Mathematics -

October 21st. 1878 -

Professor Brennan -

Algebra, Geometry + Trigonometry.

Professor Read

Physics.

Trigonometry—.

$\cos 3x = 4\cos^3 x - 3\cos x.$ (2) $\cos 3x = \cos(2x+x) = 2\cos^2 x \cos x - \cos x -$

$2\sin x \cos x \sin x = 2\cos^3 x - \cos x - 2\sin^2 x \cos x$

$2\sin^2 x = 1 - \cos 2x \quad 1 - 2\sin^2 x = 2\cos^2 x - 1, \; 2\sin^2 x = 2\cos^2 x - 2$

$\cos 2x = 2\cos^2 x - 1. \quad 2\cos^2 x = \cos 2x + 1$

$2\cos^3 x - \cos x - (2\cos^2 x - 2)\cos x = 4\cos^3 x - 3\cos x.$

$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}.$ (2) $\tan 3x = \frac{3\sin x - 4\sin^3 x}{4\cos^3 x - 3\cos x} =$

or $\tan(2x + \cancel{3x}\,x) = \dfrac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{\frac{1}{1} - \frac{2\tan x}{1-\tan^2 x}\frac{\tan x}{1}} = \left(\frac{2\tan x}{1-\tan^2 x} + \frac{\tan x}{1}\right)\left(\frac{1}{1} - \frac{1-\tan^2 x}{2\tan x}\frac{1}{\tan x}\right)$

$= \frac{2\tan x + \tan x - \tan^3 x}{1 - \tan^2 x} \times \frac{2\tan^2 x - 1\tan x^2}{2\tan^2 x} =$

$\frac{(3\tan x - \tan^3 x)(3\tan^2 x - 1)}{(2\tan^2 x)(1 - \tan^2 x)} = \frac{9\tan^3 x - 3\tan^5 x - 3\tan x + \tan^3 x}{2\tan^2 x - 2\tan^4 x}$

$\frac{9\tan^2 x - 3\tan^4 x - 1 + \tan x}{2\tan x - 2\tan^3 x}.$

$\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} = \dfrac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan^2 x}{1-\tan^2 x}} = \dfrac{\frac{2\tan x + \tan x - \tan^3 x}{1-\tan^2 x}}{\frac{1 - \tan^2 x - 2\tan^2 x}{1-\tan^2 x}}$

$= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\cot 3x = \frac{\cot^3 x - 3\cot x}{3\cot^2 x - 1}.$ $\cot 3x = \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}.$

$= \dfrac{\frac{\cot^2 x - 1}{2\cot x}\cot x - 1}{\frac{\cot^2 x - 1}{2\cot x} + \cot x} = \dfrac{\frac{\cot^3 x - \cot x - 2\cot x}{2\cot x}}{\frac{\cot^2 x - 1 + 2\cot^2 x}{2\cot x}} = \frac{\cot^3 x - 3\cot x}{3\cot^2 x - 1}.$

$\sin 4x = 4(\sin x - 2\sin^3 x)\cos x.$ (2) $\sin 4x = \sin(2x + 2x) =$

$(2\sin x \cos x)(\cos^2 x - \sin^2 x) + (\cos^2 x - \sin^2 x)(2\sin x \cos x) =$

$2(2\sin x \cos x)(\cos^2 x - \sin^2 x) = 2(2\sin x \cos^3 x - 2\sin^3 x \cos x) =$

$\cancel{2}(\sin x \cos^2 x - 2\sin^3)\cos x.$

$= (2\sin x \cos x)(1 - 2\sin^2 x) + (1 - 2\sin^2 x)(2\sin x \cos x) =$

$2(2\sin x \cos x - 4\sin^3 x \cos x) = 4(\sin x - 2\sin^3 x)\cos x$

Differential Coefficients.

1. $y = x^5 - 3x^3 + x - 10$. $dy = 5x^4dx - 9x^2dx + dx$. $\frac{dy}{dx} = 5x^4 - 9x^2 + 1$. $\frac{d^2y}{dx^2} = 20x^3 - 18x$.
$\frac{d^3y}{dx^2} = (60x^2 - 18)\,dx$ whence $\frac{d^3y}{dx^3} = 60x^2 - 18$. $\frac{d^4y}{dx^4} = 120x$.

2. $y = 5x^2 - 3x$. $dy = 10x\,dx - 3dx$. when $x = 1$, y increases 7 times x, $x = 2$, $y = 17x$ —

3. $y = x^5 + 2x^4 - x + 10$. $dy = 5x^4dx + 8x^3dx - dx$. $\frac{dy}{dx} = 5x^4 + 8x^2 - 1$.
$\frac{d^2y}{dx} = 20x^3dx + 16x\,dx$. $\frac{d^2y}{dx^2} = 20x^3 + 16x$. $\frac{d^3y}{dx^2} = 60x^2dx + 16dx$. $\frac{d^3y}{dx^3} = 60x^2 + 16$.
$x = 1$. $\frac{d^3y}{dx^3} = 76$. $\frac{d^2y}{dx^2} = 31$ when $x = \frac{1}{2}$. $\frac{d^3y}{dx^3} = 22\tfrac{2}{3}$ when $x = \frac{1}{3}$.

4. $y = (a+x)^m$ $dy = m(a+x)^{m-1}dx$. $\frac{dy}{dx} = m(a+x)^{m-1}$. $\frac{d^2y}{dx^2} = m(m-1)(a+x)^{(m-2)}$.
$\frac{d^3y}{dx^3} = m(m-1)(m-2)(a+x)^{m-3}$. $\frac{d^4y}{dx^4} = m(m-1)(m-2)(m-3)(a+x)^{m-4}$.
$\frac{d^5y}{dx^5} = m(m-1)(m-2)(m-3)(m-4)(a+x)^{(m-5)}$

5. $y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6 +$ etc.

6. Let $x^4 - 2x^3 + 5x^2 + x - 2 = f(x)$. Let $f'(x)$ be first diff. coefficient.

$f(x) = x^4 - 2x^3 + 5x^2 + x - 12$	$f(x) = 2x^5 + 3x^4 - 5x^2 + 10$
$f'(x) = 4x^3 - 6x^2 + 10x + 1$	$f'(x) = 10x^4 + 12x^3 - 10x$
$f''(x) = 12x^2 - 12x + 10$	$f''(x) = 40x^3 + 36x^2 - 10$
$f'''(x) = 24x - 12$	$f''(x) = 120x^2 + 72x$
$f''''(x) = 24$	$f'''(x) = 240x + 72$
$f'''''(x) = 0$.	$f''''(x) = 240$

6. $y = A + Bx + Cx^2 + Dx^3 + Ex^4 + Fx^5 + Gx^6$.
$dy = B\,dx + 2Cx\,dx + 3Dx^2dx + 4Ex^3dx + 5Fx^4dx + 6Gx^5dx$
$\frac{dy}{dx} = B + 2Cx + 3Dx^2 + 4Ex^3 + 5Fx^4 + 6Gx^5$.
$\frac{d^2y}{dx^2} = 2C + 3\times 2Dx + 3\times 4Ex^2 + 5\times 4Fx^3 + 6\times 5Gx^4$.
$\frac{d^3y}{dx^3} = 3\times 2D + 3\times 4\times 2Ex + 5\times 4\times 3Fx^2 + 6\times 5\times 4Gx^3$
$\frac{d^4y}{dx^4} = 3\times 4\times 2E + 5\times 4\times 3\times 2Fx + 6\times 5\times 4\times 3Gx^2$
$\frac{d^5y}{dx^5} = 5\times 4\times 3\times 2F + 6\times 5\times 4\times 3\times 2Gx$.

6.

$f(x) = 2x^8 - 3x^{10} + x^{12}$.	$f^{iv}(x) = 3360x^4 - 16120x^6 + 11880x^8$
$f'(x) = 16x^7 - 30x^9 + 12x^{11}$	$f^5(x) = 13440x^3 - 96720x^5 + 95040x^7$
$f''(x) = 112x^6 - 270x^8 + 132x^{10}$	$f^6(x) = 40320x^2 - 483600x^4 + 665280x^6$
$f'''(x) = 672x^5 - 2160x^7 + 1320x^9$	$f^7(x) = 80640x + 1934400x^3 + 3991680x^5$