G431

T. Hill

SYMMETRY IN SPACE.

THE universe, actual, possible and impossible, is composed of four elements, spirit, matter, space, and time, which are by no alchemy transmutable into each other. Many alchemists continue, even in this closing half of the nineteenth century, to make the attempt, and some even flatter themselves that they are succeeding; but the sturdy reply of human consciousness is, that the four elements are diverse and not transmutable; or, if any transmutation is possible, it must be confined to this, that matter may, in some manner, be an effect of spirit. But to us, finite spirits, nothing more is granted than the re-arrangement, the partial control of matter, not its creation. Matter, as we know it, is distinguished by its being the recipient and dispenser of force; which force, so far as we know it, is from spirit alone. This obedience of matter to spirit gives justification to our suspicion that it is the creation of spirit.

Space and time are without parts, and are indivisible except by a mental act. This division is suggested to us by manifest motion in matter. Force shows itself in matter by moving it; that motion calls our attention to the space and time, within which the motion is taking place; and we divide mentally this space and time, first from the remainder of the boundless contiguities, secondly into smaller parts. Thus geometry and algebra are generated, the sciences which deal respectively with space and time, those pure entities, the relation of which to the Infinite Spirit we cannot comprehend, but which we become familiar with in the finite portion embraced in our experience, in the universe and its history.

In geometry, the mind imposes upon indivisible space arbitrary boundaries of division, according to arbitrarily selected laws or conditions. These boundaries are of three kinds, surfaces, lines and points. The point is a zero of magnitude in space, but nevertheless is not nothing; which is nowhere, while the point is somewhere. This contradiction in terms, that a point should have no extension, and yet have a position, is one of those instances, in which geometry abounds, in which the mind is compelled, by the necessity of direct vision, to admit each of two truths, which are to logic mutual contradictories. The mathematician modifies the law of non-contradiction by confining it to propositions concerning finite quantities.

A lower form of a zero of magnitude in space is the line, which is extended, at each point, only in two opposite directions; and the lowest form is the surface; for which there can, at each point, be drawn a line, such that the surface extends, in every direction, only perpendicular to that line. Geometers define these lower forms of zeroes, or boundaries in space, by the further self-contradiction of imagining the movement of a point; a double contradiction, since space is itself incapable of motion, much more a zero of magnitude.

A geometrical line is defined as the path of a point, moving according to certain conditions, which always limit its motion, in each of its positions, to one of two opposite directions. Or, it may be defined as a continuous series of all the points which fulfill certain conditions, among which must be the condition that each point is contiguous only to two others, one on the opposite side to the other. So also a surface may be defined as the space in which a point moves, when, in each position which it assumes, a straight line may be drawn through it, and its motion be permitted, in any direction at right angles to that line, and in no other. Or, the surface may be defined as a series of points, through any one of which a straight line may be drawn, such that all the contiguous points lie in a direction at right angles to that line. To either of these definitions of a surface, we must add, in order to make a geometrical surface, some other conditions which the points must fulfill.

When the geometer has selected these conditions and would investigate the form which the points, so conditioned, would enclose, he is not contented with the mere act of reason; he endeavors to bring imagination to his aid; to make a sensible image of the form. If he has been blind from his birth, he imagines his fingers feeling out the form; otherwise he embodies it visibly, as in a drawing, or in a model. If he would convey a knowledge of it to others, he calls matter to his aid, and forces atoms of chalk, black lead, wood or thread, to fulfill approximately the conditions which his geometric law imposes upon the series of points. This drawing, or model, is an expression of his idea, an enunciation of his law. A geometrical figure, whether upon the blackboard, or the printed page, or in a block of wood, or a set of stretched threads, is incontrovertible evidence that a geometer has been expressing, by this means, a geometrical thought.

The laws which please the geometer most highly, are those which give us symmetrical figures, figures in which part answers to part; either on opposite sides of one line or one surface, or about more than one line or surface. This taste is not peculiar to the geometer; symmetry pleases the most savage, as it does the civilized man; and men whose whole ability lies in other directions, as well as the mathematician. A striking proof of the universality of this taste was shown in the sudden and universal popularity attained by the kaleidoscope. In a few years that toy of Brewster found its way to every parlor, and the heart of every child, ay, and every man in Christendom. But its sole magic consists in the symmetry which it imparts to a few fragments of irregular form. But that magic is sufficient to enchant all who come within its sway. We have never found any one uninterested in an extempore kaleidoscope, made by throwing open the piano, and placing brightly colored articles at one end of the folding lid.

All regularity of form is as truly an expression of thought, as a geometrical diagram can be. The particles of matter take the form in obedience to a force which is acting according to an intellectual law, imposing conditions on its exercise. It does not alter the reality of this ultimate dependence of symmetry upon thought, simply to introduce a chain of secondary causes, between the original thinking and the final expression of the thought.

Many of the geometer's *a priori* laws were, indeed, first suggested by the forms of nature. Natural symmetry leads us to investigate, first, the mathematical law which it embodies; then, the mechanical law which embodies it. Thus all the benefits which have come to our race from the pursuit, and discovery and use of the keys to physical science, have been bestowed upon us through these suggestions of geometrical thoughts in the outward creation.

But in the pursuit of mathematical knowledge, men began, at an early age, to invent and investigate *a priori* laws, laws of which they had not received any suggestion from nature. And the intellectual origin of the forms of nature was made still more manifest when these *a priori* laws, of man's invention, were, in many cases, afterwards discovered to have been truly embodied in the universe from the beginning; as, for example, Plato's conic sections in the forms and orbits of the heavenly bodies, and Euclid's division in extreme and mean ratio.

The division in the extreme and mean ratio was invented by the early geometers, without any known suggestion. It is evident that this division might be illustrated in a great variety of ways. A whole must be divided into two parts, such that the first shall bear the same relation to the second that the second does to the whole. No matter what the whole is, a division of it approximately in this manner would be an expression of the idea of extreme and mean ratio. If the whole were a quantity (distance, angle, surface, volume, value, time, velocity, &c.), and the relation were that of magnitude, the whole would be to the smaller part, as unity is to half the difference between three and the square root of five. If, on the other hand, the whole were a work of art of any kind, or a system of thought, the relation would not be one of mere magnitude; and the division would be a work of more ingenuity. But, whatever the whole, or the relation, the proper division would be an expression of the idea.

Now we have, in nature, at least three embodiments of the law of extreme and mean ratio, two of which are very striking. The botanists find that two successive leaves, counting upward on the stem, stand at an angle with each other, that is either one-half, one-third, two-fifths, three-eighths of the whole circle; or some higher approximation to this peculiar proportion. The seed vessels and buds on a spike of broad-leaved plantain afford one of the most instructive examples. They are usually set on a high approximation, so that the order is not apparent. Taking a piece of the spike, an inch or so in length, between your hands, and gently twisting reduce it to three; while a slight twist in the opposite direction brings out five rows, which a harder twist reduces to two.

The efficient cause of this arrangement we do not know. It has been ingeniously suggested that it might be produced by a simple law of the genesis of cells. Let us suppose that each cell emits a new cell at regularly recurring intervals of time, and that the new cell begins to generate cells at the expiration of two intervals after its birth. A cell developing on a plane, under this law, would produce its cells in the phyllotactic order of the leaves, in the terminal rosette of a plant. But it is difficult to see how this hypothesis can be made to include and explain the whole phenomena of the arrangement.

The final causes, although the devout mind always recognizes the impossibility of man's attaining a certainty concerning all the final causes of a phenomenon, are more obvious. It has been shown that this division of the circle insures in the only perfect way to each leaf its chance at zenith light, its best chance at air; in short, that this phyllotactic law distributes the leaves most evenly about the stem.

In the solar system, if we divide the periodic time of each planet by that of the planet next farthest from the sun, we shall have, beginning with the quotient of Uranus' year divided by that of Neptune and ending with the quotient of Mercury's year divided by that of Venus, a series of fractions agreeing very closely with the approximations of the phyllotactic law. The problem was similar. The planets would not have remained in proper subjection to the sun had they been allowed to group themselves too frequently in one rebellious line, hanging upon the golden chain of his attraction, dragging him and themselves from their proper orbits. They must be kept evenly distributed about the sun; and since they are moving, the times of their revolution, their angular velocities must be divided by the same law as that which divides the stationary angles of the leaves.

We have then in the plants a geometrical or angular illustration, and in the plants an algebraical or temporal illustration, of the mathematical idea of extreme and mean ratio. The inference seems irresistible, — these two illustrations, which cannot be imagined as having any causal or genetic connection, owe their intellectual relation to having sprung from One Mind.

This is a striking illustration, but the same inference may be drawn from every form in nature, — planet, crystal, plant and animals. All natural forms conform more or less closely to geometrical ideals; sufficiently near to suggest their ideas to men fitted to receive the suggestion; sufficiently near to show that the whole of nature, may, in one sense, be regarded as a series of drawings and models, by which to teach the mathematics to students in the school of life.

The final causes may never, however, be considered as wholly known. The perfection of the Divine workmanship is shown in the adaptation of each object in nature to a great variety of ends. The geometrical laws, on which the world is built, are adapted to all the wants and all the needs of every creature. Our human needs are innumerably various, and nature finds means to satisfy them all. Our intellect craves symmetry, and through symmetry is first led to the perception of geometric law. But we love the symmetry before we perceive the law. The sense of beauty is satisfied, even in externals, most perfectly, and fills us with most pleasure, in things that the understanding fails to analyze and define. Much has been written concerning an analysis of the beauty of outline; one great painter thinking it consists in flexure, others assigning it to a spiral, or a helix, or an ellipse; while Darwin refers it to early association, while yet a suckling, with the form of the mother's breast. I venture with diffidence to give my own opinion, that the perception of beauty in outline is the unconscious perception of geometric law, — just as the perception of harmony has been demonstrated to be the unconscious perception of arithmetical ratios in time, or algebraic law. The beauty of outline, I would say of external form, independently of expression, is in proportion to the simplicity of the geometric law, and to the variety of the outline which embodies it. Nor is it essential to the highest enjoyment of beauty that the conformity to geometric ideals should be perfect, any more than it is essential to the highest music to have the harmony perfect. On the con trary, the higher degrees of beauty are apt to be found in forms that suggest, rather than embody, the ideal; and especially in figures potentially, but not actually, symmetrical. The monotony, which might result from unbroken regularity of form, is avoided, and a new grace is given, for example, to the higher animals, by their temporary disguise of symmetry, in their varied positions and movements. In the sea shells, the same end is attained by the spiral form, which so many of them take; in which there is not an actual symmetry, but only a law of symmetry, the perfect development of which would require an infinite number of convolutions.

In the forms of vegetative life, there is the widest departure from actual symmetry, and yet a constant suggestion of its laws. The phyllotactic law secures to the tree a general regularity, and equal growth upon every side; and yet, by complication of detail, combined with occasional failure or destruction of buds, secures an endless variety of graceful forms, in each species. May we not then name beauty as another final cause, another end secured by the adoption of the division in extreme and mean ratio? The approximations are beautiful to us, and the pleasure given to us was foreseen when the law was adopted. May it not also have been felt; and may not the forms of flowers be but approximations toward the expression of an infinite beauty, hidden, from all finite sense, in the incommensurable ratio of that surd? That the external symmetry of animals may have beauty as its final cause, is rendered probable from the lack of symmetry in the viscera, which are hidden from sight.

Whatever be our speculations upon such points, this at least is manifest, that the sense and the presence of beauty are kindly adapted to each other in the world. Even shapeless matter declares its Creator's power; the perfect symmetry of crystalline forms, the potential symmetry of all the organic worlds show forth His wisdom and His love.

THOMAS HILL.