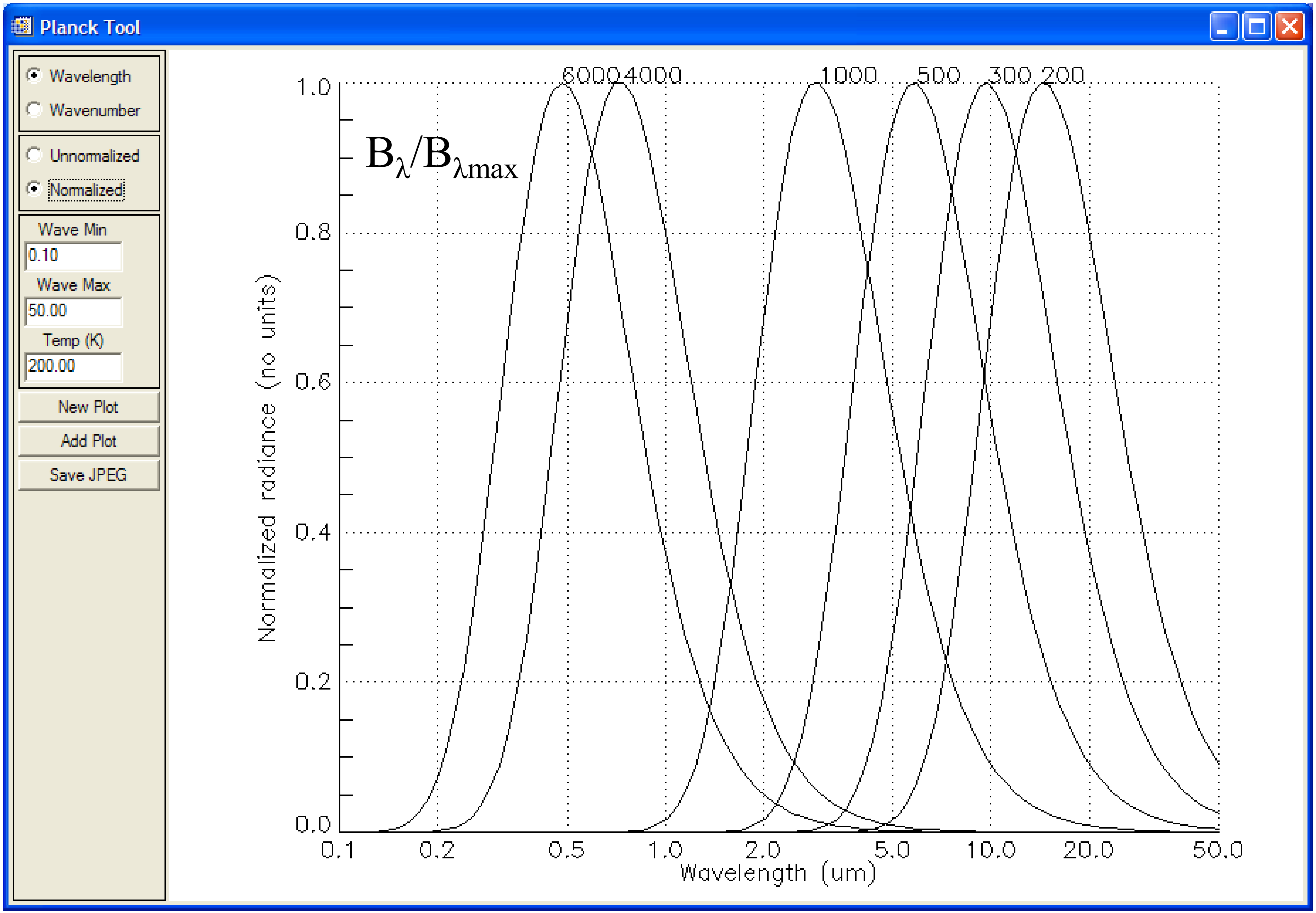

Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
0.10
Wave Max
50.00
Temp (K)
200.00
New Plot
Add Plot
Save JPEG
6000
4000
1000
500
300
200
Radiance (W/m2/sr/um)
Wavelength (um)
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0
50.0
start
Planck Lab
Runtime App
Planck Tool
59%
10:53 AM

# $B(\lambda_{max},T) \propto T^5$

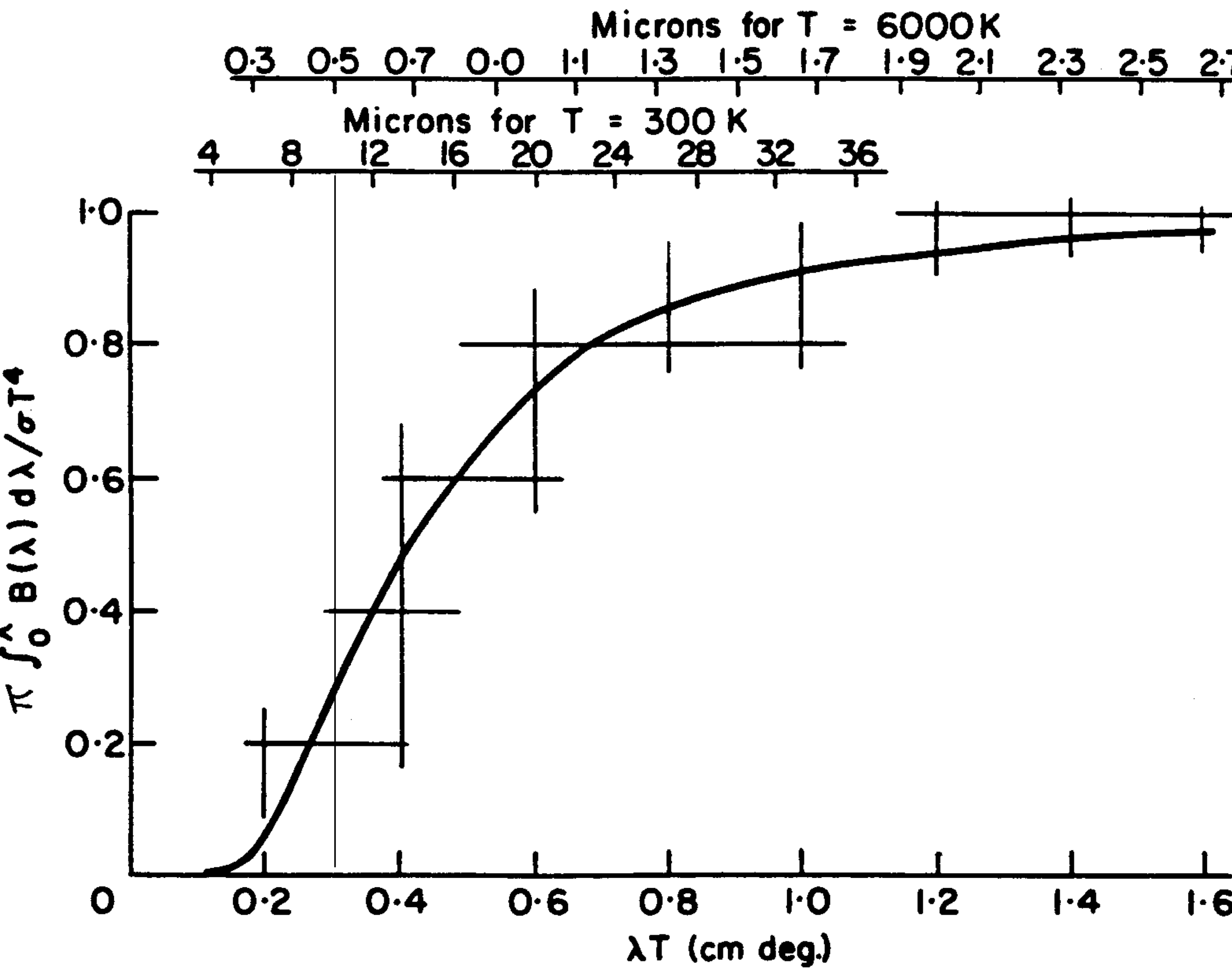
$B(\lambda max,6000) \sim 3.2 \times 10^7$
$B(\lambda max,300) \sim 1 \times 10^1$
so
$B(\lambda max,6000) / B(\lambda max,300) \sim 3 \times 10^6$

and
$(6000/300)^{5} = (20)^5 = 3.2 \times 10^6$

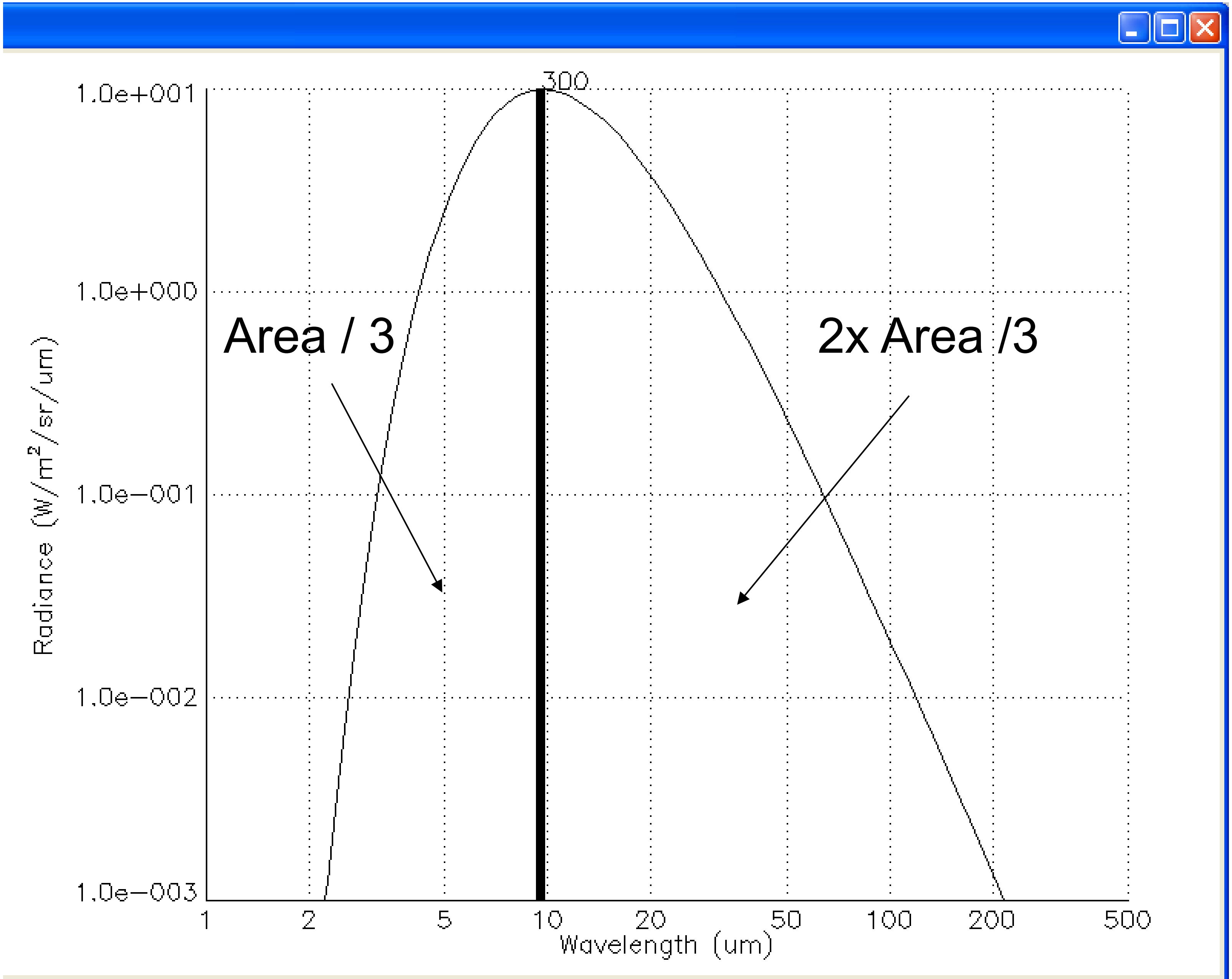
which is the same



Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
0.10
Wave Max
50.00
Temp (K)
200.00
New Plot
Add Plot
Save JPEG
6000 4000 1000 500 300 200
$B_\lambda/B_{\lambda max}$
Normalized radiance (no units)
1.0
0.8
0.6
0.4
0.2
0.0
0.1 0.2 0.5 1.0 2.0 5.0 10.0 20.0 50.0
Wavelength (um)



Microns for T = 6000 K
0·3 0·5 0·7 0·0 1·1 1·3 1·5 1·7 1·9 2·1 2·3 2·5 2·7
Microns for T = 300 K
4 8 12 16 20 24 28 32 36
1·0
0·8
0·6
0·4
0·2
0
$\pi \int_0^\lambda B(\lambda)\, d\lambda / \sigma T^4$
0·2 0·4 0·6 0·8 1·0 1·2 1·4 1·6
λT (cm deg.)



300
1.0e+001
1.0e+000
1.0e-001
1.0e-002
1.0e-003
Radiance (W/m2/sr/um)
Area / 3
2x Area /3
1
2
5
10
20
50
100
200
500
Wavelength (um)

## Using wavenumbers

**Planck's Law** $B(\nu,T) = c_1\nu^3 / [e^{c_2\nu/T} - 1]$ $(mW/m^2/ster/cm^{-1})$

where $\nu$ = # wavelengths in one centimeter (cm-1)

T = temperature of emitting surface (deg K)

$c_1$ = 1.191044 x 10-5 $(mW/m^2/ster/cm^{-4})$

$c_2$ = 1.438769 (cm deg K)

**Wien's Law** $dB(\nu_{max},T) / d\nu = 0$ where $\nu(max) = 1.95T$

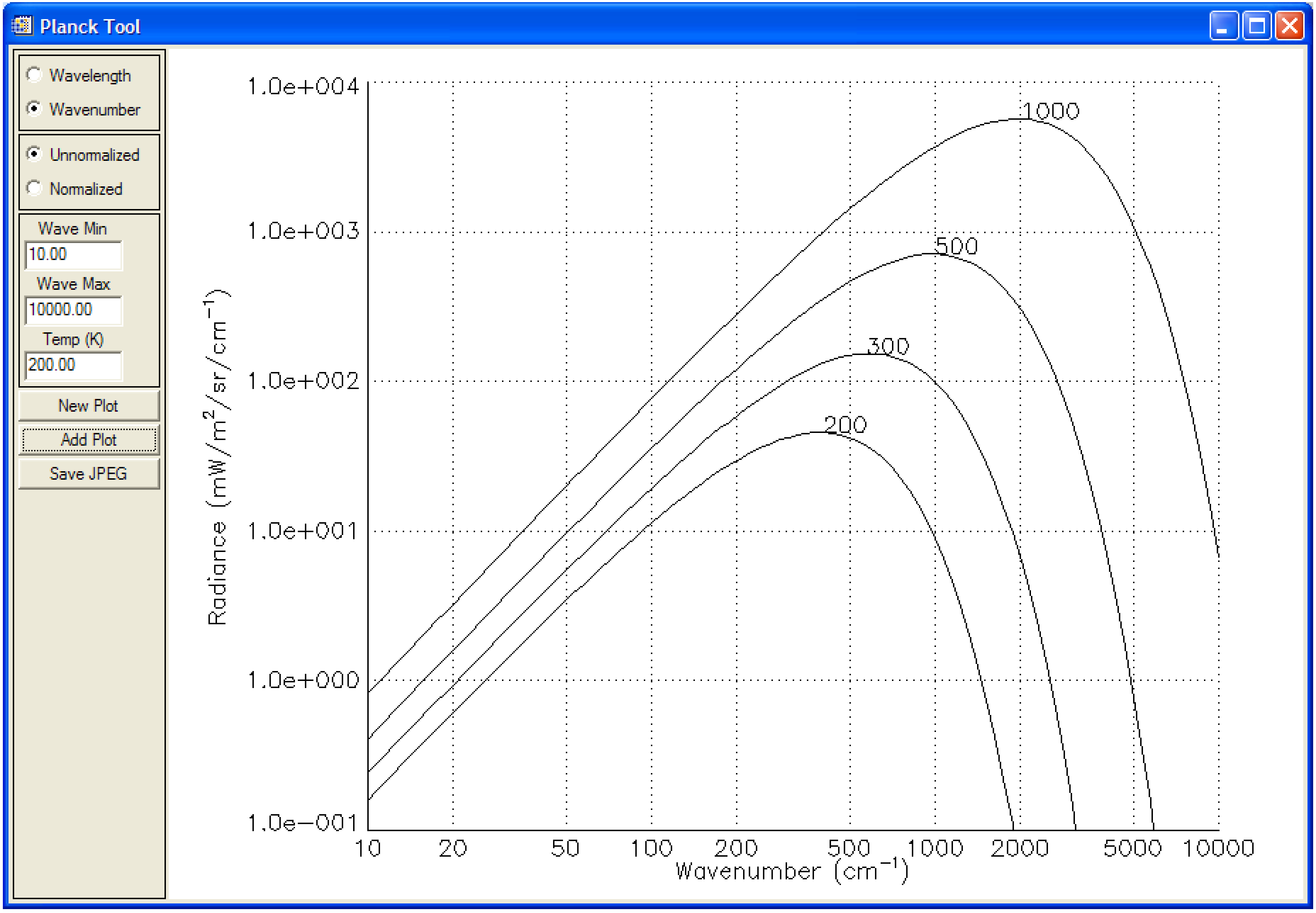
indicates peak of Planck function curve shifts to shorter wavelengths (greater wavenumbers) with temperature increase. Note $B(\nu_{max},T) \sim T^{**}3$.

**Stefan-Boltzmann Law** $E = \pi \int_o^\infty B(\nu,T)\, d\nu = \sigma T^4$, where $\sigma$ = 5.67 x 10-8 W/m2/deg4.

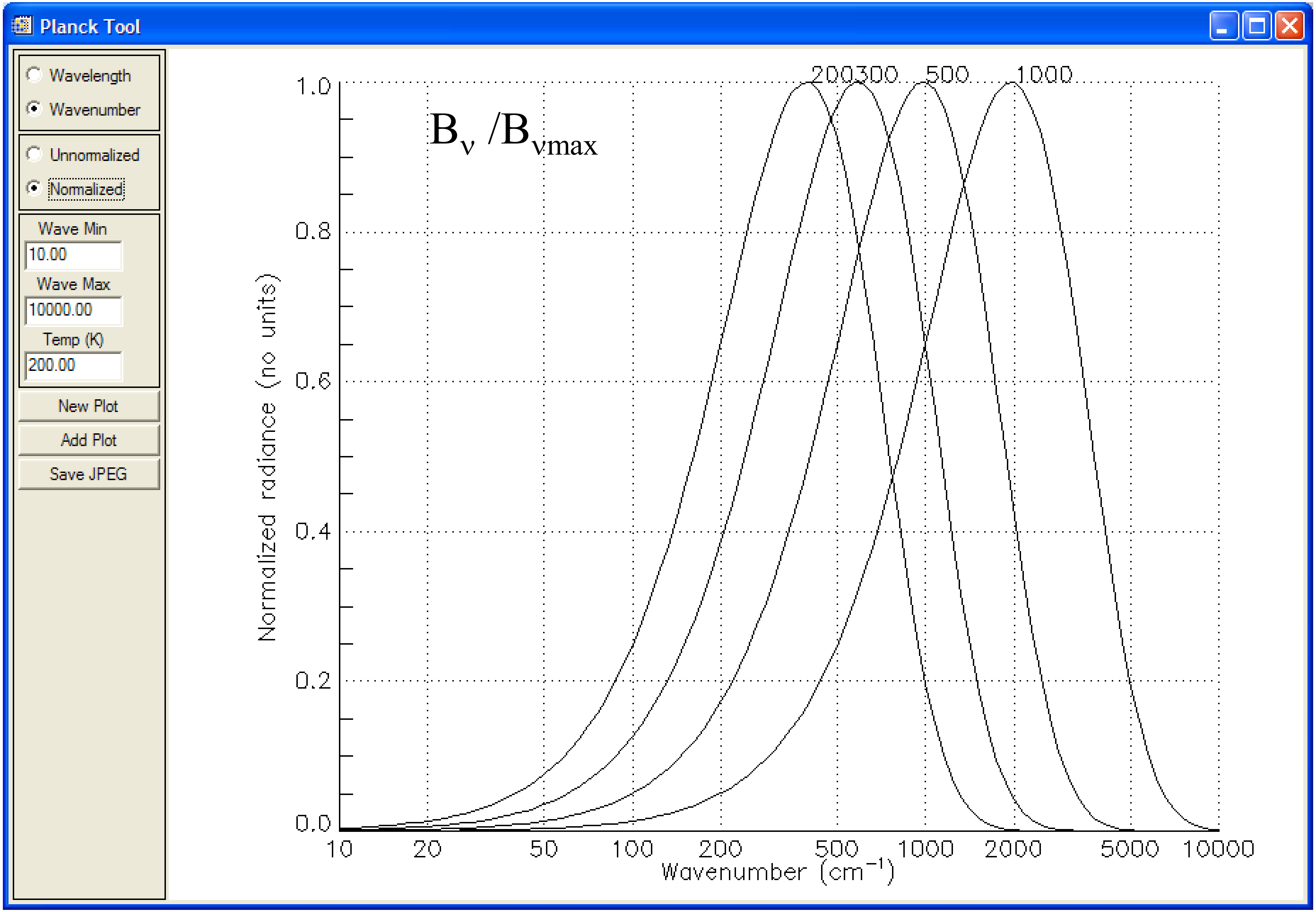
states that irradiance of a black body (area under Planck curve) is proportional to $T^4$ .

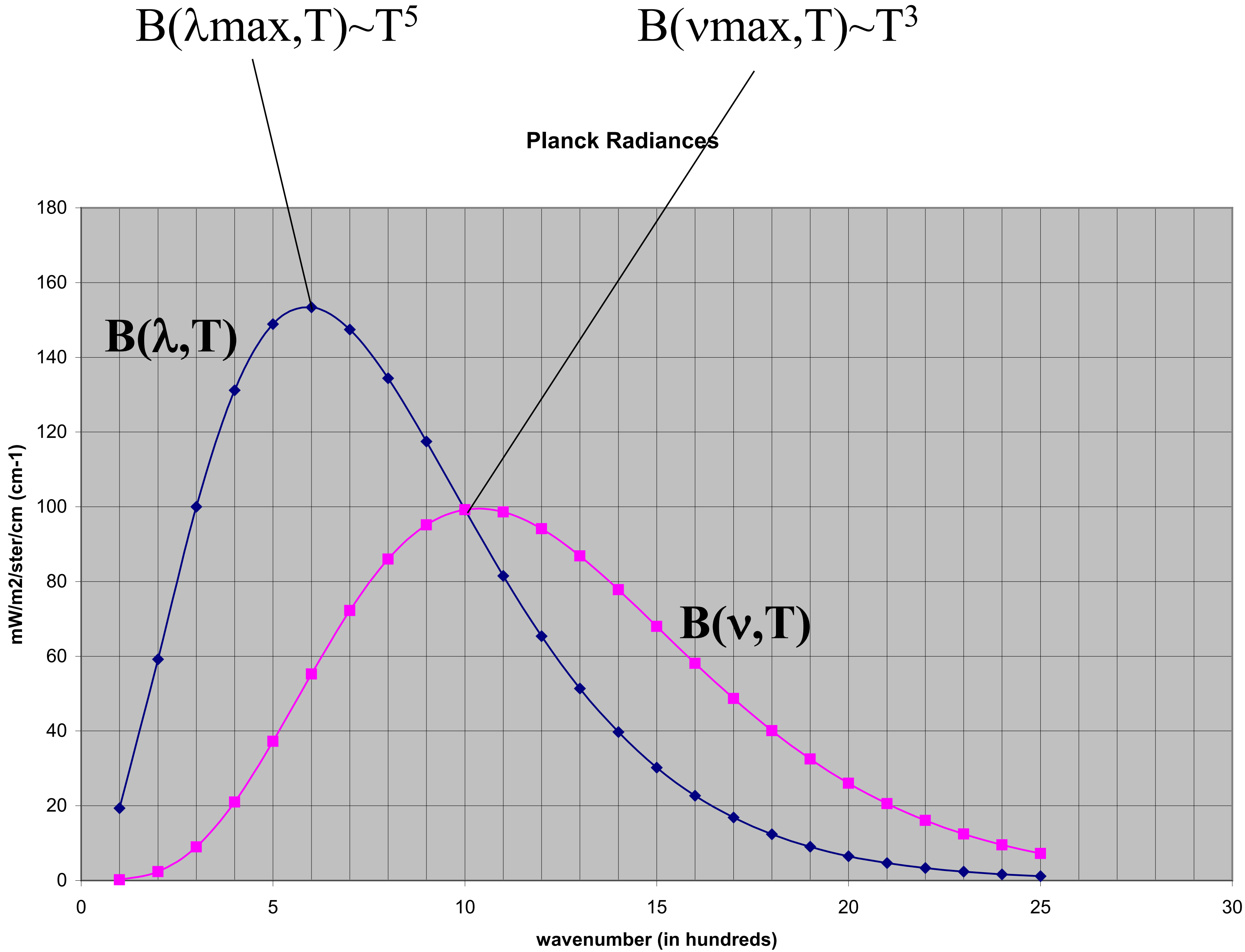
**Brightness Temperature**

$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$ is determined by inverting Planck function



Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
10.00
Wave Max
10000.00
Temp (K)
200.00
New Plot
Add Plot
Save JPEG
1.0e+004
1.0e+003
1.0e+002
1.0e+001
1.0e+000
1.0e-001
Radiance (mW/m2/sr/cm-1)
1000
500
300
200
10
20
50
100
200
500
1000
2000
5000
10000
Wavenumber (cm-1)


Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
10.00
Wave Max
10000.00
Temp (K)
200.00
New Plot
Add Plot
Save JPEG
$B_\nu / B_{\nu max}$
200
300
500
1000
1.0
0.8
0.6
0.4
0.2
0.0
Normalized radiance (no units)
10
20
50
100
200
500
1000
2000
5000
10000
Wavenumber ($cm^{-1}$)




# B(λ,T) versus B(ν,T)

**Using wavenumbers**

$$B(\nu,T) = c_1\nu^3 / [e^{c_2\nu/T} - 1]$$

$(mW/m^2/ster/cm^{-1})$

$\nu$(max in cm-1) = 1.95T

$B(\nu_{max},T) \sim T**3.$

$$E = \pi \int_o^{\infty} B(\nu,T)\, d\nu = \sigma T^4,$$

$$T = c_2\nu/[\ln(\frac{c_1\nu^3}{B_\nu} + 1)]$$

**Using wavelengths**

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

$(mW/m^2/ster/\mu m)$

$\lambda$(max in cm)T = 0.2897

$B(\lambda_{max},T) \sim T**5.$

$$E = \pi \int_o^{\infty} B(\lambda,T)\, d\lambda = \sigma T^4,$$

$$T = c_2/[\lambda \ln(\frac{c_1}{\lambda^5 B_\lambda} + 1)]$$

**Temperature sensitivity**, or the percentage change in radiance corresponding to a percentage change in temperature, $\alpha$, is defined as

$$dB/B = \alpha\, dT/T.$$

The temperature sensivity indicates the power to which the Planck radiance depends on temperature, since B proportional to $T^\alpha$ satisfies the equation. For infrared wavelengths,

$$\alpha = c_2\nu/T = c_2/\lambda T.$$

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| Wavenumber | Typical Scene Temperature | Temperature Sensitivity |
|---|---|---|
| 700 | 220 | 4.58 |
| 900 | 300 | 4.32 |
| 1200 | 300 | 5.76 |
| 1600 | 240 | 9.59 |
| 2300 | 220 | 15.04 |
| 2500 | 300 | 11.99 |

$dB/B = \alpha \, dT/T$ or $B = c\,T^{\alpha}$ where $\alpha = c2/\lambda T$ for a small temperature window around T.

$B = B(T_0) + (dB/dT)_0\,(\Delta T) + (d^2B/dT^2)_0(\Delta T)^2 + O(3)$

negligible

So to first order

$c\,(T_0 + \Delta T)^{\alpha} = c\,T_0^{\alpha} + c\,\alpha\,T_0^{\alpha-1}\,(\Delta T)$

$c\,(T_0 + \Delta T)^{\alpha} - c\,T_0^{\alpha} = c\,\alpha\,T_0^{\alpha-1}\,(\Delta T)$

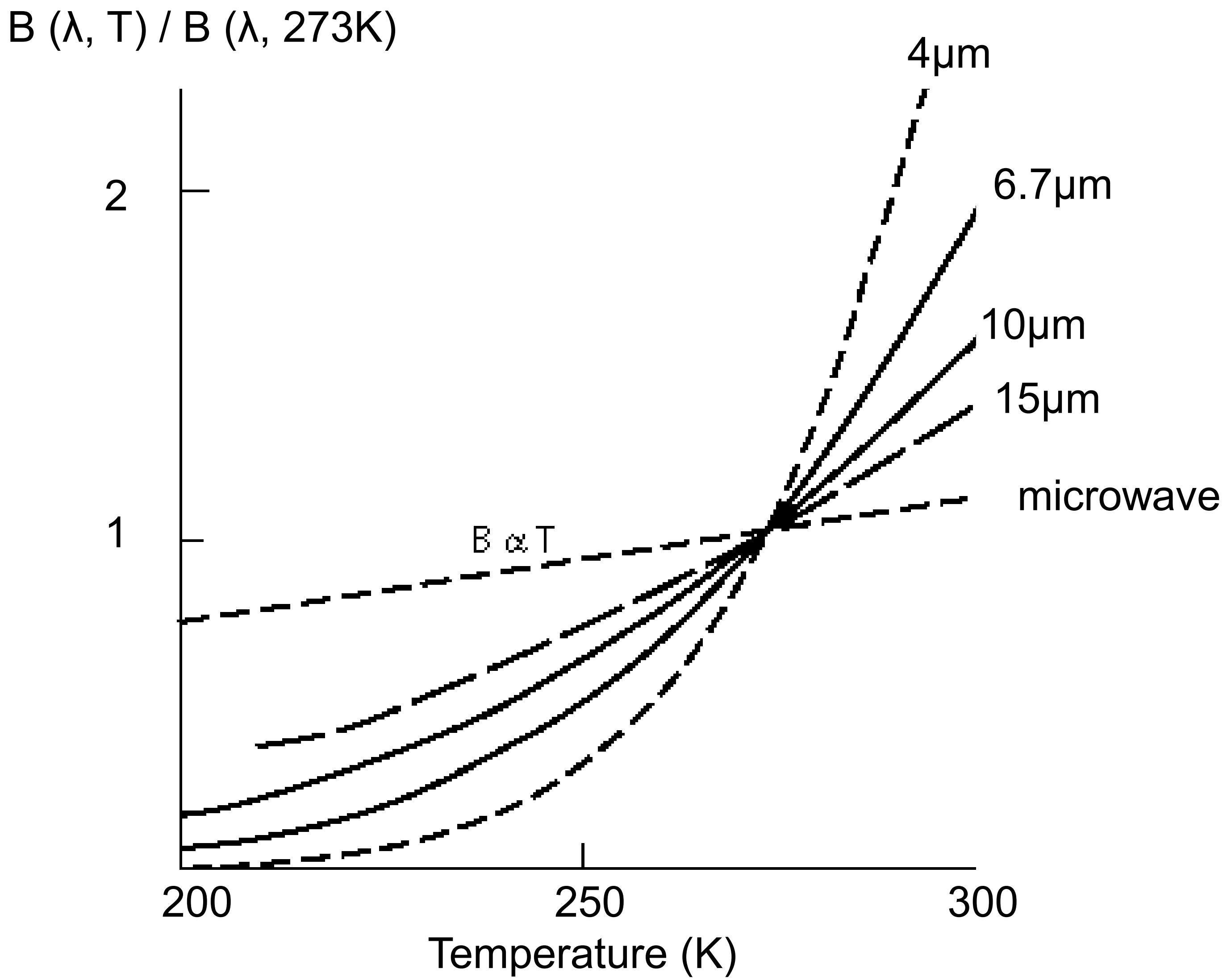
$\Delta B = c\,\alpha\,T_0^{\alpha-1}\,(\Delta T)$

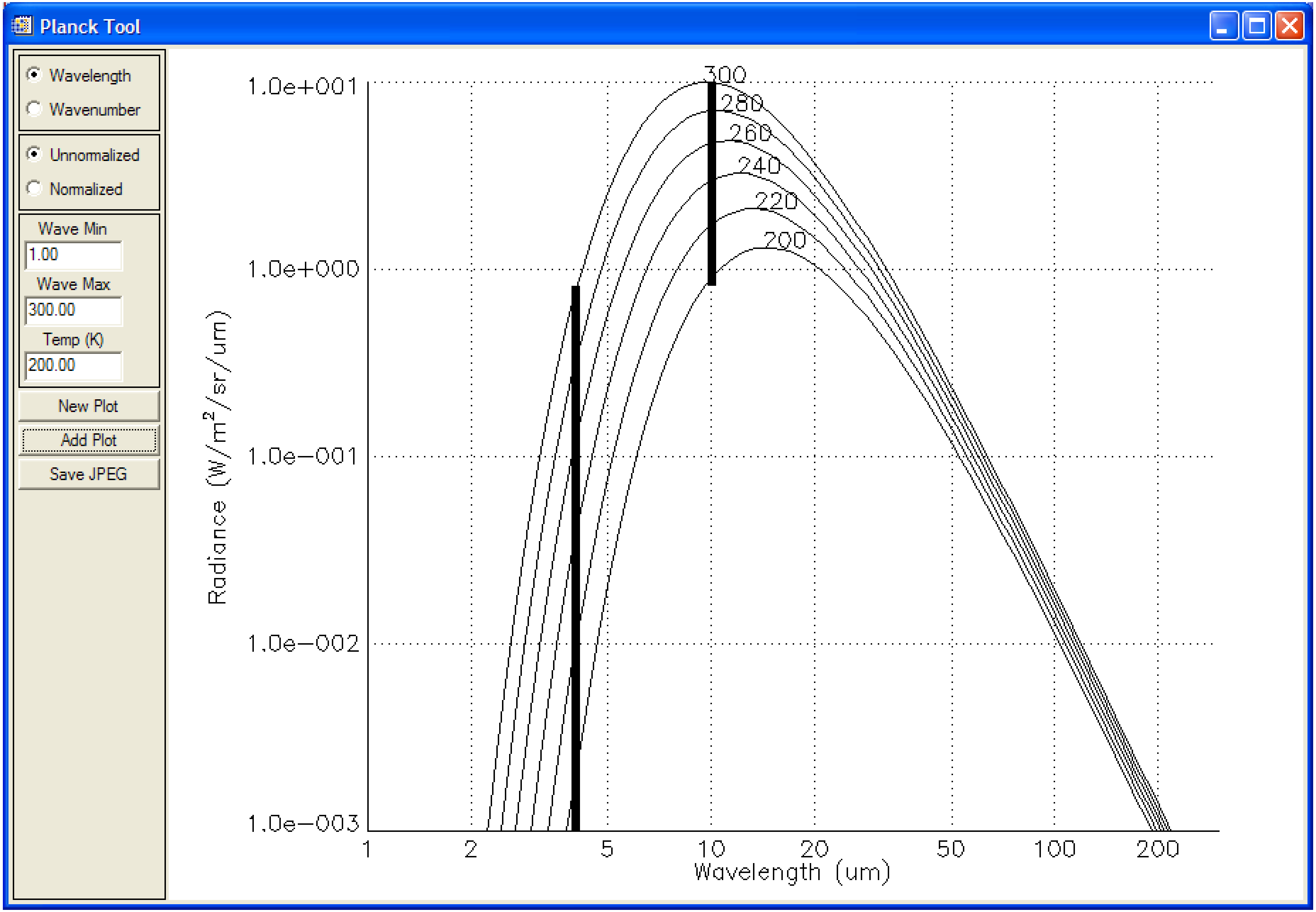
$\Delta B/B = \alpha\,\Delta T/T$

Also to first order

$(T_0 + \Delta T)^{\alpha} = T_0^{\alpha} + \alpha\,T_0^{\alpha-1}\,(\Delta T)$

# Temperature Sensitivity of B(λ,T) for typical earth scene temperatures




Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
1.00
Wave Max
300.00
Temp (K)
200.00
New Plot
Add Plot
Save JPEG
300
280
260
240
220
200
1.0e+001
1.0e+000
1.0e-001
1.0e-002
1.0e-003
Radiance (W/m2/sr/um)
1
2
5
10
20
50
100
200
Wavelength (um)

# $B(10\ um, T) / B(10\ um, 273) \propto T^4$

B(10 um,273)= 6.1

B(10 um,200)= 0.9 $\rightarrow$ 0.15

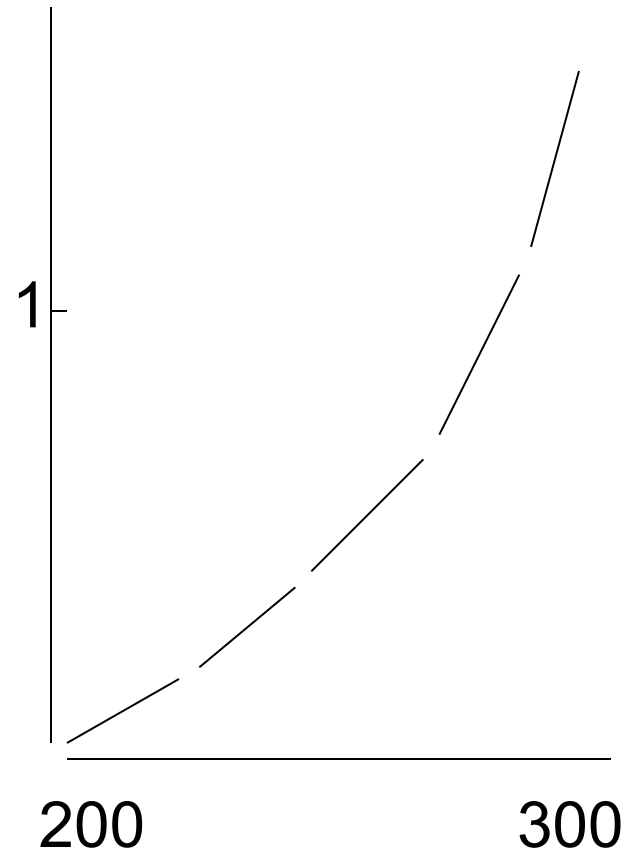
B(10 um,220)= 1.7 $\rightarrow$ 0.28

B(10 um,240)= 3.0 $\rightarrow$ 0.49

B(10 um,260)= 4.7 $\rightarrow$ 0.77

B(10 um,280)= 7.0 $\rightarrow$ 1.15

B(10 um,273)= 9.9 $\rightarrow$ 1.62



# $B(4\ um,T) / B(4\ um,273) \propto T^{12}$

$B(4\ um,273) = 2.2 \times 10^{-1}$

$B(4\ um,200) = 1.8 \times 10^{-3} \rightarrow 0.0$

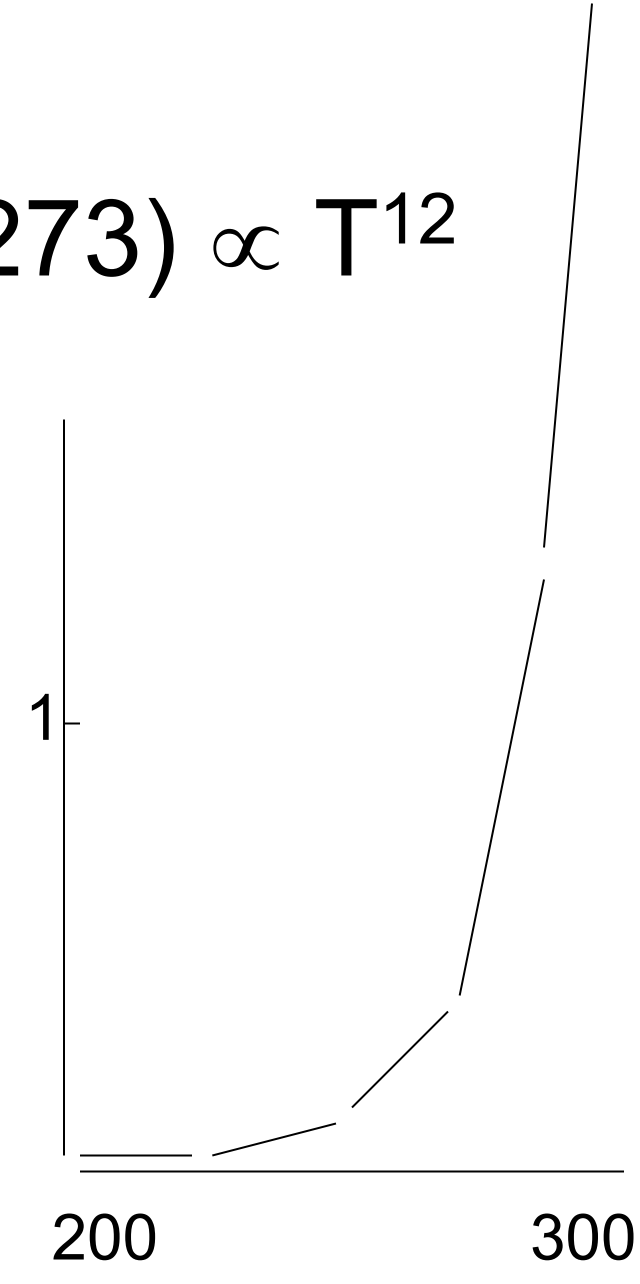
$B(4\ um,220) = 9.2 \times 10^{-3} \rightarrow 0.0$

$B(4\ um,240) = 3.6 \times 10^{-2} \rightarrow 0.2$

$B(4\ um,260) = 1.1 \times 10^{-1} \rightarrow 0.5$

$B(4\ um,280) = 3.0 \times 10^{-1} \rightarrow 1.4$

$B(4\ um,273) = 7.2 \times 10^{-1} \rightarrow 3.3$



# $B(0.3\ \text{cm}, T) / B(0.3\ \text{cm}, 273) \propto T$

$B(0.3\ \text{cm},273) = 2.55 \times 10^{-4}$

$B(0.3\ \text{cm},200) = 1.8 \rightarrow 0.7$

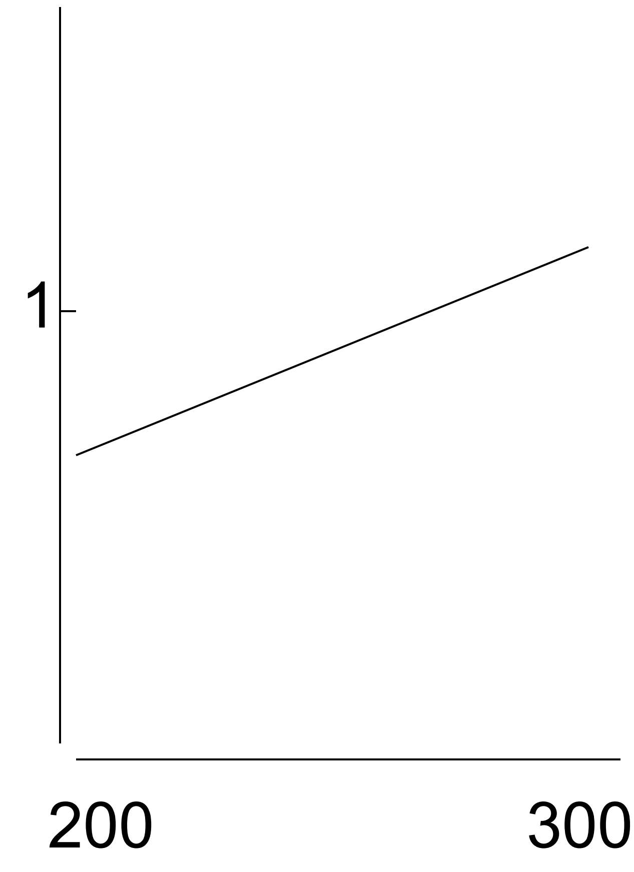
$B(0.3\ \text{cm},220) = 2.0 \rightarrow 0.78$

$B(0.3\ \text{cm},240) = 2.2 \rightarrow 0.86$

$B(0.3\ \text{cm},260) = 2.4 \rightarrow 0.94$

$B(0.3\ \text{cm},280) = 2.6 \rightarrow 1.02$

$B(0.3\ \text{cm},273) = 2.8 \rightarrow 1.1$



## Radiation is governed by Planck's Law

$$B(\lambda,T) = c_1 / \{ \lambda^5 [e^{c_2/\lambda T} - 1] \}$$

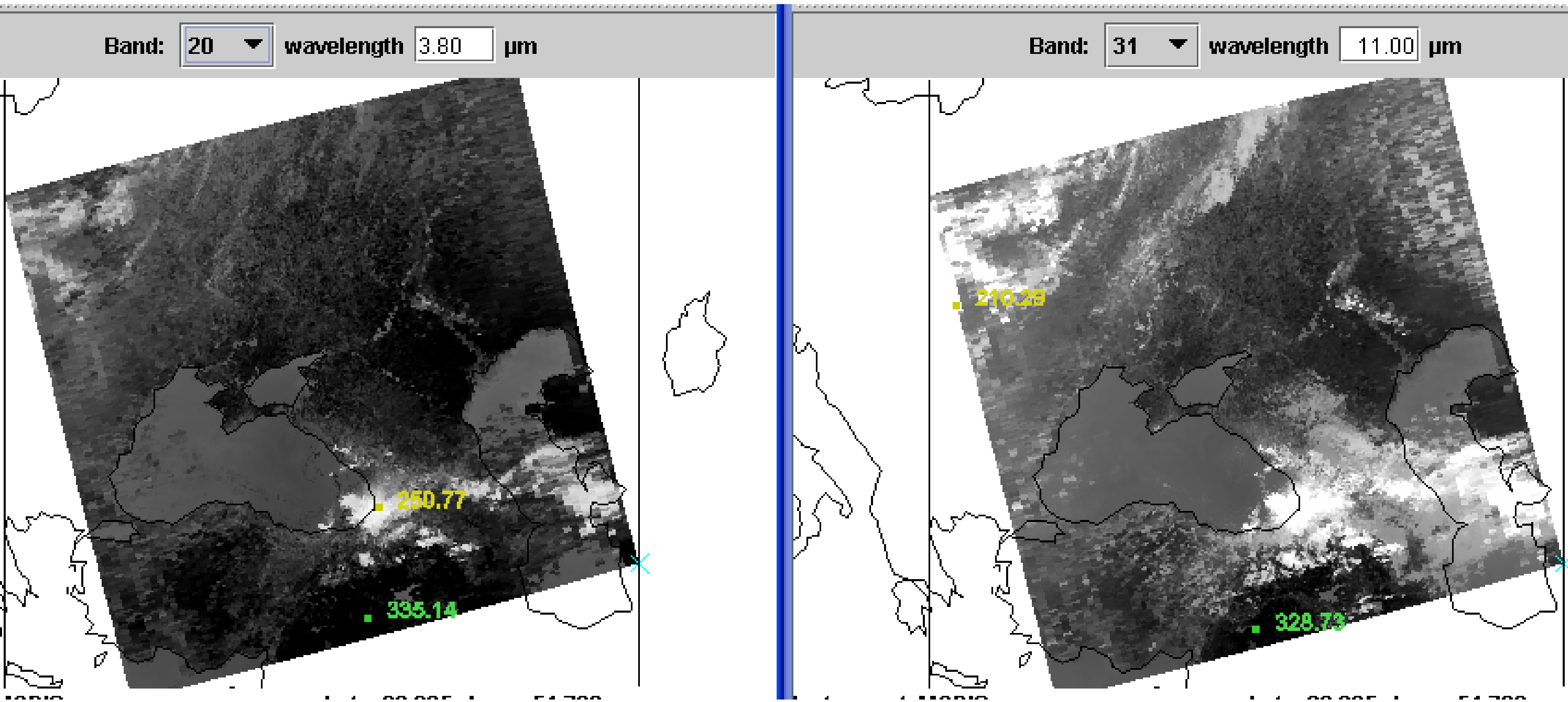
**In microwave region $c_2/\lambda T << 1$ so that**

$$e^{c_2/\lambda T} = 1 + c_2/\lambda T + \text{second order}$$

**And classical Rayleigh Jeans radiation equation emerges**

$$B_\lambda(T) \approx [c_1 / c_2] [T / \lambda^4]$$

**Radiance is linear function of brightness temperature.**




Cloud edges and broken clouds appear different in 11 and 4 um images.

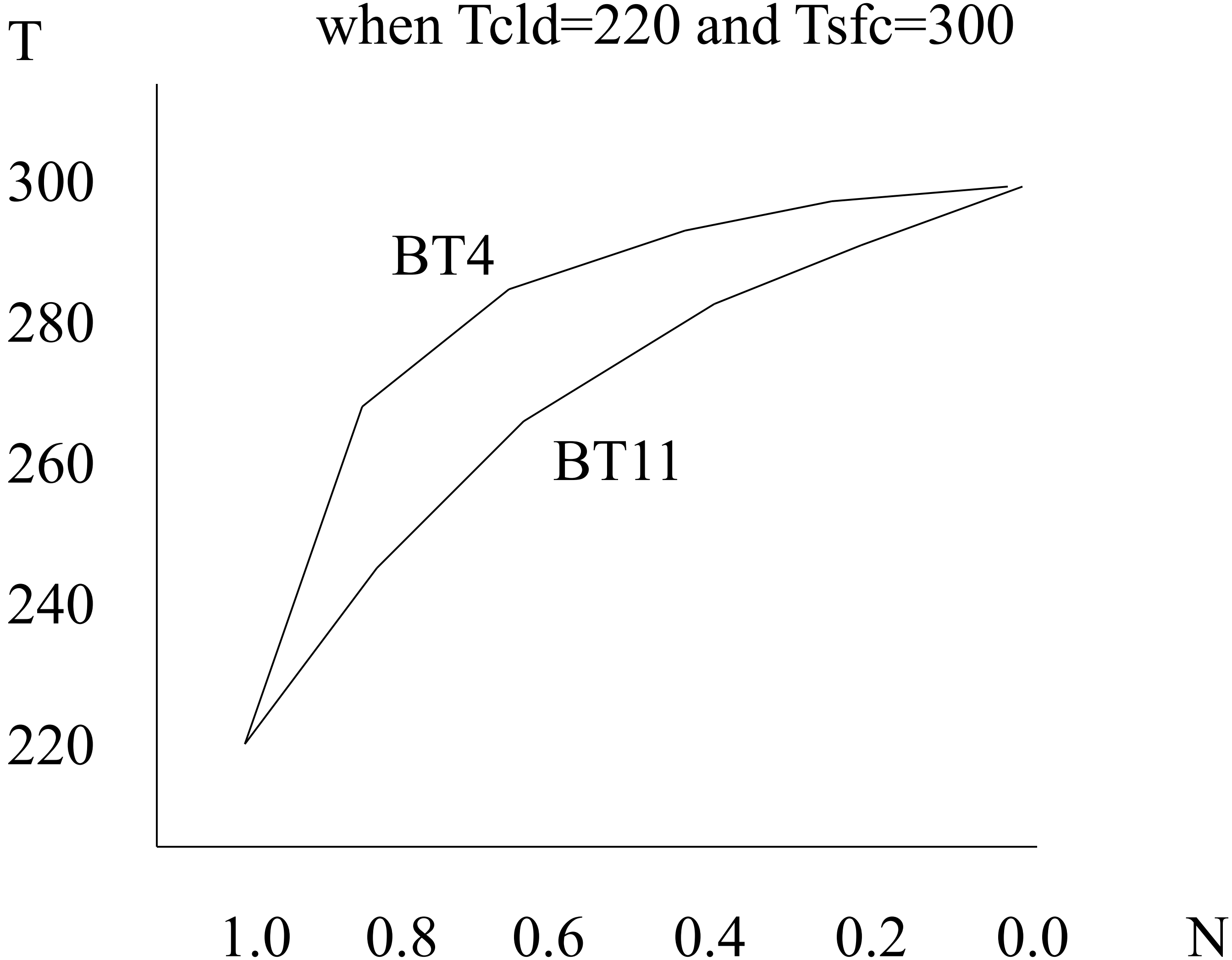
T(11)**4=(1-N)*Tclr**4+N*Tcld**4~(1-N)*300**4+N*200**4
T(4)**12=(1-N)*Tclr**12+N*Tcld**12~(1-N)*300**12+N*200**12

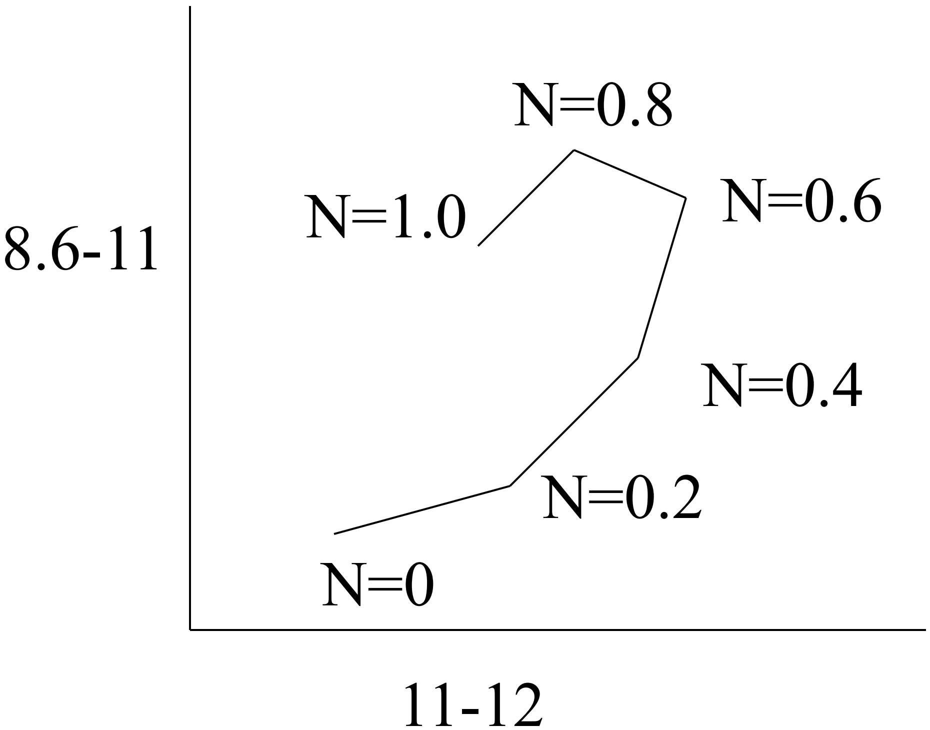
Cold part of pixel has more influence for B(11) than B(4)

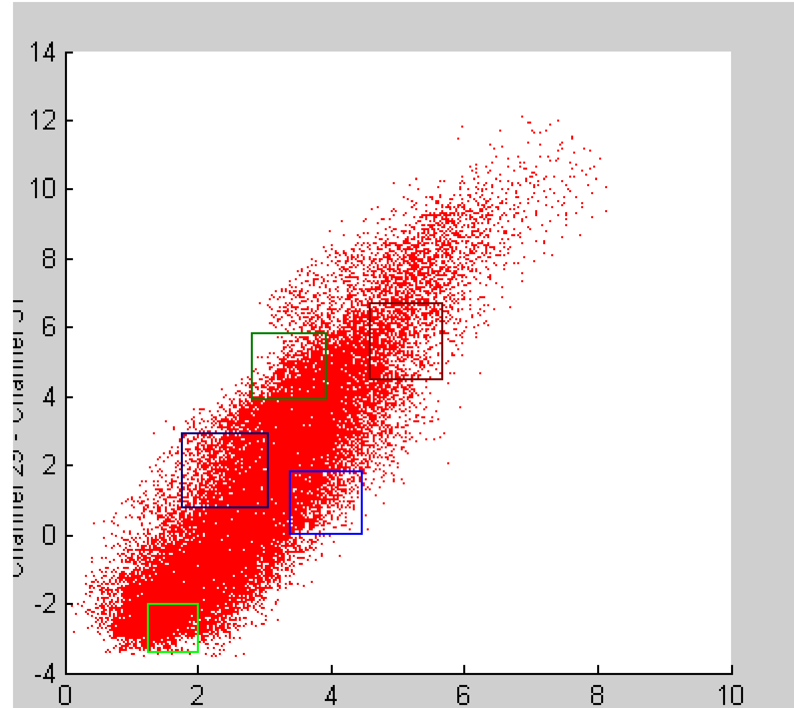
**Table 6.1** Longwave and Shortwave Window Planck Radiances (mW/m**2/ster/cm-1) and Brightness Temperatures (degrees K) as a function of Fractional Cloud Amount (for cloud of 220 K and surface of 300 K) using $B(T) = (1-N)*B(T_{sfc}) + N*B(T_{cld})$.

| Cloud Fraction N | Longwave Window Rad | Longwave Window Temp | Shortwave Window Rad | Shortwave Window Temp | $T_s - T_1$ |
|---|---|---|---|---|---|
| 1.0 | 23.5 | 220 | .005 | 220 | 0 |
| .8 | 42.0 | 244 | .114 | 267 | 23 |
| .6 | 60.5 | 261 | .223 | 280 | 19 |
| .4 | 79.0 | 276 | .332 | 289 | 13 |
| .2 | 97.5 | 289 | .441 | 295 | 6 |
| .0 | 116.0 | 300 | .550 | 300 | 0 |

SW and LW BTs for different cloud amounts when Tcld=220 and Tsfc=300





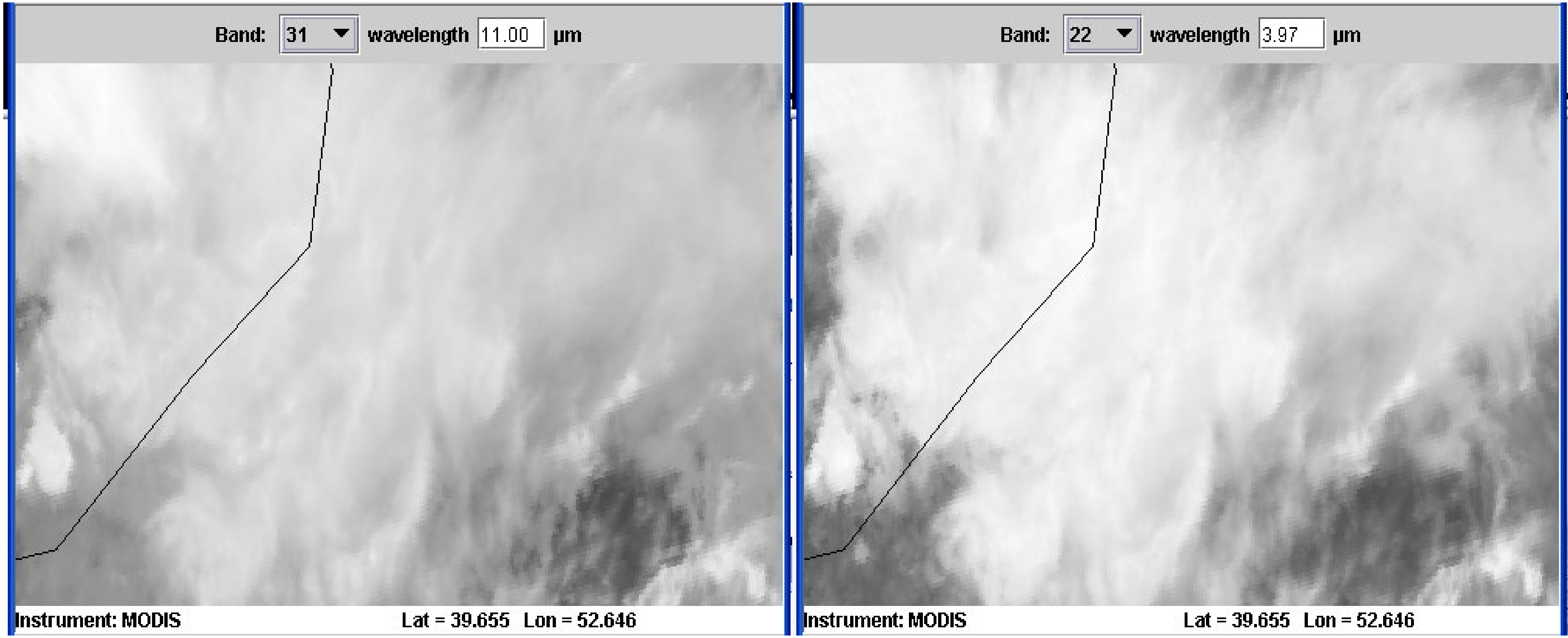




Broken clouds appear different in 8.6, 11 and 12 um images;
assume Tclr=300 and Tcld=230

$$T(11)-T(12)=[(1-N)*B11(Tclr)+N*B11(Tcld)]^{-1} - [(1-N)*B12(Tclr)+N*B12(Tcld)]^{-1}$$

$$T(8.6)-T(11)=[(1-N)*B8.6(Tclr)+N*B8.6(Tcld)]^{-1} - [(1-N)*B11(Tclr)+N*B11(Tcld)]^{-1}$$

Cold part of pixel has more influence at longer wavelengths

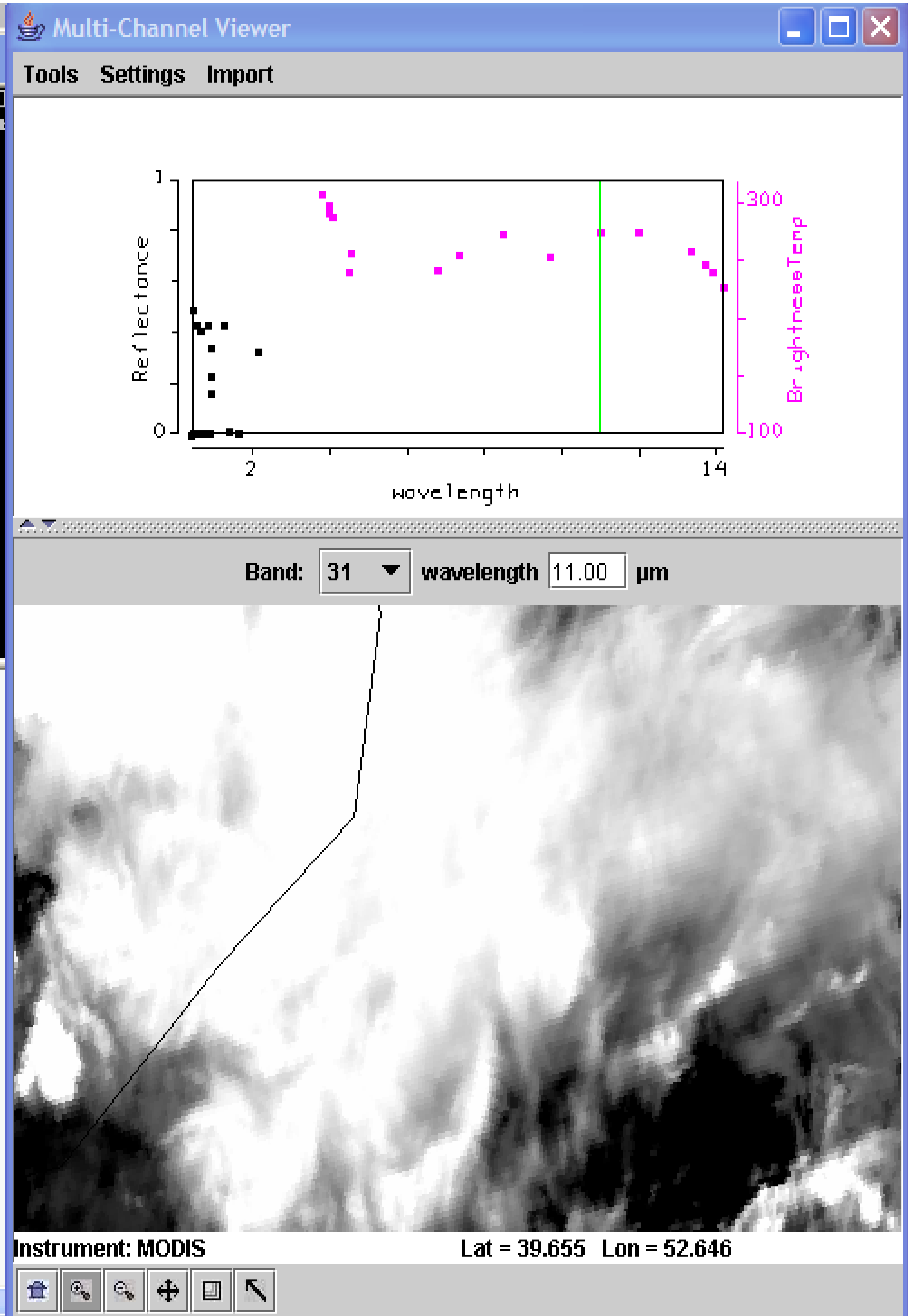



Cold clouds appear grainy in 4 um MODIS images.

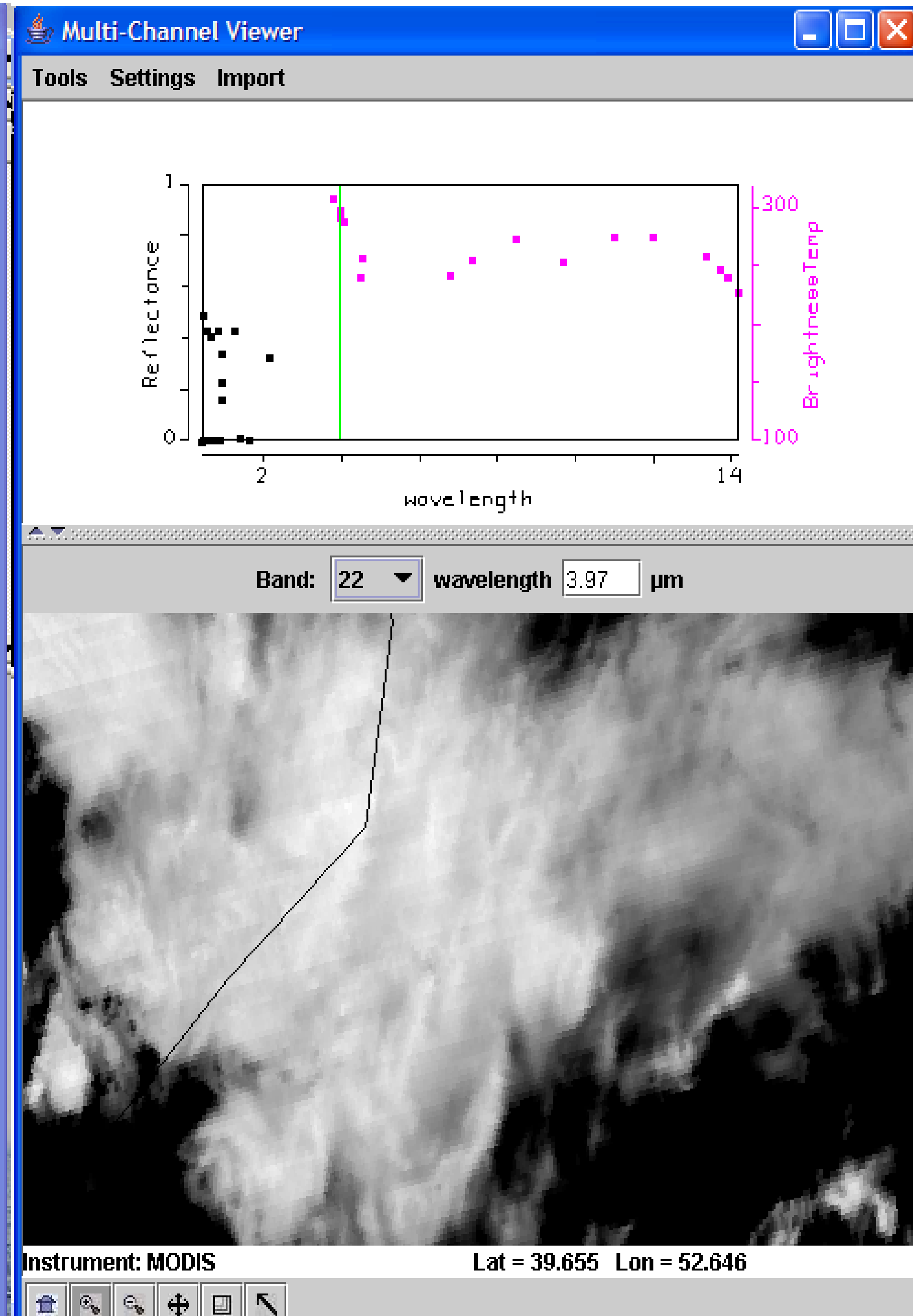
$\Delta R = Rmax / 2^{13}$ and $\Delta T = \Delta R / [dB/dT]$

dB/dT(4) is 100 times smaller at 200 K than at 300K;
Truncation error in cold scenes for 4 µm is several degrees K!

dB/dT(11) is only 4 times smaller (hence it is not noticeable).

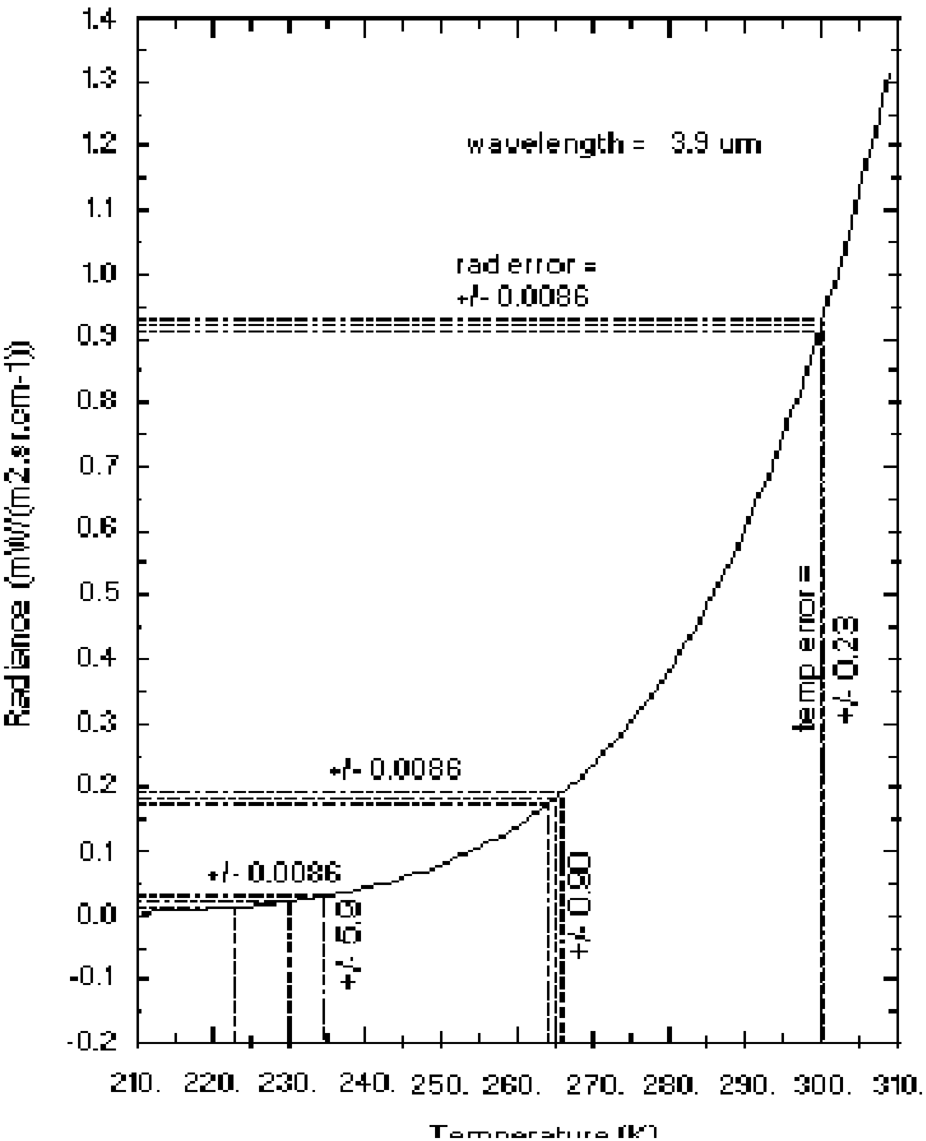


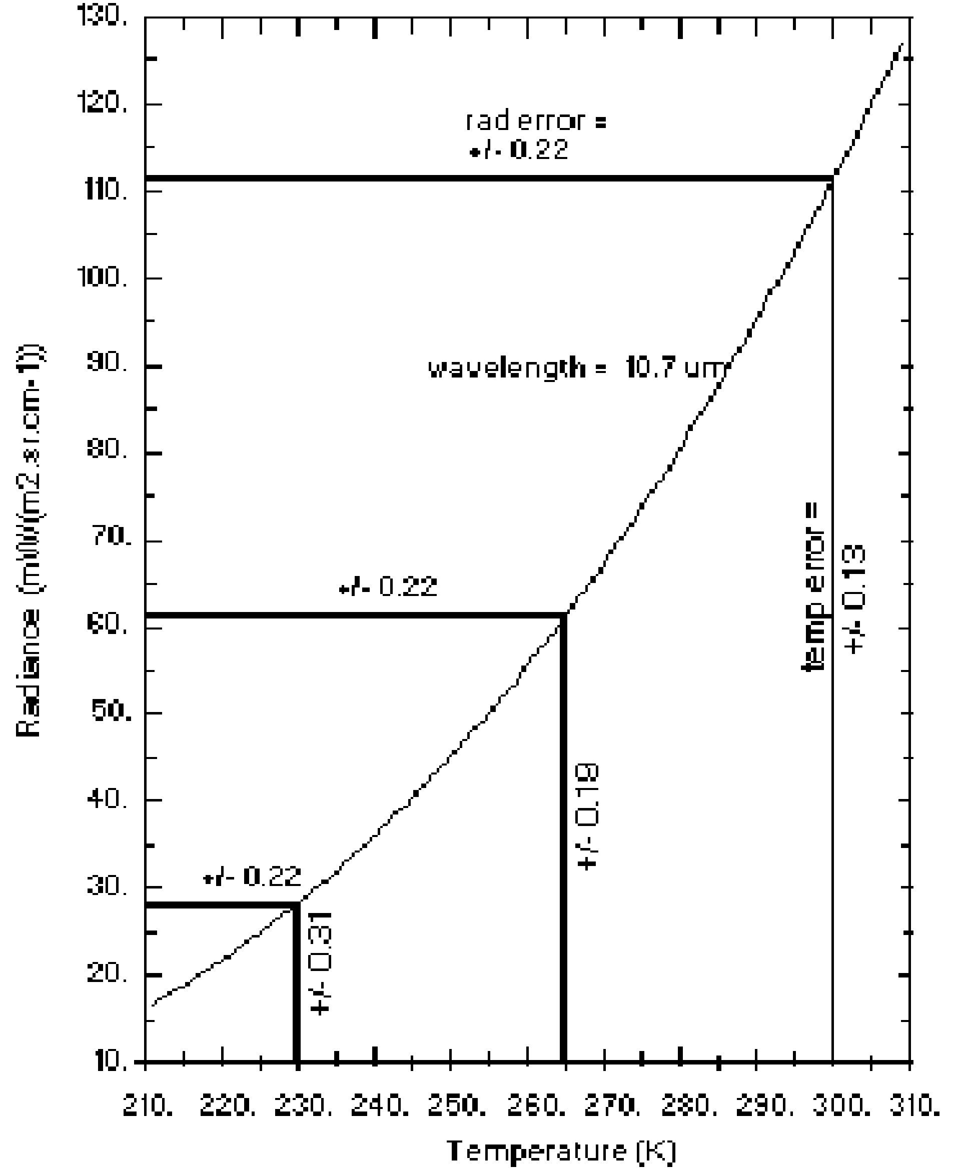
Multi-Channel Viewer
Tools
Settings
Import
Reflectance
BrightnessTemp
wavelength
Band: 31
wavelength 11.00 µm
Instrument: MODIS
Lat = 39.655 Lon = 52.646

Multi-Channel Viewer
Tools
Settings
Import
Reflectance
BrightnessTemp
wavelength
Band: 22
wavelength 3.97 µm
Instrument: MODIS
Lat = 39.655 Lon = 52.646

# NEDR vs NEDT at 4 and 11 μm



# B and dB/dT at 4 and 11 µm

w,temp,r,drdt
rad = mW/ster/m2/cm-1

| | | | |
|---|---|---|---|
| 4 | 200 | 2.87612E-03 | 2.586404E-04 |
| 4 | 210 | 6.77262E-03 | 5.524178E-04 |
| 4 | 220 | 1.475345E-02 | 1.096472E-03 |
| 4 | 230 | 3.003489E-02 | 2.042302E-03 |
| 4 | 240 | 5.762783E-02 | 3.598814E-03 |
| 4 | 250 | .1049535 | 6.040413E-03 |
| 4 | 260 | .1825299 | 9.712637E-03 |
| 4 | 270 | .3046972 | 1.503457E-02 |
| 4 | 280 | .4903518 | 2.249792E-02 |
| 4 | 290 | .7636536 | 3.266267E-02 |
| 4 | 300 | 1.154674 | 4.614974E-02 |
| 4 | 310 | 1.699957 | 6.363091E-02 |
| 4 | 320 | 2.442974 | 8.581714E-02 |
| 4 | 330 | 3.434435 | .1134449 |
| 4 | 340 | 4.732509 | .1472632 |
| 4 | 350 | 6.402821 | .1880181 |
| 4 | 360 | 8.518434 | .2364417 |
| 4 | 370 | 11.15956 | .2932373 |
| 4 | 380 | 14.41332 | .3590706 |
| 4 | 390 | 18.37315 | .4345571 |
| 4 | 400 | 23.13851 | .5202583 |

w,temp,r,drdt
rad = mW/ster/m2/cm-1

| | | | |
|---|---|---|---|
| 11 | 200 | 12.94316 | .4238625 |
| 11 | 210 | 17.68172 | .5254846 |
| 11 | 220 | 23.48358 | .6363174 |
| 11 | 230 | 30.43456 | .755097 |
| 11 | 240 | 38.60766 | .8805164 |
| 11 | 250 | 48.06268 | 1.011279 |
| 11 | 260 | 58.84691 | 1.146147 |
| 11 | 270 | 70.99543 | 1.28396 |
| 11 | 280 | 84.53237 | 1.423662 |
| 11 | 290 | 99.4718 | 1.564306 |
| 11 | 300 | 115.8188 | 1.705055 |
| 11 | 310 | 133.5708 | 1.845184 |
| 11 | 320 | 152.7184 | 1.984071 |
| 11 | 330 | 173.2464 | 2.121195 |
| 11 | 340 | 195.1349 | 2.256122 |
| 11 | 350 | 218.3603 | 2.388497 |
| 11 | 360 | 242.8955 | 2.518038 |
| 11 | 370 | 268.7109 | 2.644525 |
| 11 | 380 | 295.7752 | 2.767794 |
| 11 | 390 | 324.0556 | 2.887726 |
| 11 | 400 | 353.5183 | 3.004245 |

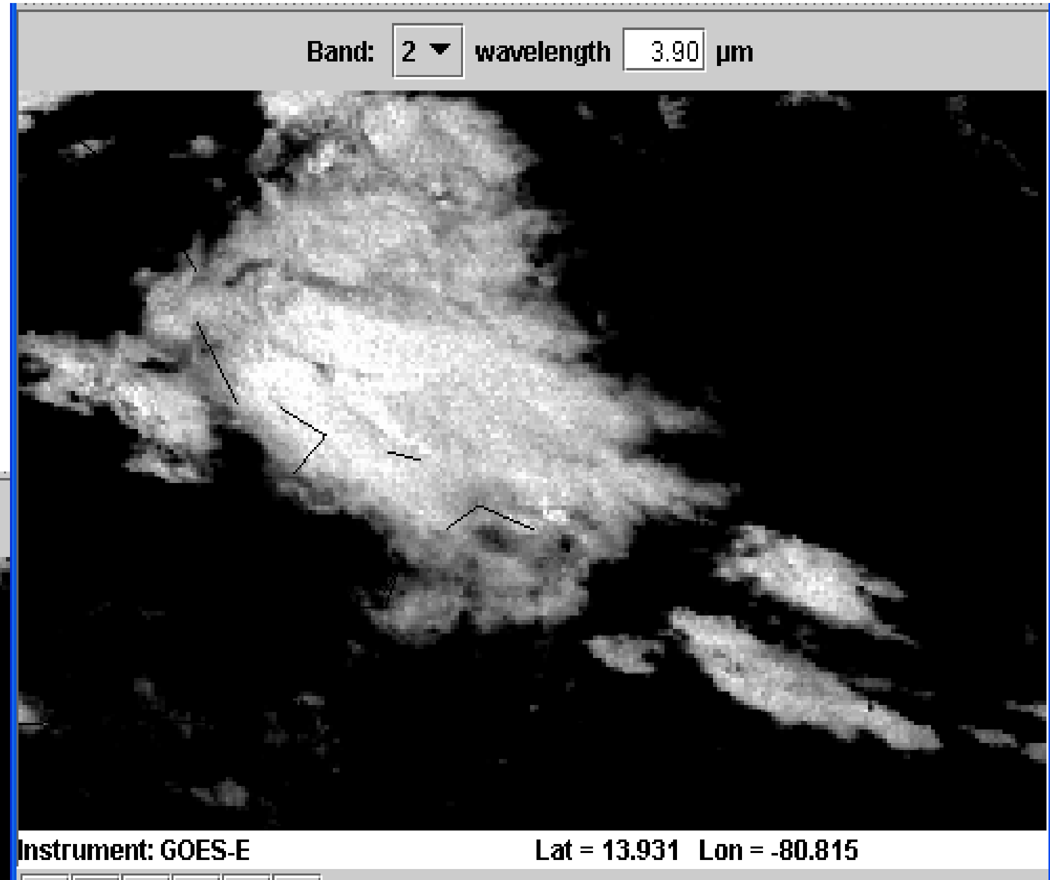
For GOES

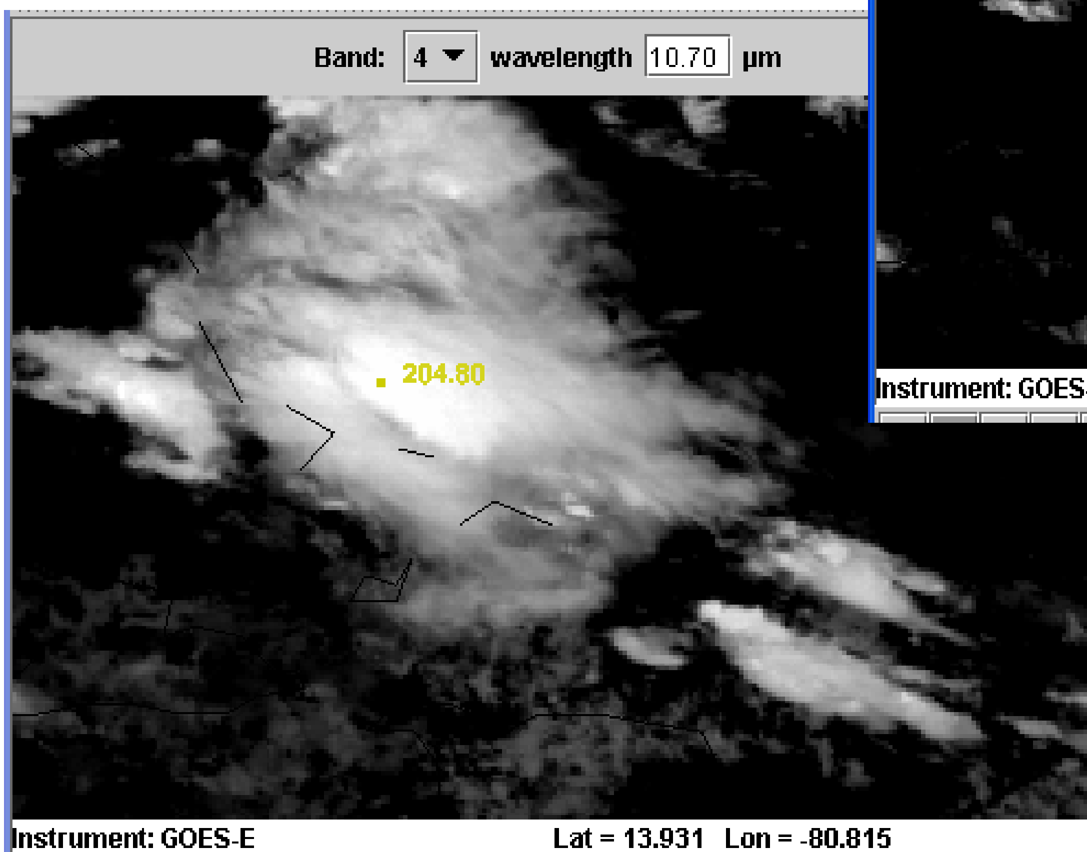
with 10 bit data it is even worse

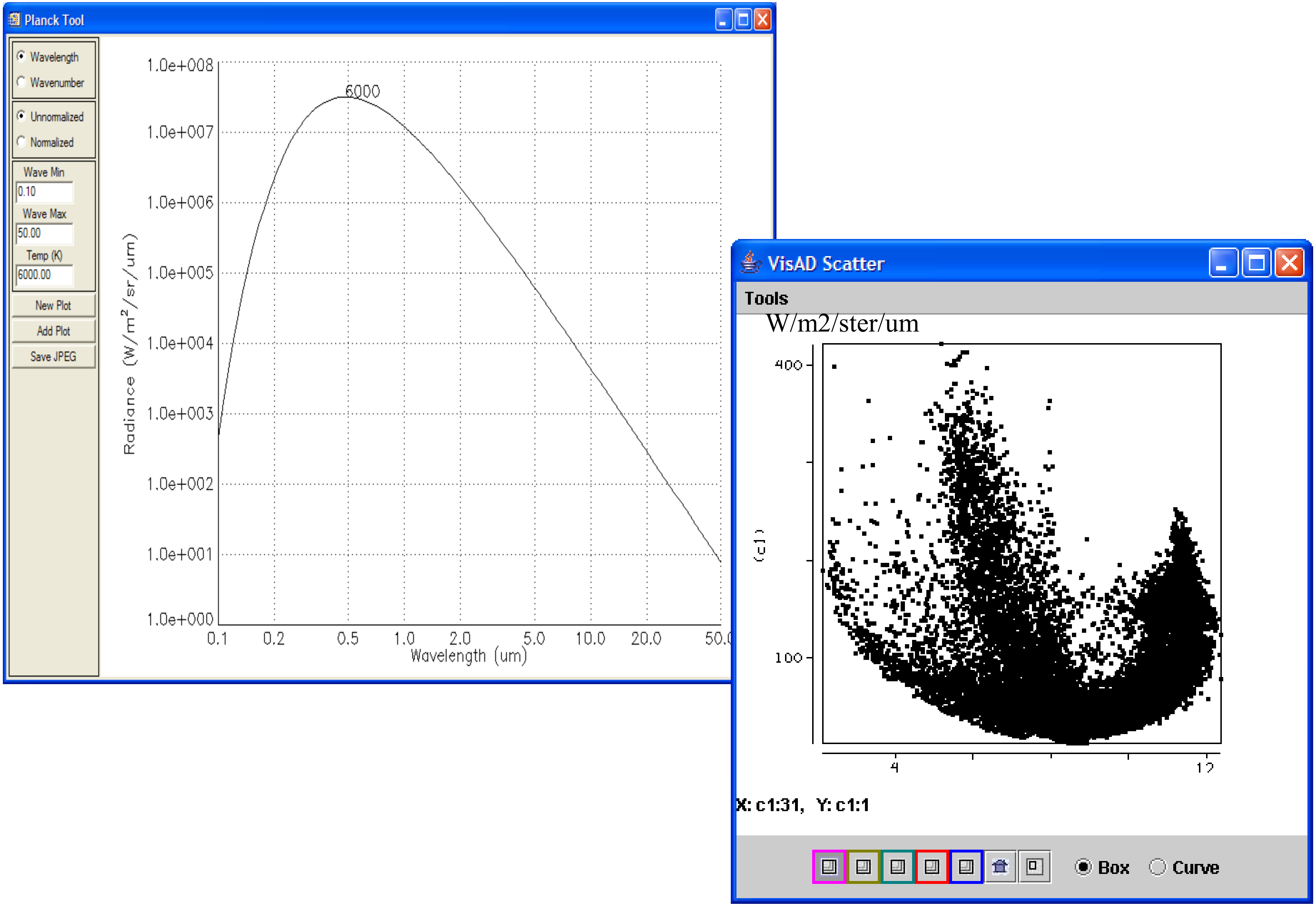
$$\Delta R = Rmax / 2^{10}$$

and

$$\Delta T = \Delta R / [dB/dT]$$





Planck Tool
Wavelength
Wavenumber
Unnormalized
Normalized
Wave Min
0.10
Wave Max
50.00
Temp (K)
6000.00
New Plot
Add Plot
Save JPEG
1.0e+008
1.0e+007
1.0e+006
1.0e+005
1.0e+004
1.0e+003
1.0e+002
1.0e+001
1.0e+000
6000
Radiance (W/m2/sr/um)
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0
50.0
Wavelength (um)
VisAD Scatter
Tools
W/m2/ster/um
400
100
(c1)
4
12
X: c1:31, Y: c1:1
Box
Curve