1990 average district size:

$$\frac{249,022,783}{435} = 572,466$$

$$= 1.74683 \text{ representatives per million.}$$

Montana with 1.244 per million is 29 % underrepresented
Wyoming with 2.193 per million is 26 % overrepresented
Rhode Island with 1.988 per million is 14 % overrepresented
...

Nine states differ from the ideal by more than ten percent.

Inequalities among states are inevitable because of the rounding problem.

If two states are nearly equal in population and one gets two seats and the other one seat, then one is almost twice as well represented as the other.

The goal of apportionment is to *minimize the disparity* in representation among the states, *to come as close to the ideal* of one person one vote as possible.

This problem has puzzled politicians and statisticians for two hundred years: Thomas Jefferson, Alexander Hamilton, Daniel Webster, J. Q. Adams, John von Neumann . . .

Simple rounding doesn't work.

State	Population	Quota of Seats	Hamilton
A	7,270,000	14.24	14
B	1,230,000	2.41	3
C	2,220,000	4.35	4
Total	10,720,000	21.00	21

Hamilton's Method (Greatest remainders) 1792.

Give each state the whole number of seats in its quota. Award any additional seats to the states with highest remainders.

Washington vetoed this seemingly natural method in favor of a rival proposal by his Secretary of State, Thomas Jefferson. (The first Presidential veto ever cast.)

State	Population	Quota	Quotient d = 484,000	Jefferson
A	7,270,000	14.24	15.02	15
B	1,230,000	2.41	2.54	2
C	2,220,000	4.35	4.59	4
Total	10,720,000	21.00	22.15	21

Jefferson's Method (Greatest Divisors) 1792.

Choose a divisor d (a target district size). Divide d into each state's population to obtain a quotient, and drop the fractional remainder. Adjust d so that the total is the required number of seats.

Webster's Method (Major Fractions) 1832.

Choose a divisor d (a target district size). Divide d into each state's population to obtain a quotient. Round fractions above .5 up, and fractions below .5 down. Adjust d so that the total is the required number of seats.

State	Population	Quota	Quotient d = 500,000	Webster
A	7,270,000	14.24	14.54	15
B	1,230,000	2.41	2.46	2
C	2,220,000	4.35	4.44	4
Total	10,720,000	21.00	21.44	21

Makes simple rounding work by adjusting the the target district size.

The Alabama Paradox.

1881: The Chief Clerk of the Census Office computed the apportionment by Hamilton's Method for all House sizes between 275 and 300.

"While making these calculations I met with the so-called Alabama paradox where Alabama was allotted 8 representatives out of a total of 299, receiving but 7 when the total became 300.. . this is to me conclusive proof that the process employed in obtaining it is defective."

The "Mississippi Paradox" of 1990.

State	Quota at 435	Ham.	Quota at 434	Ham.
N.J.	13.5355	14	13.5044	13
Mass.	10.5317	11	10.5075	10
N.Y.	31.5207	31	31.4482	31
Miss.	4.5181	4	4.5077	5
	Cut-off = .5200		Cut-off = .5076	

In a smaller House, Mississippi gains a seat!

The Method of Joseph Hill (alias "Equal Proportions") 1911

Allocate seats so that no transfer of a seat between any two states reduces the percentage difference in per capita representation between them.

State	Population	Quota	Seats	No. of reps per million	Abs. Diff.
Ok.	3,157,604	5.5158	5	1.5835	
Mass.	6,029,051	10.5317	11	1.8245	.2410

Mass. is 15.2 % better represented than Ok.

Ok.	3,157,604	5.5158	6	1.9002	
Mass.	6,029,051	10.5317	10	1.6586	.2415

Ok is 14.6% better represented than Mass.

Bias

A method is unbiased if every state gets its fair share of seats over the long run.

Hill's method is systematically biased in favor of small states.

On average, Hill's method gives the smallest states about 3% more representation per capita than the largest states, even after deleting the very small states that must receive one seat in any case.

Webster's method is unbiased.

Effect of a Switch from Hill to Webster 1941-1990

The largest states would have received 5 more seats in total.

The smallest states (excluding those with quota less than 1) would have received 4 fewer seats in total.

Since 1941, the large states have "lost" 5 seats (Mass 2!
small states have gained 4 seats

because of using Hill's method instead of Webster

	Losers	Gainers
1942	Michigan	Arkansas
1950	California	Kansas
1960	Massachusetts	N. Hampshire
1970	Pennsylvania	Montana
	Connecticut	South Dakota
1980	Indiana	New Mexico
1990	Massachusetts	Oklahoma