Compliments of the Author.

MICROMETRIC MEASURES OF THE DIAMETER OF MARS GIVING BESIDE THE SIZE AND THE ELLIPTICITY OF THE PLANET APPARENT EVIDENCE OF A TWILIGHT ARC.

PERCIVAL LOWELL.

Reprint from POPULAR ASTRONOMY.

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During the summer and autumn, 1894, 464 micrometric measures of the diameter of Mars were made at this observatory.

NUMBER OF MEASURES OF THE DIAMETERS OF MARS.

Equatorial Diameters	256
Polar Diameters	208
	464

Of these	403	were made by A. E. Douglass.
	50	were made by Professor W. H. Pickering.
	11	were made by Percival Lowell.
	464	

The 403 taken by Mr. Douglass, in addition to their general value, these prove to be of peculiar interest. For on reducing them I find that beside furnishing, from their great number, relatively accurate values of the equatorial and polar diameters and of the polar flattening, they yield a by product as unexpected as it is important. Their discussion reveals what appears to be unmistakeable evidence of a twilight upon the planet, sufficiently pronounced to be visible from the Earth and actually to have been measured, unconsciously. That Mars possessed an atmosphere we had what amounted to proof positive before; but that the fact should again be brought to light in this literal manner, as a silver lining to a cloud of figures is a point of some curiosity. For the measures had no such end in view; indeed, to detect the presence of an atmosphere by measures of the diameters had not suggested itself to any one. Yet, as will be seen, the quantities upon which the evidence rests are so large as to be quite without the pale of accidental error, being ten times as large as the probable errors of observation, and twice as large as the differences that disclose the polar flattening. That up to this time they should have escaped detection is due to their having been masked by another factor affecting the size of the polar diameter, the varying irradiation from the polar cap. To the hitherto unsuspected presence of the first of these factors and to the difficulty of duly evaluating the second are attributable in all probability the discrepancies in the values previously found for the ellipticity.

The first of Mr. Douglass' measures were made on the 6th of July and the last on the 21st of November, 1894. From the 12th of October the planet was measured by him nearly every other night; and in the following manner: the longitudinal thread of the micrometer having been set to the planet's phase axis or polar axis as the case might be, the field of the micrometer was so adjusted that the stationary transverse thread lay close to, but not on, the limb of the planet's disk. The movable thread was then carried some way beyond the other limb and steadily brought back till it lay to the same side of, and at the same distance from the other limb, that the stationary thread lay from the first one. It was then read. During the process of bringing it up its motion was never reversed. In case of overrunning it was carried some distance back and again continuously brought up. Thus any possible backlash was prevented from affecting the measures.

The whole field was next shifted till the stationary thread occupied the same general position toward the second limb that it had done toward the first, and the moveable thread was brought past it till it reached a corresponding distance from the other limb, upon which it was read. The whole field was then shifted to the first position and the process repeated. Each reading thus furnished half the data to two measures.

In the measures made during September, October, and November, Mr. Douglass took the further precaution to place himself so that the line joining his eyes was kept either perpendicular or parallel to the diameter measured; a record of his position being entered opposite the measures. Such record appears in column

4 of the table: E. S. standing for "eyes parallel to single thread," that is, parallel to the longitudinal thread of the micrometer; and E. D., "eyes parallel to double (i. e. transverse) threads." E. S. therefore denotes a horizontal measurement; E. D. a vertical one. As measures were taken in both positions for each diameter at various times, we have here a comparison of some value as will appear further on.

To eliminate systematic errors as much as possible a circular disk was constructed, carefully measured, and carried to the top of Mt. Agassiz, one of the San Francisco peaks, where it was placed just under the summit, facing the Observatory. From triangulation the angles by which its position deviated from the vertical in the first place and from parallelism to the dome in the second were found to be respectively: 6° and 2° west; its distance from the dome in an air-line being 45,330 ft., whence its apparent diameters were:

vertical diameter	:	21″.08
horizontal diameter	:	21 .22

The size of the artificial disk was thus such as to subtend at that distance from the Observatory an angle very nearly equal to the planet's disk. The diameters were then observed through the telescope and micrometrically measured by Mr. Douglass.

Of the measures of the planet, the first to be reduced were those taken from Oct. 12 to Nov. 22 inclusive. They are as follows:

TABLE I.

POLAR DIAMETERS.

Measured by Mr. A. E. Douglass.

Time.		h	m	No. of Meas.	Eyepiece		Wt.	Uncor. Meas. ″	Cor. for Refrac. ″	Cor. for Irrad. on Limb. ″	Cor. for Irrad. on Term. ″	Cor. for phase. ″	Reduced to Distance Unity. ″
Oct.	12	13	45	5	862		5	21.95	21.95	21.85	21.79	21.81	9.40
	15	13	42	6	"	E. R. A.	4	21.84	21.84	21.74	21.67	21.68	9.36
	17	15		4	"	E. S.	4	21.77	21.77	21.67	21.59	21.60	9.38
	19	15	5	5	"	E. S.	2	21.92	21.92	21.82	21.73	21.73	9.46
	20	13	57	5	"	E. S.	5	21.69	21.69	21.59	21.50	21.50	6.41
	21	13	42	5	"	E. D.	5	21.69	21.70	21.60	21.51	21.51	9.42
	21	14		5	"	E. S.	4	21.61	21.61	21.51	21.42	21.42	9.39
	23	13	40	5	"	E. S.	4	21.28	21.28	21.18	21.08	21.08	9.31
	24	12	15	5	"	E. S	3	21.17	21.18	21.08	20.98	21.98	9.30
	24	12	33	6	"	E. D.	3	21.08	21.09	20.99	20.89	20.89	9.27
	29	13	16	5	"	E. S.	3	20.94	20.94	20.84	20.75	20.75	9.46
	30	14	50	7	"	E. S.	3	20.78	20.78	20.68	20.59	20.59	9.46
Nov.	2	13	34	5	"	E S.	6	20.15	20.15	20.05	19.97	19 97	9.36
	4	12	40	5	"	E. S.	4	19.85	19.85	19.75	19.67	19.68	9.37
	5	13	10	5	"	E. S.	5	19.61	19.61	19.51	19.43	19.44	9.34
	5	14	38	5	"	E. S.	3	19.54	19.54	19.44	19.36	19.37	9.30
	6	12	45	5	"	E. S.	7	19.67	19.67	19.57	19.49	19.50	9.44
	9	13	35	5	"	E. S.	7	19.23	19.23	19.13	19.05	19.07	9.49
	14	14	35	5	"	E. S.	6	18.02	18.02	17.92	17.85	17.88	9.34
	15	13	53	5	"	E. S.	5	18.03	18.03	17.92	17.85	17.88	9.44
	19	12	45	5	"	E. S.	3	17.19	17.19	17.09	17.02	17.06	9.38
	20	11	50	5	"	E. S.	2	17.15	17.15	17.05	16.99	17.03	9.46
	21	11	45	5	"	E. S.	7	16.86	16.86	16.76	16.70	16.74	9.40

For cor. irr. 0″.10 with power 862 and irradiation varying as the $\sqrt[6]{}$ of the illumination.

Measures made on true diameter.

Table II.

Equatorial Diameters.

Time.	h m	No. of Meas.	Eyepiece		Wt.	Uncor. Meas. ″	Cor. for Refrac. ″	Cor. for Irrad. on Limb, ″	Cor. for Irrad. on Term. ″	Cor. for Phase. ″	Reduced to Distance Unity. ″
Oct. 12	13 15	5	862		5	22.15	22.15	22.05	21.99	22.06	9.51
15	14 2	5	"	E. S.	3	21.96	21.96	21.86	21.78	21.80	9.42
17	14 42	5	"	E. S.	4	21.98	21.99	21.89	21.79	21.79	9.44
19	14 48	5	"	E. S.	3	22.00	22.01	21.91	21.81	21.81	9.50
20	14 11	5	"	E. S.	?6	21.80	21.81	21.71	21.61	21.61	9.44
21	14 25	6	"	E. D.	4	21.74	21.75	21.65	21.56	21.56	9.45
21	14 45	5	"	E. S	5	21.57	21.58	21.48	21.39	21.39	9.37
23	14 15	7	"	E. S.	3	21.45	21.46	21.36	21.28	21.30	9.40
24	11 50	6	"	E. S.	3	21.38	21.38	21.28	21.21	21.24	9.42
24	12 50	5	"	E. D.	5	21.32	21.33	21.23	21.16	21.19	9.40
29	13 3	5	"	E. D.	3	20.91	20.92	20.82	20.77	20.88	9.52
30	15	5	"	E. D.	2	20.54	20.55	20.45	20.40	20.54	9.43
Nov. 2	14 12	5	"	E D.	6	20.03	20.04	19.94	19.89	20.10	9.42
4	13 10	5	"	E. D.	4	19.87	19.88	19.78	19.74	20.01	9.53
5	13 36	5	"	E. D.	4	19.49	19.50	19.40	19.36	19.66	9.44
5	14 15	5	"	E. D.	3	19.46	19.48	19.38	19.34	19.64	9.43
6	13 20	6	"	E. D.	8	19.59	19.60	19.50	19.46	19.79	9.58
9	13 53	5	"	E. D.	7	19.04	19.06	18.96	18.92	19.34	9.62
14	14 18	7	"	E. D	6	17.90	17.93	17.83	17.80	18.37	9.59
15	13 36	7	"	E. D.	6	17.69	17.71	17.61	17.58	18.17	9.58
19	13	5	"	E. D.	5	16.76	16.78	16.68	16.66	17.36	9.54
20	12 20	5	"	E. D.	6	16.65	16.67	16.57	16.55	17.28	9.60
21	11 58	4	"	E. D.	1	16.34	16.35	16.25	16.23	16.97	9.53

For cor. irr. 0″.10 with power 862.

Measures made on true diameter.

No. of Meas. denotes the number of the differences in the readings between the movable and the fixed wire, the movable one having been placed first on one side of the fixed one, and then on the other.

Before discussing the table the corrections in the several columns deserve notice. Of these, the first is that arising from refraction. This is the correction due to the differential effect of refraction upon the planet's opposite limbs at the extremities of the particular diameter measured. It depends upon the altitude of the planet at the time of observation, and upon the inclination at that moment of the particular diameter to the vertical. In about half the measures only was it large enough to make itself perceptible to hundredths of a second of arc.

The correction for abberation, similarly a differential effect, was so utterly insignificant throughout as not to appear at all.

The next correction is that due to irradiation. As this is the chief correction into which uncertainty enters we will consider it at some length, the more so that no investigation of it, has, to my knowledge, ever been made. It is an important correction, but one for which no possible value can destroy the quantities upon which the evidence for the twilight rests, or even effect appreciably the amount of the polar flattening. This will be evident farther on. Its effect is primarily upon the absolute values of the diameters; and secondarily upon the amount, not the existence, of the twilight arc.

It is to be noticed, to start with, that the irradiation is a function of the illumination and of the observer. For a given observer, therefore, it is a function of the illumination, and for a given illumination it is a personal constant. It is independent of the size of the object and diminishes relatively with the magnification. Other things equal, the greater the power used, the smaller the correction arising from it.

Toward its determination two different tests were made, in each case upon both Professor W. H. Pickering and myself; in the one the effect should have been less than in the case of Mars, in the other greater. As in both cases the observers substantially agreed, the results may be accepted as having some impersonal value.

The first test was made upon a railroad switch-head, a white circular disk with a smaller black circle painted upon it. The size of these circles was unknown to the observers.

Their estimates were

(W. H. P.)	(white rim)	1.;	(diameter black circle)	1 3
(P. L.) (two sets—mean)	"	1.;	" " "	1 265

The disks and their distance were then measured and gave:

for diameter black circle :	202 mm.
for radius white rim :	126 mm.
for ratio :	$\frac{1}{16}$
for distance from eye :	57 yds.

Therefore 1 mm equalled 3″.9

For the amount of the irradiation in seconds of arc, x, assume the amount of the irradiation of the white rim against the general background of earth of a brown color to have been ⅔ of that of the rim against the black circle. We have, then, for the first observer, the following equation to determine x.

$$\frac{252 \text{ mm.} + \frac{10}{3}x}{212 \text{ mm.} - \frac{6}{3}x} = \frac{2.0}{1.3}; \text{ from which } x = 9.2 \text{ mm. or } 36''$$

for the second observer $x =$ 40″

The second test was on the Moon (Nov. 22, 1894), when the old moon was seen on the new moon's arms. In this case the irradiation proved for both observers to be 1/7 of the radius of the old moon or about 157″.

The illumination of the surface of Mars bears to the illumination of the lunar surface the ratio $\frac{1}{(1.5)^2} \cdot \frac{26}{17}$ to unity; the first factor denoting the ratio due to their respective distances from the Sun, and the second that due their respective albedoes. In round numbers this $= (2/3)^2\ 3/2$ or 2/3. But in the telescope, owing to the loss of light in passing through the lenses, first of the object glass and then of the eyepiece, this is further to be diminished by the ratio of .82 to unity; which gives .55 for the relative illumination of the Martian surface as compared with the lunar.

Irradiation being a question of contrast and the contrast between the Martian limb and the sky being pretty certainly greater than that of the white rim and the black circle of the switch-head, just as it is certainly less than that of the Moon's bright limb and the sky, to which the contrast between the limbs of the old and of the new Moon closely approximates, the value for Mars should lie between the two determinations. Just where it falls between the two is a matter of estimate inasmuch as the exact functional relation between the illumination and the irradiation is not known. If we assume it to be, for short distances from the edge of the bright body, as the square root of the illumination, we have the irradiation from the Martian disk in the telescope 3/4 of that of the lunar to the naked eye; if as the fourth root .87, if as the sixth .91. Perhaps we shall not be far out if we take .8 for the value. This gives 127″ for Mars. But this again should be slightly diminished on account of the lightening of the field in the telescope due to the inevitable scattering of light by the surfaces of the lenses and the diminished contrast in consequence. If we assume the field to be relatively twice as bright for Mars, we get about 110″ for the irradiation. With a power of 862,110″ becomes 0″.125 For a lower power it is proportionately greater, for a higher one proportionately less. All the measures on and after September 23d were made with a power of 862; those before with powers from 440 to 1308. I have calculated two tables, one on the basis of an irradiation of 0″.10 and one on that of 0″.15; the first table is given in its entirety, of the second only the means. Lastly, account has been taken of the time at which the measure was made, whether during the night, the dawn, or the daylight, and the correction applied in accordance. Such is the correction for irradiation upon the planet's limb. The double of it, therefore, would need to be subtracted from the measures of a disk similarly placed to that of Mars and fully illuminated.

But here we come to a correction upon this correction. Except at the moment of opposition the disk of the planet showed, not two bright limbs, but a limb and a terminator. Now it is evident at once that the irradiation upon the terminator must be very different from that upon the limb, inasmuch as the light fades away to nothing at the one while it has its full value at the other.

To determine the amount of the consequent correction needed at the terminator, it is to be observed that if Mars presented a dead surface like a plaster cast, we should have the edges less brilliant than the centre and the following formula to express the difference

$$\sqrt[n]{c}\ (\sin \alpha)^{\frac{1}{n}} - [\cos \gamma - \cos (\alpha + \gamma)]$$

If again the illumination were equal all over we should have for values of the phase angle appearing in these measures:

$$m \left(\frac{\sin \alpha}{\sin (\alpha + \gamma)}\right)^{\frac{1}{n}} - \left[\cos \gamma - \cos (\alpha + \gamma)\right]$$

where γ is the areocentric angle between the Sun and the Earth; α the angle between the terminator and the point of the illuminated surface whose irradiation is sought; m, the ratio of the irradiation at the limb to the radius of the disk; n, denotes the ratio of the irradiation to the illumination; and c is a constant determined so as to make the irradiation at the limb unity.

But the actual surface of Mars is neither of these, being actually brighter near the limb than elsewhere. If this effect were due, as it probably is, to illuminated particles in the air, the first term would depend, omitting the effects of temperature, etc., upon

$$\sqrt[n]{\int_{\varphi_1}^{\varphi_2} \sec^2 \varphi,\ c,\ e^{-\alpha \sec \varphi}\ d\varphi}$$

where the limits φ_1 and φ_2 depend upon the chord intercepted between the top of the atmosphere and the tangent to the surface at the point, and the integral is the amount of atmosphere necessary for visibility. The term is then to be multiplied by the ratio of the illumination of the disk at the time to that at opposition.

The integral cannot be directly found, being attainable only by a series. As we do not know that this fully expresses the fact, it will be sufficient to estimate approximately the amount of the correction for each varying phase angle. This will be found in column 9. Substitution in the column of any other value, even to the whole extent of the correction for irradiation at the limb—a value which would imply equal illumination there, which is not only theoretically impossible, but visibly not the case—would not suffice to affect, substantially, the result, as will appear on performing the operation and comparing the values obtained with those given below.

The corrections for irradiation at the centre of the limb and at the extremities of the phase diameter have been taken as the same. This would be approximately true in the case of an atmospherically enveloped planet. It would not be true for a bare one, as is evident from the case of the Moon. When the Moon shows a crescent the greater irradiation at the center of the limb is manifest to the naked eye in its distortion of the limb beyond a perfect semi circle. The above consideration has of course no effect upon the question of a twilight arc; but only upon the polar flattening.

Fourth comes the correction for phase. Inasmuch as the phase axis and the polar axis did not in general coincide, there entered into its determination beside the amount of the lacking lune, the angle of inclination of the two axes. So that the amount of the defalcation had to be calculated in accordance for each night.

Fifth comes the correction for tilt. As the planet's polar axis was inclined to the Earth, the diameter measured for the polar one was not the true polar diameter. The diameter measured for the equatorial diameter, although also not in fact an equatorial diameter, was always exactly equivalent to one since its extremities were always 90° distant from the pole. To get the correction to be applied to the polar diameter, the polar flattening was taken at 1/200, a value sufficiently near the truth, and gave 0″.007 at the beginning of the measures, 0″.006 in the middle, and 0″.007 again at the end. This being, for the interval of a few days practically constant was applied to the several means into which the resulting columns were divided. It does not therefore appear in Table I.

No correction is needed for astigmatism, since the measures themselves correct it, inasmuch as image, field, and wire are all three proportionately affected.

In the last column appear the measures reduced to distance unity.

So soon as the measures had been corrected and reduced, two results made themselves evident, both so large as to be almost unmistakeable, before taking the means. One was the polar flattening. This was more or less expected. The other was not; but it was even more noticeable. It was a systematic increase in the size of the equatorial diameter from opposition to the times when the last measures were made in November. The November measures were evidently larger than the October ones; while, on the other hand, the polar measures showed no corresponding increase Struck by this fact and suspecting its cause, instead of taking the mean of all the measures for each diameter, I divided them into sets according to their proximity in date to the time of opposition, and took the mean of these sets.

Allowing for tilt as well as for the other corrections the means are as follows:

MEASURES POLAR DIAMETER.

Mean Oct. 15th to Oct. 23d both dates inc.	9″ 379
" " 12th, 24th, 30th. both dates inc	9 .375
" Nov. 2d to Nov. 21st, both dates inc	9 .390

MEASURES EQUATORIAL DIAMETER.

Mean Oct. 15th to Oct. 23d both dates inc	9″.420
" " 12th, 24th and 30th both dates inc	9 .457
" Nov. 2d to Nov. 21st both dates inc	9 .545

Opposition occurred on October 20th. The first set in each schedule, therefore, was made within four days of opposition; the second, within eleven days of it; the last from fourteen to thirty-two days after it. That there is a systematic increase in the equatorial measures is apparent. That it is not paralleled by a corresponding increase in the polar ones shows instantly that it can hardly have been due to systematic error in the observer, since in that case both sets of measures would in all probability have been affected.

Now as all the measures had previously been corrected for refraction, irradiation, phase and tilt, the means of each diameter should have agreed with themselves. The polar did so in a very satisfactory manner; the equatorial not only did not, but they differed in proportion to the distance in time from the date of opposition. Now the only factor that increased in proportion to the distance in time from opposition was the phase. The direct effect in the way of decreasing the equatorial diameter had already, as we have seen, been allowed for; what is more it is a correction susceptible of some exactness, since it depends upon the motions and relative distances of the Earth and Mars, quantities very accurately known. Besides these quantities there is nothing which enters into the calculation but the position of the pole of Mars, and this would have to be, not only some 35 Martian degrees in error to explain the discrepancy, but, as we shall see later, would have had to have shifted obligingly to an opposite error during July and August to account for the measures taken then. In other words no such discrepancy exists.

In the case of a bare globe this direct effect would be the only effect phase could have upon the equatorial diameter; not so, however, in the case of a body not bare. If a planet possessed an atmosphere, that atmosphere would cause the phenomenon of twilight, and to an observer at a distance the effect of the twilight would be to prolong the terminator beyond what would otherwise be its limit. There would thus result a seeming increase in the equatorial diameter as the disk passed from the full to the gibbous phase. Not only would this be the case, but contrarywise in the absence of an atmosphere, the measures of the equatorial diameter, should not only have shown no increase with increase of phase, but should actually have shown a decrease, inasmuch as it would be impossible for an observer to see quite out to the edge of the disk under its diminishing illumination.

TABLE II.—EARLY DIAMETERS.

POLAR.

Measured by Mr. A. E. Douglass.

	Time. h	m	No of Meas	Eye-piece.	Wt.	Seeing Set on Phase Axis	Uncor. Meas. ″	Cor. Irr. on Limb. ″	Cor. Irr. on Term. ″	Cor. for Refrac ″	Cor. for Phase. ″	At Dist. Unity Appar. ″	At Dist. Unity True. ″
July 6	17	10	4	617	8	5	12.12	12.07	12.02	12.03	12.03	10.01	10.00
" 8	16	42	4	"	10	3	12.83	12.76	12.69	12.69	12.69	10.01	10.40
" 20	16	10	3	"	6	2	13.02	12.95	12.88	12.89	12.89	9.68	9.67
" 22	18	8	5	1305	6	3	12.93	12.92	12.90	12.90	12.90	9.54	9.53
Aug. 11	16	10	5	"	6	4	14.97	14.94	14.90	14.91	14.91	9.47	9.46
" 14	14	56	5	"	7	4	15.31	15.24	15.17	15.18	15.18	9.40	9.39
" 21	17	55	5	617	7	6	15.92	15.88	15.84	15.84	15.84	9.25	9.24
Sept. 20	14		5	"	4	3	20.59	20.45	20.31	20.32	20.32	9.43	9.42
" 23	14	35	5	862	5	3	20.85	20.75	20.65	20.66	20.66	9.42	9.41
Oct. 5	14	36	7	"	4	3	21.77	21.67	21.57	21.58	21.58	9.37	9.36
						EQUATORIAL.							
July 6	17		4	617	6	5	9.93	9.88	9.88	9.88	11.74	9.76	9.76
" 8	16	25	6	"	6	3	10.49	10.42	10.42	10.42	12.39	10.15	10.15
" 18	18	30	5	"	6	5	10.67	10.64	10.64	10.65	12.56	9.58	9.58
" 22	17	55	5	1305	8	3	10.75	10.73	10.73	10.74	12.66	9.38	9.38
Aug. 11	15	45	5	"	6	3	12.79	12.74	12.74	13.74	15.83	10.04	10.04
" 14	14	45	5	"	7	5	13.79	13.72	13.72	13.72	15.74	9.75	9.75
" 21	17	45	5	617	7	6	14.02	13.98	13.98	13.99	15.85	9.26	9.26
Sept. 20	14	15	5	"	5	3	19.69	19.55					
" 23	14	12	5	862	6	3	20.11	10.01					
Oct. 5	14	15	7	"	4	4	21.83	21.73					

For irr. 0″.10.

The effect which would be due to a visible twilight is thus strikingly reflected in the measures. Nor is this all. Comparison with the measures made in July, August, and September and the first part of October brings the same effect even more prominently into view. The table of these measures is opposite.

Now if we take the means of these measures also in chronological sets and put them beside the later ones, we get the following most curious corroboration:

TABLE III.—MEANS.

	Cor. for Ref. irr., tilt and phase. Irr. 0".10	Prob. Error. "	Angle of Phase. °	Irr. 0".15
POLAR DIAMETERS.				
July (6 to 22 inc.)	9.976	0.13	0	9 933
Aug. (11 to 21 inc.)	9.362	0.04	0	9.325
Sept. (20 to Oct. 5 inc.)	9.401	0.012	0	9 355
Oct. (12 and 24 to 30 inc)	9.375	0.028	1	9.336
Oct. (15 to 23 inc.)	9.379	0.011	2.5	9 339
Oct. (12 and 24 to 30 inc.)	9 375	0.028	1	9.336
Nov. (2 to 21 inc)	9.390	0.012	4	9.350
EQUATORIAL DIAMETERS.				
July (6 to 22 inc.)	9.691	0.11	46.5	9.672
Aug. (11 to 21 inc.)	9.666	0 15	41	9.645
Sept. (20 to Oct. 5)	9.523	0.010	20.5	9 490
Oct. (12 and 24 to 30 inc.)	9.457	0.016	7	9.417
Oct. (15 to 23 inc)	9.429	0 010	1	9 385
Oct. (12 and 24 to 30 inc.)	9 457	0 016	7	9.417
Nov. (2 to 21 inc.)	9.545	0.015	19	9.514

The effect here is certainly striking. On the one hand we see that except for July the values of the polar diameter remained substantially the same from beginning to end; on the other hand, that the equatorial diameter increased on *both* sides of opposition, that it increased in proportion to its distance from opposition, and that the probable errors of the means are much less than the values of the variations.

The smallness of the polar means for October 12th, 24th and 30th is doubtless to be attributed to the fact that some of the measures then taken were abnormally small.

That the probable errors are so much larger in the earlier measures is partly explained by the relatively small size of the disk at the time; for the probable errors would vary inversely as the square of the disk's diameter.

By way of applying still further correction to the above result we may now take into account the position of the observer's eyes although the consequent correction is too small to be practically appreciable.

The result is not without interest on its own account, for it discloses a sort of psychic analogae to astigmatism. Astigmatism proper cannot enter into the values obtained because it must affect equally the space passed over by a turn of the micrometer screw and the image itself. It therefore cancels out of the result. Nevertheless, measures of double stars show a difference according as the distance between them is measured vertically or horizontally to the line joining the observer's eyes. To determine whether there is any such effect in the present case we notice that among the measures are four pairs made under identical conditions except for the manner of setting the eyes. These pairs are as follows:

Polar Diameters.				Equatorial Diameters			
	h m				h m		
Oct. 21	13 42	E. D.	9.42	Oct. 21	14 25	E. D.	9.45
Oct. 21	14 00	E. S.	9.39	Oct. 21	14 45	E. S.	9.37
Oct. 24	12 15	E. S.	9.30	Oct. 24	11 50	E. S.	9.42
Oct. 24	12 33	E. D.	9.27	Oct. 24	12 50	E. D.	9.40

On scanning these there seems to be no preponderance one way or the other since in just half the pairs the E. D. measures are the greater and in half of them the reverse. But on further inspection it appears that another personal factor is at work; for on considering the matter of time it turns out that in every pair the later measure is the smaller. If from the hint thus given the

Equatorial Diameter.

Taken before or after Polar Diameter.

		″	″
		Equa. diam. greater than polar one by	
Sept. 20	after	+ 0.10	
" 23	before		+ 0.08
Oct. 5	before		+ 0.18
" 12	before		+ 0.11
" 15	after	+ 0.06	
" 17	before		+ 0.07
" 19	before		+ 0.04
" 20	after	+ 0.03	
" 21	after	+ 0.06	
" 21	after	+ 0.01	
" 23	after	+ 0.09	
" 24	before		+ 0.12
" 24	after	− 0.13	
" 29	before		+ 0.06
" 30	after	+ 0.03	
Nov. 2	after	+ 0.06	
" 4	after	+ 0.16	
" 5	after	+ 0.10	
" 5	before		+ 0.13
" 6	after	+ 0.14	
" 9	after	+ 0.13	
" 14	before		+ 0.25
" 15	before		+ 0.14
" 19	after	+ 0.16	
" 20	after	+ 0.14	
" 21	after	+ 0.13	
		mean + 0.09	mean + 0.12

″ Difference beween the means of those taken before and after + 0.03 those taken before being this much larger than those taken after.

whole polar series be compared, as regards priority of observation, with the whole equatorial one, measure for measure, the same difference is brought to light, as appears on taking the mean of the foregoing table.

Turning now to the measures of the artificial disk upon Mt. Agassiz as observed by Mr. Douglass, we have:

DISK.

VERTICAL DIAMETERS.

	h	m	Direct Meas.		Seeing	Wt.		″	Cor. for Refr. ″
Oct. 25	5	19	"	"	3	5	E. D.	20.87	20.92
Oct. 25	5	33	"	"	2	3	E. D.	21.11	21.16
Oct. 31	4	36	"	"	2	3	E. D.	21.16	21.21
Nov. 6	5	21	"	"	2	3	E. D.	20.83	20.88
Nov. 8	5	3	"	"	½	6	E. D.	20.97	21.02
Mean from Seeing								20.98	21.03
Mean from squares of Seeing								20.96	21.01

HORIZONTAL DIAMETERS.

	h	m						″
Oct. 25	5	5	Direct Meas.		5	6	E. S.	21.09
Oct. 25	5	45	"	"	2	2	E. S.	21.14
Oct. 31	4	19	"	"	2	3	E. S.	21.36
Nov. 6	5	9	"	"	2	2	E. S.	20.91
Nov. 8	4	42	"	"	1	7	E. S.	21.01
Mean from Seeing								21.11
Mean from square of Seeing								21.10

from which we find for the horizontal diameters taken before or after the vertical diameter.

Oct. 25	before	difference + .17		difference.
Oct. 25	after		− .02	
Oct. 31	before	+ .15		0″.105
Nov. 6	before	+ .03		in favor of before.
Nov. 8	before	− .01		

but as this was on a disk of 21″ it becomes on one of 9″.4 about 0″.05. This agrees within expectation with the result found from Mars.

Returning to the four pairs of measures of Mars and using the value 0″.03 deducted from the whole series of measures of the planet for the before-after correction we get 0″.02 for the correction between E. S. and E. D., E. D. being = E. S + 0″.02.

For corroboration of this value it is not possible to use the measures of the artificial disk. For inspection of them shows that the observed measures were actually smaller than the real ones, which hints that the disk, in consequence of exposure to the weather, shrank. It becomes, therefore, probable that the disk shrank unequally and therefore its actual dimensions become unreliable.

If now we apply both these corrections to the previous table of the means of the two diameters we have, taking for basis E. S.:

	Polar.		Equatorial.	
	Irr. 0″.10		Irr. 0″.10	
July	9.976	9.98	9.691	9.69
August	9.362	9.36	9.666	9.67
Sept. 20, 23, Oct. 5	9.406	9.41	9.518	9.52
Oct. 12, 24, 30	9.379	9.38	9.452	9.45
Oct 15, 23	9.379	9.38	9.429	9.43
Oct. 12, 24, 30	9.379	9.38	9.452	9.45
Nov. 2, 21	9 380	9.38	9.540	9.54

The same relative result would have come out in the table for the irradiation correction 0″.15.

Professor W. H. Pickering's measures, although relatively few in number, confirm so far as they go Mr. Douglass'. They are as follows:

Table IV.—Early Diameters.

Measured by Professor Pickering.

Time.			Meas.	Eye-piece.	Wt.	Seeing set on Phase axis.	Uncor Meas.	Cor. Irr. on Limb	Cor Irr. on Term	Cor. for Refrac.	Cor. for Phase.	At Dist. Appar.	Unity True.
							Polar.						
	h	m					″	″	″	″	″	″	″
July 11	16	52	5	617	4	3	12.36	12.33	12.30	12.31	12.31	9.88	9.87
21	18	40	5	"	5	4	12.98	12.96	12.94	12.94	12.94	9.64	9.63
Aug. 7	17	52	5	"	7	7	14.16	14.14	14.12	14.12	14.12	9.30	9.29
24	15	20	5	"	5	4	16.67	16.60	16.53	16.54	16.54	9.44	9.43
							Equatorial.						
July 11	17	10	5	617	3	3	10.54	10.50	10.50	10.50	12.47	10.01	10.01
21	18	30	5	"	5	4	10.91	10.88	10.88	10.89	12.84	9.58	9.58
21	18	48	5	"	5	4	11.08	11.05	11.05	11.06	13.04	9.73	9.73
Aug 7	16	28	3	"	7	7	12.62	12.55	12.55	12.55	14.55	9.52	9.52
24	15	5	5	"	4	4	15.06	14.92	14.92	14.92	16.80	9.59	9.59
28	18	30	5	"	3	3	15.25	15.21	15.21	15.22	16.97	9.36	9.36

Table V.

Means.

Professor W. H. Pickering.

Polar Diameter reduced to dist. unity (all corrections made except for the twilight band)..9″ 373

Equatorial Diameter reduced to dist. unity (all corrections made except for the twilight band)..9″.580

Polar Diameter, polar cap away from limb..9″.14

Polar Diameter. polar cap on the limb..9″.49

Twilight arc 11°.

To determine the extent of the twilight thus disclosed by the measures: if we call E the phase angle; T the angle between the radius to the sunset point and the radius prolonged, to the point of the atmosphere last illuminated; *a*, the true equatorial radius reduced to distance unity. and *b*, the corresponding amount of its excess, we have the following equation to determine T:

$$\frac{1 + \cos \mathrm{E} + \sin \mathrm{E} \,.\, \tan \mathrm{T}}{1 + \cos \mathrm{E}} \,.\, 2a = 2a + b$$

T, neglecting temperature, etc., being determined from

$$\int_{\varphi_1}^{\varphi_2} \sec^2 \varphi \, \mathrm{C} \,.\, \mathrm{E}^{-a \sec \varphi d\varphi}$$

where the limits φ_1 and φ_2 are such as to make the integral constantly equal to the amount of illumination necessary for visibility. Whence

$$\tan \mathrm{T} = \frac{b \,.\, 1 + \cos \mathrm{E}}{2a \,.\, \sin \mathrm{E}}$$

As b is the mean value of the excess of the equatorial diameter between certain times, we must, to determine T take the mean value of

$$\frac{1 + \cos \mathrm{E}}{\sin \mathrm{E}}$$

between the same times.

This mean value is in form,

$$\int_{\mathrm{E}_1}^{\mathrm{E}_2} \frac{1 + \cos \mathrm{E}}{\sin \mathrm{E}} d\mathrm{E} \div \int_{\mathrm{E}_1}^{\mathrm{E}_2} d\mathrm{E}$$

But as $d\mathrm{E}$ is not constant if dt be taken constant, its value in terms of dt must be introduced from the elliptic motions. A short cut to the result may be taken, however, by deducing this mean value direct from the phase corrections.

By so doing we get an approximate value of T, which applied to the measures of October 15th to 23d—since even these were not made exactly at opposition—gives a new value for b and a closer approximation to $2a$. By repeating this process we get values for both as near as we please to their true values until $\mathrm{E} = \mathrm{T}$. After this, when $\mathrm{E} < \mathrm{T}$, the equation ceases to be true, b diminishing not to nothing, but to a minimum with irradiation 0″ 10 of about 0″.025 at distance unity, the amount namely of the twilight seen in profile on the limb. This also is affected by irradiation, since the twilight itself would be swamped in the irradiation effect of the limb, and at the same time would cause an irradiation of its own. We can thus get an approximate value for the twilight arc which is evidently about 5°, the double of which or 10° is the angle which determines the duration of the twilight. On the Earth this angle is 18°.

The value for the twilight band deduced from these observations does not measure the full breadth of that band. It gives only a minimal value for it. For although Mars shows us a disk which is always more than half-full, in which aspect an illuminated fringe of atmosphere would be more perceptible to an observer placed without than to one placed within it, provided both were at the same distance off, in the case before as the observer is a great deal farther off. In consequence, what would be quite recognizable to one standing upon the planet's surface would be too faint to be seen by him at a distance of forty millions of miles. The detection, therefore, of any twilight on Mars hints that the extent of that twilight is greater than appears; how much greater we cannot at present say.

A second possible cause affecting the extent of the twilight is the constitution of the Martian atmosphere. That atmosphere is practically cloudless; if also it be clearer than our own, the twilight would be relatively less, for the amount of twilight is, among other things, a question of the clearness of the air. It is the particles suspended in the air that reflect the light; the less particles, therefore, the less the twilight.

Polar Flattening.

From these measures we deduce for the polar flattening $\frac{a-b}{a}$ the value 1/190

This value receives corroboration from two other sources. It is, in the first place, happily accordant with what theory would lead us to expect. Tisserand has found that with the known rotation of Mars and supposing homogeneity, the planet's flattening should be 1/175 of the equatorial diameter, while if the strata varied in density after the manner of those of the Earth, the polar flattening should be 1/227 of it. Now assuming Mars to have been developed in general accordance with nebular hypothesis, his strata would neither be homogeneous on the one hand, nor on the other, would they vary in density from the surface to the center so marked as is the case with those of the Earth. For Mars, being a smaller body, the pressure due to gravity would be less, somewhere between that of the Earth and that of homogeneity (which is nothing) and the polar flattening should therefore be somewhere between 1/227 and 1/175 of the equatorial diameter. 1/190 is, therefore, not far from the value probable *a priori*.

Secondly, since this paper was written Hermann Struve has found from the speed of rotation of the apsides of Deimos and Phobos the value 1/190 for the polar flattening. It is interesting to have this result agree thus closely with theory, as it furnishes so much more reason for believing in the general evolution of our solar system.

Any value much less than 1/190 would require that Mars should have solidified an unconscionably long time ago. For as we see, the Earth still rotates very nearly as she did when she solidified, and although in the case of Mars this must have happened relatively long before, yet on the other hand, Mars lacks relatively the tools for the purpose, possessing neither suitable satellites on the one side, nor sufficient seas on the other.

For the absolute dimensions of the diameters we may best take the correction for irradiation at 0″.125 and thus obtain:

Polar Diameter	9″.32
Equatorial Diameter	9 .37
Ellipticity	$\frac{1}{190}$
Twilight arc (minimal value)	about 10°

Lowell Observatory,
Dec., 1895.

MICROMETRIC MEASURES

of the Diameters of Mars

giving beside the Size and the Ellipticity of the Planet

Apparent Evidence of a Twilight Arc.

During the summer and autumn, 1894, ~~Mr. Douglass and Professor W.H.Pickering made~~ ~~341~~ 464 micrometric measures of the diameter of Mars, were made at this observatory. [insert 1 bis] ¶. The 403 taken by Mr. Douglass In addition to their general value, ~~these~~ prove to be of peculiar interest. For on reducing them I find that beside furnishing, from their great number, relatively accurate values of the equatorial and polar diameters and of the polar flattening, they yield a by-product as unexpected as it is important. Their discussion reveals what appears to be unmistakeable evidence of a twilight upon the planet, sufficiently pronounced to b visible from the Earth and actually to have been measured, unconsciously. That Mars possessed an atmosphere we had what amounted to proof positive before; but that the fact should again be brought to light in this literal manner, as a silver lining to a cloud of figures, is a point of some curiosity. For the measures had no such end in view; indeed, to detect the presence of an atmosphere by measures of the diameters had not suggested itself to any one. Yet, as will be seen, the quantities upon which the evidence rests are so large as to be quite without the pale of accidental error, being ten times as large as the probable errors of observation, and twice as large as the differences that disclose the polar flattening. That up to this time they should have escaped detection is due to their having been masked by another factor affecting the size

Number of Measures

of the

Diameters of Mars.

Equatorial Diameters	256	
Polar Diameters	208	
	464	
of these	403	were made by ~~Mr~~. A. E. Douglass.
	50	" " " Prof. W. H. Pickering
	11	" " " ~~Mr~~. Percival Lowell.
	464	

of the polar diameter, the varying irradiation from the polar cap. To the hitherto unsuspected presence of the first of these factors and to the difficulty of duly evaluating the second are attributable in all probability the discrepancies in the values previously found for the ~~polar flattening~~ ellipticity.

The first of Mr. Douglass' measures were made on the 6th of July and the last on the 21st of November, 1894. From the 12th of October the planet was measured by him nearly every other night; and ~~The measures I shall first discuss were all made by Mr" Douglass~~ in the following manner: the longitudinal thread of the micrometer having been set to the planet's phase axis or polar axis as the case might be, the field of the micrometer was so adjusted that the stationary transverse thread lay close to, but not on, the limb of the planet's disk. The moveable thread was then carried some way beyond the otherlimb and steadily brought back till it lay to the same side of, and at the same distabce from, the other limb that the stationary thread lay from the first one. It was then read. During the process of bringing it up its motion was never reversed. In case of overrunning it was carried some distance back and again continuously brought up. Thus any possible backlash was prevented from affecting the measures.

The whole field was next shifted till the stationary thread occupied the same general position towardthe second limb that it had done toward the first, and the moveable ~~one~~ thread was brought past it till it reached a corresponding distance from the other limb, upon which it was read. The whole field was then shifted to the first position and the process repeated. Each reading thus furnished half the data to two measures.

Later in the paper I shall consider those made by Professor Pickering which will be found to confirm the result. But here at the outset it may be well to note that whether the results of many observers are to be preferred to those of one is, omitting personalities, a question entirely of what is to be determined. If the determination be ine of absolute quantity, the more observers the better, provided tey be good; but if on the other hand, the determination be of relative magnitudes, the number of observations, not the number of observers, is the important matter.

In the measures made during ~~of~~ September, October, and November, Mr. Douglass took the further precaution to place himself so that the line joining his eyes was kept either perpendicular or parallel to the diameter measured; ~~and entered~~ a record of his position being entered opposite the measures. Such record appears in column 4 of the table: E.S. ~~st~~ standing for "eyes parallel to single thread", that is, parallel to the longitudinal thread of the micrometer; and E.D., "eyes parallel to double (i.e. transverse) threads'. E.S. therefore denotes a horizontal measurement; E.D. a vertical one. As measures were taken in both positions for each diameter at various times, we have here a comparison of some value as will appear further on.

To eliminate systematic errors as muchas possible a circular

TABLE I.

POLAR DIAMETERS.

Measured by Mr. A.E. Douglass

For cor. irr. 0".10 with power 862 and irradiation varying as the √ of the illumination –

	Time	No.of Meas.	Eyepiece	Wt.	Uncor. Meas.	Cor.for Refrac.	Cor.for Irrad.on Limb	Cor.for Irrad. on Term.	Cor.for Phase	Reduced to Distance Unity.	
~~Sept. 20~~	~~14h~~	~~5~~	~~617 E.S.~~	~~4~~	~~20".59~~	~~20".60~~	~~20".53~~	~~20".46~~	~~20".46~~	~~9".50~~	~~Measures made on phase diameter.~~
~~" 23~~	~~14h 35m~~	~~5~~	~~862 E.D.~~	~~5~~	~~20".85~~	~~20".86~~	~~20".8?7~~	~~20".76~~	~~20".76~~	~~9".47~~	~~" " " " "~~
~~Oct. 5~~	~~14h36m~~	~~7~~	~~"~~	~~4~~	~~21".77~~	~~21".78~~	~~21".73~~	~~21".68~~	~~21".68~~	~~9".42~~	
" 12	13h 45m	5	"	5	21".95	21".95	21".85	21".79	21".81	9".40	*Measures made on* true *diameter*
" 15	13h 42m	6	" E.R.A.	4	21".84	21".84	21".74	21".67	21".68	9".36	
" 17	15h	4	" E.S.	4	21".77	21".77	21".67	21".59	21".60	9".38	
" 19	15h 5m	5	" E.S.	2	21".92	21".92	21".82	21".73	21".73	9".46	"
" 20	13h 57m	5	" E.S.	5	21".69	21".69	21".59	21".50	21".50	9".41	
" 21	13h 42m	5	" E.D.	5	21".69	21".70	21".60	21".51	21".51	9".42	
" 21	14h	5	" E.S.	4	21".61	21".61	21".51	21".42	21".42	9".39	
" 23	13h 40m	5	" E.S.	4	21".28	21".28	21".18	21".08	21".08	9".31	"
" 24	12h 15m	5	" E.S.	3	21".17	21".18	21".08	20".98	20".98	9".30	"
" 24	12h 33m	6	" E.D.	3	21".08	21".09	20".99	20".89	20".89	9".27	
" 29	13h 16m	5	" E.S.	3	20".94	20".94	20".84	20".75	20".75	9".46	
" 30	14h 50m	7	" E.S.	3	20".78	20".78	20".68	20".59	20".59	9".46	"
Nov. 2	13h 34m	5	" E.S.	6	20".15	20".15	20".05	19".97	19".97	9".36	
" 4	12h 40m	5	" E.S.	4	19".85	19".85	19".75	19".67	19".68	9".37	
" 5	13h 10m	5	. E.S.	5	19".61	19".61	19".51	19".43	19".44	9".34	
" 5	14h 38m	5	" E.S.	3	19".54	19".54	19".44	19".36	19".37	9".30	"
" 6	12h 45m	5	" E.S.	7	19".67	19".67	19".57	19".49	19".50	9".44	
" 9	13h 35m	5	" E.S.	7	19".23	19".23	19".13	19".05	19".07	9".49	
" 14	14h 35m	5	" E.S.	6	18".02	18".02	17".92	17".85	17".88	9".34	
" 15	13h 53m	5	" E.S.	5	18".03	18".03	17".92	17".85	17".88	9".44	"
" 19	12h 45m	5	" E.S.	3	17".19	17".19	17".09	17".02	17".06	9".38	
" 20	11h 50m	5	" E.S.	2	17".15	17".15	17".05	16".99	17".03	9".46	
" 21	11h 45m	5	" E.S.	7	16".86	16".86	16".76	16".70	16".74	9".40	

TABLE II.

EQUATORIAL DIAMETERS.

~~Sept. 20~~	~~14h 15m~~	~~5~~	~~617 E.S.~~	~~5~~	~~19".69~~	~~19".69~~	~~19".58~~	~~19".52~~	~~20".50~~	~~9".53~~	~~Measures made on phase diameters.~~
~~" 23~~	~~14h 12m~~	~~5~~	~~862 E.D.~~	~~6~~	~~20".11~~	~~20".11~~	~~20".01~~	~~19".99~~	~~20".82~~	~~9".50~~	~~" " " " "~~
~~Oct. 5~~	~~14h 15m~~	~~7~~	~~"~~	~~4~~	~~21".83~~	~~21".83~~	~~21".73~~	~~21".69~~	~~21".99~~	~~9".55~~	~~" " " true "~~
" 12	13h 15m	5	"	5	22".15	22".15	22".05	21".99	22".06	9".51	*Measures made on true diameter*
" 15	14h 2m	5	" E.S.	3	21".96	21".96	21".86	21".78	21".80	9".42	
" 17	14h 42m	5	" E.S.	4	21".98	21".99	21".89	21".79	21".79	9".44	
" 19	14h 48m	5	" E.S.	3	22".00	22".01	21".91	21".81	21".81	9".50	"
" 20	14h 11m	5	" E.S.	?6	21".80	21".81	21".71	21".61	21".61	9".44	

TABLE II. (Cont.)

EQUATORIAL DIAMETERS.

For cor. irr 0".10 with power 862

	Time	No. of Meas.	Eyepiece	Wt.	Uncor. Meas.	Cor. for Refrac.	Cor. for Irrad. on Limb	Cor. for Irrad. on Term.	Cor. for Phase	Reduced to Distance Unity.
Oct. 21	14h 25m	6	862 E.D.	4	21".74	21".75	21".65	21".56	21".56	9".45
" 21	14h 45m	5	" E.S.	5	21".57	21".58	21".48	21".39	21".39	9".37
" 23	14h 15m	7	" E.S.	3	21".45	21".46	21".36	21".28	21".30	9".40
" 24	11h 50m	6	" E.S.	3	21".38	21".38	21".28	21".21	21".24	9".42
" 24	12h 50m	5	" E.D.	5	21".32	21".33	21".23	21".16	21".19	9".40
" 29	13h 3m	5	" E.D.	3	20".91	20".92	20".82	20".77	20".88	9".52
" 30	15h	5	" E.D.	2	20".54	20".55	20".45	20".40	20".54	9".43
Nov. 2	14h 12m	5	" E.D.	6	20".03	20".04	19".94	19".89	20".10	9".42
" 4	13h 10m	5	" E.D.	4	19".87	19".88	19".78	19".74	20".01	9".53
" 5	13h 36m	5	" E.D.	4	19".49	19".50	19".40	19".36	19".66	9".44
" 5	14h 15m	5	" E.D.	3	19".46	19".48	19".38	19".34	19".64	9".43
" 6	13h 20m	6	" E.D.	8	19".59	19".60	19".50	19".46	19".79	9".58
" 9	13h 53m	5	" E.D.	7	19".04	19".06	18".96	18".92	19".34	9".62
" 14	14h 18m	7	" E.D.	6	17".90	17".93	17".83	17".80	18".37	9".59
" 15	13h 36m	7	" E.D.	6	17".69	17".71	17".61	17".58	18".17	9".58
" 19	13h	5	" E.D.	5	16".76	16".78	16".68	16".66	17".36	9".54
" 20	12h 20m	5	" E.D.	6	16".65	16".67	16".57	16".55	17".28	9".60
" 21	11h 58m	4	" E.D.	1	16".34	16".35	16".25	16".23	16".97	9".53

No. of Meas. denotes the number of the differences in the readings between the movable and the fixed wire, the movable one having been placed first on one side of the fixed one, and then on the other –

E. D = Eyes parallel to double wire, that is the measuring wires

E. S = Eyes parallel to single wire, that is the one at right angles to the measuring wires –

disk was constructed, carefully measured, and carried to the top of Mt Agassiz, one of the San Francisco peaks, where it was placed just under the summit facing the Observatory. ~~The size of the disk was such as to subtend at that distance an angle very nearly equal to the planet's disk.~~ From triangulation the angles by which its position deviated from the vertical in the first place and from parallelism to the dome in the second were found to be respectively: 6° and 2° west; ~~A point in the town of Flagstaff, the dome and the disk were then severally triangulated from a base line both horizontally and vertically and gave for the air-line~~ its distance ~~between~~ from the dome in an air-line being 330 ~~and the disk 45~~ ft.~~, and for the elevation of the disk above the dome . From this the~~ Whence its apparent diameters were ~~of the disk came out~~:

vertical diameter ; 21".08

horizontal diameter ; 21".22

The diameters were then observed through the telescope and micrometrically measured by Mr. Douglass. The size of the artificial disk was thus such as to subtend at that distance from the Observatory an angle very nearly equal to the planet's disk.

Of the measures of the planet, the first to be reduced were those ~~from~~ taken from Oct. 12 to Nov 22 inc. They are as follows:

Tables I ~~and II~~.

Before discussing the ~~resulting values~~ table the corrections in the several columns deserve notice. Of these, the first is that arising from refraction. This is the correction due to the differential effect of refraction upon the planet's opposite limbs at the extremities of the particular diameter measured. It depends upon the altitude of the planet at the time of observation, and upon the inclination at that moment of the particular diameter to the vertical. In about half the measures only was it large enough to make itself perceptible to hundredths of a second of arc.

The correction for abberration, similarly a differential effect was so utterly indignificant throughout as not to appear at all.

The next correction is that due to irradiation. As this is the chief correction into which uncertainty enters we will consider it at some length, the more so that no investigation of it, has, to my knowledge, ever been made. It is an important correction, but one for which no possible value can destroy the quantities upon which the evidence for the twilight rests, or even effect appreciably the amount of the polar flattening. This will be evident farther on. Its effect is primarily upon the absolute values of the diameters; and secondarily upon the amount, not the existence, of the twilight arc.

It is to be noticed, to startwith, that the irradiation is a function of the illumination and of the observer. For a given observer, therefore, it is a function of the illumination, andfor a given illumination it is a personal constant. It is independent of the size of the object and diminishes relatively with the magnification. Other things equal, ~~therefore,~~ the greater te power used, the smaller the correction arising from it.. Toward its determination two different tests were made, in each case upon both Professor W.H.Pickering and myself; in the one the effect should have been less than in the case of Mars, in the other greater. As in both cases the observers substantially agreed, the results may be accepted as having some impersonal value.

The first test was made upon a railroad switch-head, a white circular disk with a smaller black circle painted upon it. The size of these circles was unknown to the observers.

Their estimates were

(W.H.P.)	(white rim)	1.;	(diameter black circle) 1.3
(P.L.) (two sets - mean)	"	1.;	" " " 1.265

The disks and their distance were then measured and gave:

for diameter black circle : 202 mm.

for radius white rim : 126 mm.

for ratio : $\frac{1}{16}$

for distance from eye : 57 yds.

Therefore 1 mm. equalled 3".9

For the amount of the irradiation in seconds of arc, x, assume the amount of the irradiation of the white rim against the general background of Earth of a brown color to have been 2/3 of that of the rim against the black circle. We have, then, for the first observer, the following equation to determine x.

$\frac{252 \text{ mm.} + 10/3 . x}{212 \text{ mm.} - 6/3 . x} = \frac{2.0}{1.3}$; from which x = 9.2 mm. or 36" ;

for the second observer x = 40" .

The second test was on the Moon (Nov.22, 1894), when the old moon was seen n the new moon's arms. In this case the irradiation proved for both observers to be 1/7 of the radius of the old moon or about 157".

The illumination of the surface of Mars bears to the illumination of the lunar surface the ~~following~~ ratio $\frac{1}{(1.5)^2} \cdot \frac{26}{17}$ to unity; the first factor denoting the ratio due to their respective distances form the Sun, and the second that due their respective albedoes. In round numbers this = $(2/3)^2 \cdot 3/2$ or 2/3. But in the telescope owing to the loss of light in passing through the lenses, first of the object glass and then of the eye-piece, this is further to be diminished by the ratio of .82 to unity; which gives ~~.57~~ .55 for the relative illumination of the Martian surface as compared with the lunar.

Irradiation being a question of contrast and the contrast between the Martian limb and the sky being pretty certainly greater than that of the white rim and the black circle of the switch-head, just as it is certainly less than that of the moon's bright limb and the sky, to which the contrast between the limbs of the old and of the new moon closely approximates, the value for Mars should lie between the two determinations. Just where it falls between the two is a matter of estimate inasmuch as the exact functional relation between the illumination and the irradiation is not known. If we assume it to be, for short distances from the edge of the bright body, as the square root of the illumination, we have the irradiation from the Martian disk in the telescope 3/4 of that of the lunar to the naked eye; if as the fourth root .87, if as the sixth .91. Perhaps we shall not be far out if we take .8 for the value. This gives 127" for Mars. But this again should be slightly diminished on account of the lightening of the field in the telescope due to the inevitable scattering of light by the surfaces of the lenses and the diminished contrast in consequence. If we assume the field to be relatively twice as bright for Mars, we get about 110" for the irradiation. With a power of 862 110" becomes 0".125. For a lower power it is proportionately greater; for a higher one proportionately less. All the measures on and after September 23d were made with a power of 862; those before with powers from 440 to 1308. I have calculated two tables, one on the basis of an irradiation of 0".10 and one on that of 0".15; the first table is given in its entirety, of the second only the means.

Lastly account has been taken of the time at which the measure was made, whether during the night, the dawn, or the daylight, and the correction applied in accordance. Such is the correction for irradiation upon the planet's limb. The double of it, therefore, would need to be subtracted from the measures of a disk similarly placed to that of Mars and fully illuminated.

But here we come to a correction upon this correction. Except at the moment of opposition the disk of the planet showed, not two bright limbs, but a limb and a terminator. Now it is evident at once that the irradiation upon the terminator must be very different from that upon the limb, inasmuch as the light fades away to nothing at the one while it has its full value at the other.

To determine the amount of the consequent correction needed at the terminator, it is to be observed that if Mars presented a dead surface like a plaster cast, we should have the edges less brilliant than the centre and the following formula to express the difference

$$\sqrt[n]{c\,(\sin\alpha)^{\frac{1}{n}}} - [\cos\gamma - \cos(\alpha+\gamma)]$$

If again the illumination were equal all over we should have for values of the phase angle appearing in these measures:

$$m\left(\frac{\sin\alpha}{\sin(\alpha+\gamma)}\right)^{\frac{1}{n}} - [\cos\gamma - \cos(\alpha+\gamma)]$$ where

γ is the areocentric angle between the Sun and the Earth;

α, the angle between the terminator and the point of the illuminated surface whose irradiation is sought;

m, the ratio of the irradiation at the limb to the radius of the disk

n, denotes the ratio of the irradiation to the illumination;

and c is a constant determined so as to make the irradiation at the limb unity.

But the actual surface of Mars is neither of these, being actually brighter near the limb than elsewhere. If this effect were due, as it probably is, to illuminated particles in the air, the first term would depend omitting the effect of temperature etc, upon $\sqrt[n]{\int_{\varphi_1}^{\varphi_2} \sec^n\varphi . c . \varepsilon^{-a\sec\varphi}\, d\varphi}$ where the limits φ_1 and φ_2 depend upon the chord intercepted between the top of the atmosphere and the tangent to the surface at the point, and the integral is the amount of atmosphere necessary for visibility. The term is then to be multiplied by the ratio of the illumination of the disk at the time to that at opposition. The integral cannot be directly found, being attainable only by a series. As we do not know

The corrections for irradiation at the centre of the limb and at the extremities of the phase diameter have been taken as the same. This would be approximately true in the case of an atmospherically enveloped planet. It would not be true for a bare one, as is evident from the case of theMoon. When the Moon shows a crescent the greater irradiation at the centre of the limb is manifest to the naked eye in its distortion of the limb beyond a perfect semicircle. The above consideration has of course no effect upon the question of a twilight arc; but only upon the polar flattening.

that this fully expresses the fact, it will be suffucient to estimate approximately the amount of the correction for each varying phase angle. This will be found in column 9. Substitution in the column of any other value, even to the whole extent of the correction for irradiation at the limb - a value which would imply equal illumination there, which is not only theoretically impossible, but visibly not the case - would not suffice to affect, substantially, the result, as will appear on performing the operation and comparing the values obtained with those given below. [10bis

Fourth comes the correction for phase. Inasmuch as the phas axis and the polar axis did not in general coincide, there entered into its determination beside the amount of the lacking lune, the angle of inclination of the two axes. So that the amount of the defalcation had to be calculated in accordance for each night.

Fifth comes the correction for tilt. As the planet's polar axis was inclined to the Earth, the diameter measured for the polar one was not the true polar diameter. The diameter measured for the equatorial diameter, although also not in fact an equatorial diameter, was always exactly equivalent to one since its extremities were always 90 distant from the pole. To get the correction to be applied to the polar diameter, ~~an approximate value of~~ the polar flattening was taken at 1/200, ~~was taken which was~~ a value sufficiently near the truth, and gave 0".007 at the beginning of the measures, 0".006)n the middle, and 0".007 again at the end. This being, for the interval of a few days practically constant was applied to the several means into which the resulting columns were divided. It does not therefore appear in Table I

No correction is neededfor astigmatism, ~~as~~ since the measures themselves correct it, inasmuch as image, field and wire are all three proportionately affected.

In the last column appear the measures reduced to distance unity.

So soon as the measures had been corrected and reduced, two results made themselves evident, both so large as to be almost unmistakeable before taking the means. One was the polar flattening. This was more or less expected. The other was not; but it was even more noticeable. It was a systematic increase in the size of the equatorial diameter from opposition to the times when the last measures were made in November. The November measures were veidently larger than the October ones; while, on the other hand, the polar measures showed no corresponding increase. Struck by this fact and suspecting its cause, instead of taking the mean of all the measures for each diameter, I divided them into sets according to their proximity in date to the time of opposition, and took the mean of these sets.

Allowing for tilt as well as for the other corrections the means are s follows:

Measures Polar Diameter.

Mean Oct.15th to Oct.23d both dates inc.	9".379	
" " 12th 24th & 30th " " "	9".375	some abnormally small measures made in this set.
" Nov.2d to Nov.21st " " "	9".390	

Measures Equatorial Diameter.

Mean Oct.15th to Oct.23d	both dates inc.	9".420
" " 12th & 24 & 30th	" " "	9".457
" Nov.2d to Nov.21st	" " "	9".545

Opposition occurred on October 20th. The first set in each schedule, therefore, was made within four days of opposition; the second, within eleven days of it; the last from fourteen to thirty-two days after it. That there is a systematic increase in the equatorial measures is apparent. That it is not parallelled by a corresponding increase in the polar ones shows instantly that it can hardly have been due to systematic error in the observer, since in that case both sets of measures would in all probability have been affected.

Now as all the measures had previously been corrected for refraction, irradiation, phase and tilt, the means of each diameter should have agreed with themselves. The polar did so in a very satisfactory manner; the equatorial not only did not, but they differed in proportion to the distance in time from the date of opposition. Now the only factor that increased in proportion to the distance in time from opposition was the phase. The direct effect in the way of decreasing the equatorial diameter had already, as we have seen, been allowed for; what is more it is a correction susceptible of ~~great accuracy~~ some exactness, since it depends upon the motions and relative distances of the Earth and Mars, quantities very accurately known. Besides these quantities there is nothing which enters intothe calculation but the position of theppole of Mars, and this would have to be, not only some

35 Martian degrees in error to explain the discrepancy, but, as we shall see later, would have had to have shifted obligingly to an opposite error during July and August to account for the measures taken then. In other words no such discrepancy exists.

In the case of a bare globe this direct effect would be the only effect phase could have upon the equatorial diameter; not so, however, in the case of a body not bare. If a planet possessed an atmosphere, that atmosphere would cause the phenomenon of twilight, and to an observer at a distance the effect of the twilight would be to prolong the terminator beyond what would otherwise be its limit. There would thus result a seeming increase in the equatorial diameter as the disk passed from the full to the gibbous phase. Not only would ~~they do~~ this be the case, but ~~it is furthermore worth noting that~~ contrarywise in the absence of anatmosphere, the measures of the equatorial diameter ~~as the phase increased~~, sholld not only have shown no increase with increase of phase, but should actually have shown a decrease, inasmuch as it would be impossible for an observer to see quite out to the edge of the disk under ~~the~~ its diminishing illumination.

The effect which would be due to a visible twilight ~~upon the planet~~ is thus strikingly reflected in the measures. Nor is this all. Comparison with the measures made in July, August and September and the first part of October brings the same ~~coincidence~~ effect even more prominently into view. The table of these measures is as follows:

Table II

Table II.

EARLY DIAMETERS. *Measured by* Mr. A. E. Douglass. For irr. 0".10

POLAR.

Time	No. of Meas.	Eyepiece	Wt.	Seeing	Set on	Uncor. Meas.	Cor. Irr. on Limb	Cor. Irr. on Term.	Cor. for Refrac.	Cor. for Phase	At dist. Appar.	Unity True
July 6 17h 10m	4	617	8	5	Phase axis	12".12	12".07	12".02	12".03	12".03	10".01	10".00
July 8 16h 42m	4	" "	10	3	" "	12".83	12".76	12".69	12".69	12".69	10".01	10".40
" 20 16h 10m	3	"	6	2	" "	13".02	12".95	12".88	12".89	12".89	9".68	9".67
" 22 18h 8m	5	1305	6	3	" "	12".93	12".92	12".90	12".90	12".90	9".54	9".53
Aug. 11 16h 10m	5	"	6	4	" "	14".97	14".94	14".90	14".91	14".91	9".47	9".46
" 14 14h 56m	5	"	7	4	" "	15".31	15".24	15".17	15".18	15".18	9".40	9".39
" 21 17h 55m	5	617	7	6	" "	15".92	15".88	15".84	15".84	15".84	9".25	9".24
Sept 20 14h	5	"	4	3	" "	20".59	20".45	20".31	20".32	20".32	9".43	9".42
" 23 14h 35m	5	862	5	3	" "	20".85	20".75	20".65	20".66	20".66	9.42	9".41
Oct. 5 14h 36m	7	"	4	3	" "	21".77	21".67	21".57	21".58	21".58	9".37	9".36

EQUATORIAL.

Time	No. of Meas.	Eyepiece	Wt.	Seeing	Set on	Uncor. Meas.	Cor. Irr. on Limb	Cor. Irr. on Term.	Cor. for Refrac.	Cor. for Phase	At dist. Appar.	Unity True
July 6 17h	4	617	6	5	" "	9".93	9".88	9".88	9".88	11".74	9".76	9".76
" 8 16h 25m	6	"	6	3	" "	10".49	10".42	10".42	10".42	12".39	10".15	10".15
" 18 18h 30m	5	"	6	5	" "	10".67	10".64	10".64	10".65	12".56	9".58	9".58
" 22 17h 55m	5	1305	8	3	" "	10".75	10".73	10".73	10".74	12".66	9".38	9".38
Aug. 11 15h 45m	5	"	6	3	" "	12".79	12".74	12".74	13".74	15".83	10".04	10".04
" 14 14h 45m	5	"	7	5	" "	13".79	13".72	13".72	13".72	15".74	9".75	9".75
" 21 17h 45m	5	617	7	6	" "	14".02	13".98	13".98	13".99	15".85	9".26	9".26
Sept 20 14h 15m	5	"	5	3	" "	19".69	19".55					
" 23 14h 12m	5	862	6	3	" "	20".11	10".01					
Oct. 5 14h 15m	7	"	4	4	" "	21".83	21".73					

Table III.

MEANS.

Polar Diameters.

	cor. for ref., irr., tilt and phase Irr. 0".10	Prob. error	Angle of Phase	Irr. 0".15
July (6 to 22 inc.)	9.976	0".13	0°	9.933
Aug. (11 to 21 inc.)	9.362	0".04	0°	9.325
Sept. (2o to Oct. 5 inc.)	9.401	0".012	0°	9.355
Oct. (12 & 24 to 30 inc.)	9.375	0".028	1°	9.336
Oct. (15 to 23 inc.)	9.379	0".011	2°.5	9.339
Nov. (2 to 21 inc.)	9.390	0".012	4°	9.350
Oct. (12 + 24 to 30 inc.)	9.375	0".028	1°	9.336

Equatorial Diameters.

July (6 to 22 inc.)	9.691	0".11	46°.5	9.672
Aug. (11 to 21 inc.)	9.666	0".15	41°	9.645
Sept. (20 to Oct. 5)	9.523	0".010	20°.5	9.490
Oct. (12 & 24 to 30 inc.)	9.457	0".016	7°	9.417
Oct. (15 to 23 inc.)	9.429	0".010	1°	9.385
Oct. (12 & 24 to 30 inc.)	9.457	0".016	7°	9.417
Nov. (2 to 21 inc.)	9.545	0".015	19°	9.514

Now if we take the means of these measures also in chronological sets and put them beside the later ones, we get the following most curious ~~verification:~~ corroboration

Table III

The effect here is ~~unmistakeable.~~ certainly striking. On the ~~first place~~ one hand we see that except for ~~apparently for the excessive irradiation of the polar cap if~~ July ~~(not allowed for in the correction applied for irradiation which was the same throughout)~~ the values of the polar diameter remained substantially the same from beginning to end; on the ~~second~~ other hand that the equatorial diameter increased on <u>both</u> sides of opposition, ~~thus showing that a certain class of errors did not enter into the effect; thirdly,~~ that it increased in ~~due~~ proportion to its distance from opposition, ~~in other words, with increase of phase;~~ and ~~lastly on calculating~~ that the probable errors of the~~se~~ means ~~it will be seen that the probable errors were too small to account for~~ are much less than the values of the variations.

Even the anomalies from a perfect curve are largely susceptible of explanation. The excessive value for the polar diameter f found in July is in great part due to irradiation from the then existent polar cap. In August the same cap had diminished so as not to affect the extremity of thephase axis upon which the measures were then taken. And the same was true later till at the time when the measures began to be taken on the polar axis the cap had itself disappeared.

The smallness of the polar means for October 12, 24 and 30th is doubtless to be attributed to the fact that some of the measures then taken were abnormally small.

That the probable errors are so much larger in the earlier measures is partly explained by the relatively small size of the disk at the time; for the probable errors would vary inversely as the square of the disk's diameter.

By way of applying still furthercorrection to the above result we may now take into account the position of the observer's eyes although the consequent correction is too small to be practically appreciable.

The result is not without interest on its own account, for it discloses a sort of psychic analogae to astigmatism. Astigmatism proper cannot enter into the values obtained because it must affect equally the space passed over by a turn of the micrometer screw and the image itself. It therefore cancels out of the result. Neverthe-less, measures of double stars show a difference according as the distance between them is measured vertically or horizontally to the line joining the observer's eyes. To determine whether there is any such effect in the present case we notice that among the measures are four pairs made under identiacl conditions except for the manner of setting the eyes. These pairs are as follows:

Polar Diameter		Equatorial Diameter	
Oct.21 13h 42m E.D.	9.42	Oct.21 14h 25m E.D.	9.45
Oct.21 14h 00m E.S.	9.39	Oct.21 14h 45m E.S.	9.37
Oct.24 12h 15m E.S.	9.30	Oct.24 11h 50m E.S.	9.42
Oct.24 12h 33m E.D.	9.27	Oct.24 12h 50m E.D.	9.40

~~Now~~ On scanning these there seems to be no ~~decided effect~~ preponderance one way or the other since in just half the pairs the E.D. measures are the greater and in half of them the reverse. But on further inspection it appears that another personal factor is at work; for on considering the matter of time it turns out that in every pair the later measure is the smaller. If from the hint thus given the whole polar series be compared, as regards priority of observation, measure for measure, with the whole equatorial one, the same difference ~~emerges~~ is brought to light, as appears on taking the measn of the following table.

Equatorial Diameter taken before or after Polar Diameter.

		~~Difference in meas. of~~ equat. diam. greater than polar one by	
Sept. 20	after	+0".10	
" 23	before	~~+0".08~~	+0".08
Oct. 5	before		+0".18
Oct. 12	before		+0".11
" 15	after	+0".06	
" 17	before		+0".07
" 19	before		+0".04
" 20	after	+0".03	
" 21	after	+0".06	
" 21	after	+0".01	
" 23	after	+0".09	
" 24 b	before		+0".12
" 24	after	−0".13	
" 29	before		+0".06
" 30	after	+0".03	
Nov. 2	after	+0".06	
" 4	after	+0".16	
" 5	after	+0".10	
" 5	before		+0".13
" 6	after	+0".14	
" 9	after	+0".13	
" 14	before		+0".25
" 15	before		+0".14
" 19	after	+0".16	
" 20	after	+0".14	
" 21	after	+0".13	
	mean	+0".09	mean +0".12

Difference between the means of those taken before and after ~~Mean of all~~ +0".03; those taken before being this much larger than those taken after

Turning now to the measures of the artificial disk upon Mt. Agassiz as observed by Mr Douglass, we have ~~which are as follows:~~

DISK.

Vertical Diameters.

Date	Time		Seeing	Wt.			Cor. for Refr.
Oct. 25	5h 19m	Direct Meas.	3	5	E.D.	20".87	20".92
Oct. 25	5h 33m	" "	2	3	E.D.	21".11	21".16
Oct. 31	4h 36m	" "	2	3	E.D.	21".16	21".21
Nov. 6	5h 21m	" "	2	3	E"D.	20".83	20".88
Nov. 8	5h 3m	" "	1/2	6	E.D.	20".97	21".02
Mean from Seeing						20".98	21".03
Mean from squares of Seeing						20".96	21".01

Horizontal Diameters.

Date	Time		Seeing	Wt.		
Oct. 25	5h 5m	Direct Meas.	5	6	E.S.	21".09
Oct. 25	5h 45m	" "	2	2	E.S.	21".14
Oct. 31	4h 19m	" "	2	3	E.S.	21".36
Nov. 6	5h 9m	" "	2	2	E.S.	20".91
Nov. 8	4h 42m	" "	1	7	E.S.	21".01
Mean from Seeing						21".11
Mean from square of Seeing						21".10

we find: Horiz.diam.taken before or after vert.diam.

Oct.25	before	difference	+ .17		
Oct.25	after			-.02	
Oct.31	before		+ .15		Mean 0.105
Nov. 6	before		+ .03		
Nov. 8	before		-.01		

but as this was on a disk of 21" it becomes on one of 9".4 about 0".05 This agrees within expectation with the result found fom Mars.

Returning to the four pairs of measures of Mars and using the value 0".03 deducted from the whole series of measures of the planet for the before-after correction we get

0".02 for the corection between E.S. and E.D., E.D. being = E.S. + 0".02

For corroboration of this value it is not possible to use the measures of the artificial disk.

Table

Inspection of them ~~will~~ shows that ~~either~~ the measures were ~~greater~~ than the ~~fact or~~ that the disk, in consequence of exposure to the weather shrank. ~~As the latter is the more probable supposition,~~ it becomes, ~~also~~ probable that the disk shrank unequally and therefore its actual dimensions become unreliable.

If now we apply both these correctionsto the previous table of the means of the two diameters we have, taking for basis E.S.:

measured by — Table

Prof. Pickering.

EARLY DIAMETERS.

POLAR.

	Time	No. of Meas.	Eyepiece	Wt.	Seeing	Set on	Uncor. Meas.	Cor. Irr. on Limb	Cor. Irr. on Term.	Cor. for Refrac.	Cor. for Phase	At dist. Appar.	Unity True
July 11	16h 52m	5	617	4	3	Phase axis	12".36	12".33	12".30	12".31	12".31	9".88	9".87
" 21	18h 40m	5	"	5	4	" "	12".98	12".96	.2".94	12".94	12".94	9".64	9".63
Aug. 7	17h 52m	5	617	7	7	" "	14".16	14".14	.4".12	14".12	14".12	9".30	9",29
" 24	15h 20m	5	"	5	4	" "	16".67	16".60	.6".53	16".54	16".54	9".44	9".43

EQUATORIAL.

	Time	No. of Meas.	Eyepiece	Wt.	Seeing	Set on	Uncor. Meas.	Cor. Irr. on Limb	Cor. Irr. on Term.	Cor. for Refrac.	Cor. for Phase	At dist. Appar.	Unity True
July 11	17h 10m	5	617	3	3	" "	10".54	10".50	.0".50	10".50	12".47	10".01	10".01
" 21	18h 30m	5	"	5	4	" "	10".91	10".88	.0".88	10".89	12".84	9".58	9".58
" 21	18h 48m	5	"	5	4	" "	11".08	11".05	.1".05	11".06	13".04	9".73	9".73
Aug. 7	16h 28m	3	"	7	7	" "	12".62	12".55	.2".55	12".55	14".55	9".52	9".52
" 24	15h 5m	5	"	4	4	" "	15".06	14".92	.4".92	14".92	16".80	9".59	9".59
" 28	18h 30m	5	"	3	3	" "	15".25	15".21	.5".21	15".22	16".97	9".36	9".36

Table V

Means

Prof. W. H. Pickering

Polar Diameter reduced to dist. unity (all corrections made except for the twilight band) 9".373

Equatorial Diameter " " " , " corrections made except for the twilight band) 9".580

Polar Diameter, ~~when the~~ polar cap ~~was~~ away from ~~the~~ limb 9".14

" " , ~~when the~~ polar cap ~~was~~ on the limb 9".49

Twilight arc 11°

	Polar		Equatorial	
	Irr. 0".10		Irr. 0".10	
July	9.976	9.98	9.691	9.69
Aug.	9.362	9.36	9.666	9.67
Sept.20,23,Oct.5	9.406	9.41	9.518	9.52
Oct.12,24,30.	9.379	9.38	9.452	9.45
Oct.15, 23	9.379	9.38	9.429	9.43
Oct.12, 24, 30.	9.379	9.38	9.452	9.45
Nov. 2, 21.	9.380	9.38	9.540	9.54

The same relative result would have come out in the table for the irradiation correction 0".15

Professor W.H.Pickering's measures, although relatively few in number, confirm so far as they go Mr. Douglass'. They are as follows:

Table IV + V

To determine the extent of the twilight thus disclosed by the measures; if we call E the phase angle; T the angle between the radius to the sunset point and the radius prolonged, to the point of the atmosphere last illuminated; *a*, the true equatorial radius reduced to distance unity, and *b*, the corresponding amount of its excess, we have the following equation to determine T.

neglecting temperature etc, $$\frac{1+\cos E + \sin E . \tan T}{1+\cos E} . 2a = 2a + b$$

T being determined from $\int_{\varphi_1}^{\varphi_2} \sec^2\varphi \, c . \varepsilon^{-a \sec\varphi} d\varphi$ where the limits φ_1 & φ_2 are such as to make the integral constantly equal to the amount of illumination necessary for visibility.

whence

$$\tan T = \frac{b . 1+\cos E}{2a . \sin E}$$

As b is the mean value of the excess of the equatorial diameter between certain times, we must, to determine T take the mean value of $\frac{1+\cos E_1}{\sin E_1}$ between the same times.

This mean value is in form,

$$\int_{E_1}^{E_2} \frac{1+\cos E_1}{\sin E_1} dE_1$$

But as dE is not constant if dt be taken constant, its value in terms of dt must be introduced from the elliptic motions. A short cut to the result may be taken, however, by deducing this mean value direct from the phase corrections.

By so doing we get an approximate value of T, which applied to the measures of October 15th to 23d - since even these were not made exactly at opposition - gives a new value for b and a closer approximation to 2a. By repeating this process we get values for both as near as we please to their true values until E = T. After this, when E < T, the equation ceases to be true, b diminishing not to nothing but to a minimum with irradiation 0".10 of about 0".025 at distance unity, the amount namely of the twilight seen in profile on the limb. This also is affected by irradiation, since the twilight itself would be swamped in the irradiation effect of the limb, and at the same time would cause an irradiation of its own. We can thus get an approximate value for the twilight arc which is evidently about 5°, the double of which or 10° is the angle which determines the duration of the twilight. On the Earth this angle is 18°.

The value for the twilight band deduced from these observations does not measure the full breadth of that band. It gives ~~rather~~ only a minimal value for it. For although Mars shows us a disk which is always more than half-full, in which aspect an illuminated fringe of atmosphere would be more perceptible to an observer placed without than to one placed within it, provided both were at the same distance off, In the case before us the observer is a great deal farther off. In consequence, what would be quite recognizable to one standing upon the planet's surface would be too faint to be seen by him at a distance of forty millions of miles. The detection, therefore, of any twilight on Mars hints that the extent of that twilight is greater than appears; how much greater we cannot at present say. ¶ A second possible cause affecting the extent of the twilight is the constitution of the Martian atmosphere. That atmosphere is practically cloudless; if also it be clearer than our own, the twilight would be relatively less, for the amount of twilight is, among other things, a question of the clearness of the air. It is the particles suspended in the air that reflect the light; the less particles therefore, the less the twilight.

Polar Flattening.

From these measures we deduce for the polar flattening

$\frac{a-b}{a}$ the value $\quad 1/190$

This value receives corroboration from two other sources. It is, in the first place, ~~also~~ happily accordant with what theory would lead us to expect. Tisserand has found that with the known rotation of Mars and supposing homogeneity, the planet's flattening should be 1/175 of the equatorial diameter, while if the strata varied in density after the manner of those of the Earth, the polar flattening should be 1/227 of it. Now assuming Mars to have been developed in general accordance withthe nebular hypothesis, his strata would neither be homogeneous on the one hand, nor on the other, would they vary in density from the surface to the centre so markedly as is the case with those of the Earth. For Mars, being a smaller body, the pressure due to gravity would be less, somewhere between that of the Earth and that of homogeneity (which is nothing), and the polar flattening should therefore be somewhere between 1/227 and 1/175 of the equatorial diameter. 1/190 is, therefore, not far from the value probable a priori. ¶ Secondly, since this paper was written Hermann Struve has found from the speed of rotation of the apsides of Deimos and Phobos the value 1/190 forthe polar flattening. It is interesting to have this result agree ths closely with theory, as it furnishes so much more reason for believing in the general evolution of our solar system.

Any value much less than 1/190 would require that Mars should havesolidfied an unconscionably long time ago. For as we see, the Earth still rotates very nearly as she did when she solidified, and although in the case of Mars this must have happened relatively long before, yet on the other hand, Mars lacks relatively the tools for the purpose, possessing neither suitable satellites on the one side, nor sufficient seas on the other.

For the absolute dimensions of the diameters we may best take the correction for irradiation at 0".125 and thus obtain;

Mic.Measures of Diameters.

Oct.20th.

12h	Polar						
	uncor.						
to		cor.refr.	co.irr.	cor.phase		at dist.unity	
	22".54	22".54	22".44	22".35	22".35	9".758	
12h 30m	Equatorial						
	22".67	22".68	22".58	22".48	22".48	9".815	

Polar Flattening 1/173

Had the last weight been taken equal to the others the result would have been 1-203 for the polar flattening.

polar diameter 9".32
Equatorial diameter 9".37
Ellipticity $\frac{1}{190}$
Twilight arc (minimal value) about 10°

Percival Lowell

Lowell Observatory
Dec. 1895.